ANALYSIS OF THE RADAR RETURN SIGNAL
FROM ROTATING AIRCRAFT BLADES

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ABSTRACT

It is well known that moving objects tend to cause a Doppler frequency shift of the radar return signal. However, rotating objects tend to cause a modulation of the return signal. Almost all aircraft have rotating parts, viz. rotor blades, propeller blades, or jet engine compressor and turbine blades, therefore almost all aircraft can cause this modulation.

This thesis presents a detailed mathematical analysis of the radar return signal from rotating aircraft blades. The thesis is concerned with the analysis of the return signal and frequency spectrum of rotor and propeller blades when there is both phase and amplitude modulation of the return signal.

A detailed survey of the physical configurations, physical parameters and radar features of rotor and propeller aircraft ranging in size from light to heavy aircraft is also presented. This survey includes those features which relate to the aircraft in general, and those which relate to the rotating aircraft blades in particular.

The extraction of aircraft features from the return signal is also discussed. The main emphasis is on the extraction of features which relate to the rotor or propeller blades of the aircraft, but the extraction of other features from the return signal is also considered.

Some of the practical considerations which are associated with the analysis of the return signal from rotating aircraft blades are also discussed. Four main subjects are considered: the radar parameters, window functions, relative acceleration and aircraft classification.
ACKNOWLEDGEMENTS

I would like to thank my first supervisor, Dr. Bernard Mulgrew, for his guidance and inspiration, my second supervisor, Prof. Peter M. Grant, for his advice and enthusiasm, my industrial supervisor, Mr. John F. Roulston, for his encouragement and sponsorship, and many students and members of staff in the Signal Processing Group and the Department of Electrical Engineering at the University of Edinburgh, for many useful comments and discussions. I would also like to thank the Science and Engineering Research Council (SERC) and GEC Ferranti Defence Systems Limited (GFDSL) for funding this work. Finally, I would like to thank my wife, Lesley, for her patience and support.
DEDICATION

To Lesley
## ABBREVIATIONS

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<tr>
<td>AM</td>
<td>amplitude modulation</td>
</tr>
<tr>
<td>ASCC</td>
<td>Air Standards Co-ordinating Committee</td>
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<td>ATCRBS</td>
<td>Air Traffic Control Radar Beacon System</td>
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<td>CFAR</td>
<td>continuous false-alarm rate</td>
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<td>COHO</td>
<td>coherent oscillator</td>
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<tr>
<td>DFT</td>
<td>discrete Fourier transform</td>
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<td>FFT</td>
<td>fast Fourier transform</td>
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<td>FM</td>
<td>frequency modulation</td>
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<td>IF</td>
<td>intermediate frequency</td>
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<td>IFF</td>
<td>Identification Friend or Foe</td>
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<td>IFFT</td>
<td>inverse fast Fourier transform</td>
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<td>JEM</td>
<td>jet engine modulation</td>
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<td>LF</td>
<td>local frequency</td>
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<tr>
<td>NACA</td>
<td>National Advisory Committee for Aeronautics</td>
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<tr>
<td>NASA</td>
<td>National Aeronautics and Space Administration</td>
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<tr>
<td>NATO</td>
<td>North Atlantic Treaty Organisation</td>
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<tr>
<td>NOE</td>
<td>nap-of-the-earth</td>
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<tr>
<td>PDF</td>
<td>probability density function</td>
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<tr>
<td>PFLP-GC</td>
<td>Popular Front for the Liberation of Palestine - General Command</td>
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<td>PLF</td>
<td>Palestinian Liberation Front</td>
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<tr>
<td>PRF</td>
<td>pulse repetition frequency</td>
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<td>PRI</td>
<td>pulse repetition interval</td>
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<tr>
<td>Abbreviation</td>
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<tr>
<td>PSNR</td>
<td>peak signal-to-noise ratio</td>
</tr>
<tr>
<td>RF</td>
<td>radio frequency</td>
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<tr>
<td>SNR</td>
<td>signal-to-noise ratio</td>
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<td>SSR</td>
<td>Secondary Surveillance Radar</td>
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<td>STALO</td>
<td>stable local oscillator</td>
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SYMBOLS

\( a \) relative acceleration of the target

\( a_{err} \) error relative acceleration of the target

\( a_{est} \) estimated relative acceleration of the target

\( A_r \) magnitude of the return signal

\( A_t \) magnitude of the transmitted signal

\( B_a \) Cyclic Doppler frequency spread of the target spectrum, due to relative acceleration

\( B_1 \) bandwidth of the notched sidebands

\( B_2 \) bandwidth of the significant sidebands

\( f \) cyclic frequency

\( f(t) \) input signal

\( F(f) \) Fourier transform of \( f(t) \)

\( f_c \) cyclic frequency of the transmitted signal

\( f_d \) cyclic Doppler frequency shift of the target

\( f_{da} \) Cyclic Doppler frequency shift of the target spectrum, due to relative acceleration

\( f_r \) cyclic frequency of rotation of the rotor

\( g(t) \) output signal

\( G(f) \) Fourier transform of \( g(t) \)

\( G_{ps}(\tau_q) \) power cepstrum of \( g(t) \)

\( L_1 \) distance of the blade roots from the centre of rotation

\( L_2 \) distance of the blade tips from the centre of rotation
\(N\) number of blades
\(N_1\) number of notched sidebands
\(N_2\) number of significant sidebands
\(N_3\) number of significant Bessel functions
\(R\) range of the target
\(t\) time
\(T_o\) observation interval
\(v\) relative velocity of the target
\(v_b\) rotational velocity of the blade tips
\(v_c\) relative velocity of the blade tips
\(v_r(t)\) return signal
\(v_t(t)\) transmitted signal
\(\alpha_m\) modulation index
\(\Delta f\) blade-passing frequency
\(\Delta t\) blade-passing interval
\(\lambda\) radar wavelength
\(\theta\) angle between the plane of rotation and the line of sight from the radar to the centre of rotation, where \(\theta\) is positive in the direction of thrust, and negative in the direction of drag
\(\tau_q\) quefrency
\(\phi_p\) blade pitch angle
\(\omega\) radian frequency
\(\omega_c\) radian frequency of the transmitted signal
\(\omega_d\) radian Doppler frequency shift of the target
\(\omega_r\) radian frequency of rotation of the rotor
CHAPTER ONE

INTRODUCTION

1.1 Introduction

It is well known that moving objects tend to cause a Doppler frequency shift of the radar return signal. However, rotating objects tend to cause a modulation of the return signal. Almost all aircraft have rotating parts, viz. rotor blades, propeller blades, or jet engine compressor and turbine blades, therefore almost all airborne targets can cause this modulation. The modulation is a form of target noise, and depends on a number of target parameters. More specifically, the modulation is a form of phase and amplitude modulation, and causes modulation sidebands about the centre frequency of the target. In some cases, it is possible to classify different types of target by analysing the modulation in the return signal [1].

A useful understanding of complex physical phenomena is usually found to lag considerably behind the first observation of their effects. This is the case with the return signal from rotating aircraft blades, since it has long been known that the rotation of the blades tends to modulate the return signal. However, it is only recently that any detailed mathematical analysis of the phenomenon has begun to appear in the literature. This is evidenced by the fact that most authors deal with the return signal from rotating aircraft blades on an “as is” basis, without a detailed understanding of the underlying theory.

Many different radar classification techniques have been investigated, and much work has been published in this area, e.g. [2]-[21]. In each case, the classification is achieved by analysing how the transmitted radar signal is affected by the target. However, very little work has been published in the open literature on the
subject area of this thesis, and what little work has been published has tended mainly to describe the basic theory involved, not present any detailed mathematical analysis.

The classic aircraft classification technique is "Identification, Friend or Foe" (IFF), the Second World War system on which the Air Traffic Control Radar Beacon System (ATCRBS), or Secondary Surveillance Radar (SSR), is based [22]. An IFF system consists of an interrogator, which transmits a coded radar interrogation signal to the target, and a transponder, which transmits a coded radar reply signal back to the interrogator. The reply signal can provide various types of target information, e.g. altitude, identity, position, etc. [23].

The classification of aircraft, based on the return signal from the rotating aircraft blades, is sometimes called "engine classification" or "jet engine modulation (JEM) classification". However, a more accurate term for this type of classification would be "blade classification", since it is the blade parameters of the aircraft which determine the aircraft features, not the aircraft engines, per se. For example, in theory, two rotor aircraft which have different blade parameters but identical engines can be classified from one another. However, two rotor aircraft which have identical blade parameters but different engines cannot. The exception to this rule is the case of jet aircraft, in which the blades are part of the engine.

All of the work in the thesis is based on simulated data. This is mainly because of the difficulty which was encountered in obtaining real data, due to military classification, but also partly because of the difficulty which may have been encountered in publishing the thesis, also due to military classification, if real data had been used.

All of the work in the thesis is concerned with the analysis of the return signal from rotating rotor and propeller blades, not with that from rotating compressor and turbine blades. This is because at most aspect angles, the blades on jet aircraft are shielded by the engine cowling, to some extent, and in some cases there is no
line of sight from the radar to the blades, i.e. the return signal from the blades is received only after multiple reflections, and this will result in a more complicated return signal. Moreover, with jet aircraft, the transmitted signal may be reflected from more than one compressor or turbine stage, i.e. the signal may propagate further into the engine than just the first set of compressor or turbine blades. This will cause interference between the return signals from the different stages, which will again result in a more complicated return signal. For these reasons, it is felt that the analysis of the return signal from rotating compressor and turbine blades would best be done together with the analysis of real data. Despite this, most of the results in the thesis can also be extended to also apply to the case of rotating compressor and turbine blades.

Although the thesis is of interest mainly with respect to aircraft feature extraction and aircraft classification, the presence of sidebands in the frequency spectrum is also important with respect to cell-averaging continuous false-alarm rate (CFAR) detectors. With cell-averaging CFAR detectors, the false-alarm rate of the radar, i.e. the rate at which targets are incorrectly reported as being present, when none are actually present, is reduced by performing cell-averaging of the signals in adjacent range-Doppler cells [24]. However, the presence of sidebands in the frequency spectrum may affect the performance of the cell-averaging CFAR detector, since the presence of sidebands in adjacent Doppler bins may affect the cell-averaging, and therefore may affect the false-alarm rate. (This is analogous to the way in which the performance of the cell-averaging CFAR detector may be affected by the presence of time sidelobes in the return signal, due to pulse compression. In this case, the presence of time sidelobes in adjacent range gates may affect the cell-averaging, and therefore may affect the false-alarm rate.)

The presence of sidebands in the frequency spectrum are also important with respect to target tracking. Target acquisition and tracking is sometimes achieved using the Doppler frequency shift of the airframe of the target [25]. However, the presence of sidebands in the frequency spectrum may affect the performance of the
tracker, since the tracker may lock onto and track one or more of the sidebands [26]. (Again, this is analogous to the way in which the performance of the tracker may be affected by the presence of time sidelobes in the return signal, due to pulse compression.)

1.2 Pulse Doppler Radar

Since we will be analysing both the return signal and the frequency spectrum of the rotating aircraft blades, we will assume that the radar is a pulse Doppler radar. This section presents a brief outline of the theory of pulse Doppler radar.

A pulse Doppler radar can be defined as a radar which transmits its electromagnetic energy as a series of pulses, in order to obtain range information, and which uses the Doppler effect, in order to obtain velocity information. Pulse Doppler radars are used mainly in airborne applications which require the detection of moving targets in ground clutter.

Figure 1.1 shows a block diagram of a typical pulse Doppler radar. The coherent oscillator (COHO) is a stable oscillator, and provides the coherent intermediate frequency (IF) reference signal. The stable local oscillator (STALO) is also a stable oscillator, and provides the local frequency (LF) signal. The COHO signal is mixed with the STALO signal to produce the radio frequency (RF) signal, which is then modulated by the pulse modulator. This signal is amplified by the RF power amplifier, and after passing through the duplexer, is then transmitted by the antenna. The return signal is received by the antenna, and after passing through the duplexer, is then amplified by the RF amplifier. This signal is mixed with the STALO signal to produce the IF signal, which is then amplified by the IF amplifier. This signal is applied to the phase detector in order to produce real and imaginary components, which are then passed on for further signal processing. Finally, the target information is shown on the display.

The radar equation gives the power of the return signal from a target, in terms
Figure 1.1
Block diagram of a typical pulse Doppler radar.
of the radar, target and environmental parameters. The radar equation is given by

\[ P_r = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^2 LR^4} \]  

(1.1)

where \( G \) = power gain of the antenna,
\( L \) = loss factor of the radar,
\( P_r \) = power of the return signal,
\( P_t \) = power of the transmitted signal,
\( R \) = range of the target,
\( \lambda \) = radar wavelength,
\( \sigma \) = radar cross section of the target.

In this thesis we will be more interested in the magnitude of the return signal than in the power, in which case Equation (1.1) can be modified to give

\[ A_r = \left( \frac{P_t G^2 R_c \lambda^2 \sigma}{(4\pi)^2 LR^4} \right)^{\frac{1}{2}} \]  

(1.2)

where \( A_r \) = magnitude of the return signal,
\( R_c \) = resistance of the receiver.

The range of the target is given by

\[ R = \frac{c \tau}{2} \]  

(1.3)

where \( c \) = speed of light,
\( \tau \) = time delay of the return signal.

The velocity of the target is given by

\[ v = -\frac{f_d \lambda}{2} \]  

(1.4)

where \( f_d \) = cyclic Doppler frequency shift of the target,
\( v \) = relative velocity of the target.

In theory, when the observation interval is infinite, the transmitted signal of a pulse Doppler radar is given by
\[ v_r(t) = \sum_{n=-\infty}^{\infty} A_r \text{rect} \left( \frac{t-nT_p}{T_w} \right) e^{j\omega_r t} \]  \hspace{1cm} (1.5)

where \( A_r \) = magnitude of the transmitted signal, 
\( t \) = time,
\( T_p \) = pulse repetition interval (PRI) of the transmitted signal,
\( T_w \) = pulse width of the transmitted signal,
\( v_r(t) \) = transmitted signal,
\( \omega_r \) = radian frequency of the transmitted signal.

The Fourier transform of Equation (1.5) is given by
\[ V_r(f) = \sum_{n=-\infty}^{\infty} A_r T_w \text{sinc} \left( \frac{n \omega_p T_w}{2} \right) \delta(f - f_c - nf_p) \]  \hspace{1cm} (1.6)

where \( f \) = cyclic frequency,
\( f_c \) = cyclic frequency of the transmitted signal,
\( f_p \) = cyclic pulse repetition frequency (PRF) of the transmitted signal,
\( V_r(f) \) = Fourier transform of \( v_r(t) \).
\( \delta(f) \) = unit impulse function,
\( \omega_p \) = radian PRF of the transmitted signal.

Figure 1.2 shows the transmitted pulse envelope of a pulse Doppler radar, when the observation interval is infinite. It can be seen that the signal consists of an infinite series of rectangular pulses of electromagnetic energy. The pulses are identical, are separated by \( T_p \), and the width of the pulses is \( T_w \).

Figure 1.3 shows the frequency spectrum of \( v_r(t) \). It can be seen that the frequency spectrum consists of an infinite series of impulse functions. The impulse functions are contained in a sinc function envelope, and are separated by \( f_p \). The sinc function is centred at \( f_c \), and the nulls of the sinc function are separated by \( f_w = 1/T_w \).

In practice, the observation interval will be finite, therefore Equation (1.5) can be modified to give
Figure 1.2.
Transmitted pulse envelope of a pulse Doppler radar when the observation interval is infinite.
Figure 1.3.
Frequency spectrum of the transmitted pulse envelope of a pulse Doppler radar when the observation interval is infinite.
\[ v_i(t) = \sum_{n=-\infty}^{\infty} A_n \text{rect} \left( \frac{t - \frac{T_o}{2}}{T_o} \right) \text{rect} \left( \frac{t - nT_p}{T_w} \right) e^{j\omega_c t} \]  

where \( T_o \) = observation interval.

The Fourier transform of Equation (1.7) is given by

\[ V_i(f) = \sum_{n=-\infty}^{\infty} A_n \frac{T_o T_w}{T_p} \text{sinc} \left( \frac{n\omega_p T_w}{2} \right) \text{sinc} \left( \frac{(\omega - \omega_c - n\omega_p)T_o}{2} \right) \]  

where \( \omega = \) radian frequency.

Figure 1.4 shows the transmitted pulse envelope of a pulse Doppler radar, when the observation interval is finite. In this case, it can be seen that the signal consists of a finite series of rectangular pulses of electromagnetic energy. Again, the pulses are identical, are separated by \( T_p \), and the width of the pulse is \( T_w \).

Figure 1.5 shows the frequency spectrum of \( v_i(t) \). It can be seen that the frequency spectrum consists of an infinite series of sinc functions. The sinc functions are contained in a sinc function envelope, and are separated by \( f_p \). The sinc functions are also centred at \( f_c \pm nf_p \), and the nulls of the sinc functions are separated by \( \Delta f = 1/T_o \).

Pulse Doppler radars can be defined as having three types of PRF: low, medium and high [27].

Low PRF is defined as a PRF in which range is unambiguous, and velocity is ambiguous. The main advantage of low PRF is that it enables targets and clutter to be separated on the basis of range.

Medium PRF is defined as a PRF in which range and velocity are both ambiguous. The main advantage of medium PRF is that, to a limited extent, it enables targets and clutter to be separated on the basis of range and velocity.

High PRF is defined as a PRF in which range is ambiguous, and velocity is unambiguous. The main advantage of high PRF is that it enables targets and clutter to be separated on the basis of velocity.
Figure 1.4.
Transmitted pulse envelope of a pulse Doppler radar when the observation interval is finite.
Figure 1.5.
Frequency spectrum of the transmitted pulse envelope of a pulse Doppler radar when the observation interval is finite.
The definitions above define low, medium and high PRFs in terms of range and velocity ambiguity [28]. However, some types of PRF do not fit into any of the categories above. For example, some radars have PRFs which are unambiguous in both range and Doppler.

Here, "unambiguous range" means that range can be determined by using time discrimination at a single PRF, and "ambiguous range" means that range can only be determined by using time discrimination at a number of PRFs, followed by a correlation process. Similarly, "unambiguous velocity" means that velocity can be determined by using frequency discrimination at a single PRF, and "ambiguous velocity" means that velocity can only be determined by using frequency discrimination at a number of PRFs, followed by a correlation process [29].

1.3 Structure of the Thesis

The structure of the thesis is as follows.

Chapter 2 presents a detailed mathematical analysis of the return signal from rotating aircraft blades. The chapter is concerned mainly with the analysis of the return signal and frequency spectrum of rotor and propeller blades when there is phase and amplitude modulation of the return signal.

A detailed survey of the physical configurations, physical parameters and radar features of rotor and propeller aircraft ranging in size from light to heavy aircraft is presented in Chapter 3. The chapter discusses those features which relate to the aircraft in general, and those which relate to the rotating aircraft blades in particular.

Chapter 4 discusses the extraction of aircraft features from the return signal. The chapter is concerned mainly with the extraction of features which relate to the rotating aircraft blades, but also discusses the extraction of other features from the return signal.
Some of the practical considerations which are associated with the analysis of the return signal from rotating aircraft blades are discussed in Chapter 5. The chapter discusses four main subjects: the radar parameters, window functions, relative acceleration and aircraft classification.

Finally, Chapter 6 describes the achievements which have been made, discusses the limitations to the work, and provides suggestions for future work.
CHAPTER TWO

ANALYSIS OF THE RADAR RETURN SIGNAL
FROM ROTATING AIRCRAFT BLADES

2.1 Introduction

This chapter presents a detailed mathematical analysis of the radar return signal from rotating aircraft blades. The chapter begins by considering the return signal from a single scatterer on a single rotating aircraft blade. From this, equations are derived which describe the return signal and frequency spectrum of the rotating aircraft blades. These equations are then used to analyse the modulation of the return signal.

The structure of the chapter is as follows.

Section 2 presents a detailed mathematical analysis of the return signal from rotating aircraft blades. It is shown that the modulation caused by the rotation of the blades is a form of phase modulation (PM), that the frequency spectrum can be approximated by a finite number of sidebands, and that each sideband can be approximated by a finite series of Bessel functions. Equations are derived which show that the modulation is dependent on six main variables, four of which are parameters of the aircraft blades, one of which is a parameter of the radar, and one of which is a function of the aspect angle. It is believed that this is the first time that a detailed mathematical analysis of the return signal from rotating aircraft blades has appeared in the literature.

Simulation results for rotating aircraft blades are presented in Section 3. The section presents results for rotating rotor and propeller blades in particular, and also presents some general results which apply to both of these types of blade. It is
shown that, in the case of propeller aircraft, if the propeller has a propeller spinner, then this will cause a frequency notch about the centre frequency. It is believed that this is the first time that an analysis of the effects of the propeller spinner on the return signal has appeared in the literature. It is also shown that aircraft which have completely different blade parameters can have identical blade features.

Section 4 presents a detailed mathematical analysis of the effects of blade pitch on the return signal. It is shown that if there is any blade pitch, then this will tend to cause a periodic variation of the cross section of each blade, which will cause amplitude modulation (AM) of the return signal from each blade. It is believed that this is the first time that a detailed mathematical analysis of the effects of blade pitch on the return signal has appeared in the literature.

Some of the ways in which practical aircraft blades and return signals differ from theoretical aircraft blades and return signals, respectively, are discussed in Section 5. It is shown that practical aircraft blades are very complex structure, and that this will affect the return signal in various ways.

Finally, Section 6 draws some conclusions.

2.2 Mathematical Analysis

Figure 2.1 shows the basic scenario. Each scatterer on each rotating blade will tend to cause a Doppler frequency shift of the return signal, in which the shift will depend on the position of the scatterer on the blade, and on the instantaneous velocity of the blade [30].

Assume that the signal transmitted by the radar is given by

\[ v_r(t) = A_r e^{j\omega_r t} \]  

(2.1)

Then the return signal from a single point scatterer which consists of an incremental chord-wise section of one blade of an aircraft rotor is given by

\[ v_s(t) = A_s e^{j\omega_s (t - t_s)} \]  

(2.2)
Figure 2.1.
Basic Scenario.
\(v_r(t)\) = return signal,
\(\tau(t)\) = time delay of the return signal.

Equation (2.2) can be expanded to give
\[v_r(t) = A_r e^{j\omega_r t - \frac{2\pi k(t)}{c}}\]  
(2.3)

where \(R(t)\) = range of the scatterer.

Equation (2.3) can be rearranged to give
\[v_r(t) = A_r e^{j(\omega_r t - \frac{4\pi R(t)}{\lambda})}\]  
(2.4)

Equation (2.4) can be expanded to give
\[v_r(t) = A_r e^{j(\omega_r t - \int_{-\frac{\lambda}{4\pi}}^{\frac{\lambda}{4\pi}} \omega_r(t) dr')}\]  
(2.5)

where \(t'\) = dummy variable,
\(v(t)\) = relative velocity of the scatterer.

Equation (2.5) can also be expanded to give
\[v_r(t) = A_r e^{\int(\omega_r t - \omega_r r \cos(\omega_r t)) dr'}\]  
(2.6)

where \(r\) = distance of the scatterer from the centre of rotation of the rotor,
\(\omega_r\) = radian frequency of rotation of the rotor.

Equation (2.6) can be solved to give
\[v_r(t) = A_r e^{\int(\omega_r t - \frac{4\pi}{\lambda}(R - vr + r \sin(\omega_r t)))}\]  
(2.7)

In order to determine the return signal from an entire blade, we integrate Equation (2.7) along the length of the blade to give
\[v_r(t) = \int_{L_1}^{L_2} A_r e^{\int(\omega_r t - \frac{4\pi}{\lambda}(R - vr + r \sin(\omega_r t)))} dr\]  
(2.8)

where \(L_1\) = distance of the blade root from the centre of rotation,
\(L_2\) = distance of the blade tip from the centre of rotation.
Figures 2.2 and 2.3 show $L_1$ and $L_2$ for a rotor and a propeller aircraft, respectively. In practice, all rotors will have more than one blade. Each blade will be identical, lying in the plane of rotation, and equispaced about the axis of rotation, therefore Equation (2.8) can be expanded to give

$$v_r(t) = \sum_{n=0}^{N-1} \int A_n e^{j(\omega t - \frac{4\pi}{\lambda} r - \frac{2\pi n}{N})} dr$$

(2.9)

where $N =$ number of blades.

If the line of sight from the radar to the centre of rotation does not lie in the plane of rotation, then Equation (2.9) can be expanded to give

$$v_r(t) = \sum_{n=0}^{N-1} \int A_n e^{j(\omega t - \frac{4\pi}{\lambda} r - \frac{2\pi n}{N})} dr$$

(2.10)

where $\theta =$ angle between the plane of rotation and the line of sight from the radar to the centre of rotation, where $\theta$ is positive in the direction of aerofoil thrust, and negative in the direction of aerofoil drag.

Equation (2.10) can be solved to give

$$v_r(t) = \sum_{n=0}^{N-1} A_n \text{sinc} \left( \frac{4\pi}{\lambda} \frac{(L_2 - L_1)}{2} \cos(\theta) \sin(\omega t + \frac{2\pi n}{N}) \right)$$

(2.11)

where $A_n = A_n (L_2 - L_1)$.

A number of assumptions are made in the derivation of Equation (2.11):

1. Each blade acts as a homogeneous, linear, rigid antenna [31].
2. Each blade is always visible to the radar, i.e. there is no shielding of the blades.
3. The rotor is in the far field of the radar.

The assumptions above result in a good theoretical model of practical rotors, however the following points should be noted:
Figure 2.2.
Diagram showing $L_1$ and $L_2$ for a rotor aircraft.
Figure 2.3.
Diagram showing $L_1$ and $L_2$ for a propeller aircraft.
1. Each blade is actually a rotating aerofoil, having camber, taper, twist, etc., therefore the cross section of each section of blade will change with its distance from the centre of rotation, with its angular position, with respect to the radar, and with the aspect angle of the rotor.

2. At some aspect angles shielding of the blades will occur.

3. The rotor will almost always be in the far field of the radar. However, if the rotor is not in the far field of the radar then the rotor can still modulate the return signal, but the modulation will not be described by Equation (2.11).

The Fourier transform of Equation (2.11) is difficult to derive and is more easily determined from Equation (2.10).

Equation (2.10) can be rearranged to give

\[ v_r(t) = \sum_{n=0}^{N-1} L_2 A_r^e \int_{-L_1}^{L_1} r^{-\frac{\pi}{N}} e^{j(\omega_r t - \frac{2\pi n}{N})} dr \]  

where \( A_r^e = A_r e^{j(\omega_r + \omega_d)t - \frac{4\pi N}{\lambda}} \),

\( \omega_d = -\frac{4\pi}{\lambda} \) \( \nu \) = radian Doppler frequency shift of the aircraft.

Equation (2.12) can be expressed as a Bessel series to give

\[ v_r(t) = \sum_{k=-\infty}^{\infty} \sum_{n=0}^{N-1} L_2 A_r^e (\frac{4\pi}{\lambda} r \cos(\theta)) e^{jk(\omega_r t - \frac{2\pi n}{N})} dr \]  

where \( J_k(x) = \sum_{l=0}^{\infty} (\frac{x}{2})^k (-1)^l \frac{(k + l)!}{(k + l)!l!} \) Bessel function of the 1st kind and \( k \)th order, since [32]

\[ e^{jr \sin(\alpha)} = \sum_{k=-\infty}^{\infty} J_k(x)e^{jk\alpha} \]  

Equation (2.13) can be expanded to give

\[ v_r(t) = \sum_{k=-\infty}^{\infty} \int_{-L_1}^{L_1} A_r^e J_k\left(-\frac{4\pi}{\lambda} r \cos(\theta)\right) e^{jk(\omega_r t - \frac{2\pi n}{N})} dr \]  

\[ (e^{j\frac{2\pi 0k}{N}} + e^{j\frac{2\pi 1k}{N}} + e^{j\frac{2\pi 2k}{N}} + \cdots + e^{j\frac{2\pi (N-1)k}{N}}) \]
Equation (2.15) can be rearranged to give

$$v_r(t) = \sum_{k = -\frac{L_2}{L_1}}^{\infty} \int A_i J_k \left( -\frac{4\pi}{\lambda} r \cos(\theta) \right) e^{i\omega t} d\theta$$

$$(2.16)$$

since

$$\sum_{k = -\frac{L_2}{L_1}}^{\infty} e^{\frac{2\pi i k}{N}} + e^{\frac{2\pi i k}{N}} + e^{\frac{2\pi i k}{N}} + \cdots + e^{\frac{2\pi i (N-1) k}{N}} = N \frac{\sin(\pi k N)}{\sin(\pi k)}$$

$$(2.17)$$

Equation (2.16) can be simplified to give

$$v_r(t) = \sum_{k = -\frac{L_2}{L_1}}^{\infty} \int A_i N J_k \left( -\frac{4\pi}{\lambda} r \cos(\theta) \right) e^{i\omega t} d\theta$$

$$(2.18)$$

since

$$N \frac{\sin(\pi k N)}{\sin(\pi k)} e^{\frac{\pi (N-1) k}{N}}$$

$$(2.19)$$

Equation (2.18) can be solved to give

$$v_r(t) = \sum_{k = -\frac{L_2}{L_1}}^{\infty} \sum_{l = 0}^{\infty} \frac{2(-1)^{l+1} A_i N}{4\pi \cos(\theta)} \left( J_{N|k|+2l+1} \left( \frac{4\pi}{\lambda} L_2 \cos(\theta) \right) - J_{N|k|+2l+1} \left( \frac{4\pi}{\lambda} L_1 \cos(\theta) \right) \right) e^{i\omega t}$$

$$(2.20)$$

where $u(t) = $ unit step function,

since [33]

$$\int_0^x J_k(x) dx = 2 \sum_{l = 0}^{\infty} J_{k+2l+1} (x)$$

$$(2.21)$$

and [34]

$$J_k(-x) = J_{-k}(x) = (-1)^k J_k(x)$$

$$(2.22)$$

Equation (2.20) can be rearranged to give

$$v_r(t) = \sum_{k = -\frac{L_2}{L_1}}^{\infty} c_{Nk} e^{j(\omega_s + \omega_d - NK \omega)} t$$

$$(2.23)$$
where \( c_{nk} = \sum_{i=0}^{\infty} \frac{2(-1)^{Nk+i}A_rN}{4\pi\lambda} \cos(\theta) \)

\[ (J_{N|k|+2l+1}(\frac{4\pi L_2\cos(\theta)}) - J_{N|k|+2l+1}(\frac{4\pi L_1\cos(\theta)})e^{-\frac{j4\pi k}{\lambda}} \].

The Fourier transform of Equation (2.23) is given by

\[ V_r(f) = \sum_{k=-\infty}^{\infty} c_{nk} \delta(f - f_c - f_d - Nkf_r) \tag{2.24} \]

where \( f_c = \) cyclic frequency of rotation of the rotor,

\( V_r(f) = \) Fourier transform of \( v_r(t) \).

It can be seen from Equation (2.24) that \( V_r(f) \) consists of an infinite number of sidebands, and that each sideband consists of an infinite series of Bessel functions [35]. Although the number of sidebands and the number of Bessel functions in each sideband are infinite, it is possible to make some simplifying approximations. In practice, it is found that \( J_k(x) \approx 0 \) if \( k > x \), and \( x \gg 1 \) [36]. If we assume that \( x \gg 1 \), and the summation in Equation 2.24 exists, then Equation (2.24) can be modified to give

\[ V_r(f) = \sum_{k=-\frac{N_2}{2}}^{\frac{N_2}{2}} c_{nk} \delta(f - f_c - f_d - Nkf_r) \tag{2.25} \]

where \( c_{nk} = \sum_{i=0}^{\frac{N_3}{2}} \frac{2(-1)^{Nk+i}A_rN}{4\pi\lambda} \cos(\theta) \)

\[ (J_{N|k|+2l+1}(\frac{4\pi L_2\cos(\theta)}) - J_{N|k|+2l+1}(\frac{4\pi L_1\cos(\theta)})e^{-\frac{j4\pi k}{\lambda}} \],

\[ N_2 = \left[ \frac{8\pi L_2\cos(\theta)}{N\lambda} \right] + 1 \] = number of significant sidebands,

\[ N_3 = \left[ \frac{N_2}{2} - \left| Nk \right| - 1 \right] + 1 \] = number of significant Bessel functions,

where \([x]\) denotes the integral part of \( x \).

Note that the above argument assumes that the sum of the terms discarded in deriving Equation 2.25 from Equation 2.24 can be ignored.
It can be seen from Equation (2.25) that $V_r(f)$ can be approximated by a finite number of sidebands, and that each sideband can be approximated by a finite series of Bessel functions [37]. It can also be seen that Equations (2.11) and (2.25) are similar to the equations which describe PM, i.e. [38]

$$g(t) = A \cos(\omega_c t + \alpha_m \sin(\omega_m t))$$ \hspace{1cm} (2.26)

where $A$ = scale factor,

$g(t)$ = output signal,

$\alpha_m$ = modulation index,

$\omega_m$ = radian modulation frequency.

and

$$G(f) = \sum_{k=-\infty}^{\infty} A' (-1)^{k} I_1(\alpha_m) \left[ \delta(f + f_c + kf_m) + \delta(f - f_c - kf_m) \right]$$ \hspace{1cm} (2.27)

where $A' = \frac{A}{2}$,

$f_m$ = cyclic modulation frequency,

$G(f)$ = Fourier transform of $g(t)$.

In fact, the rotation of each scatterer on each blade will cause pure PM of the return signal, therefore this is an electro-mechanical method of generating PM [39]. However, since there is more than one scatterer on each blade, this will cause a range of frequencies to be modulated for each blade, since each scatterer on a given blade will have a different rotational velocity. Also, since there is more than one blade on the rotor, this will cause a phase angle between the scatterers on different blades.

When $L_1 = 0$, where $L_1$ is the distance of the blade roots from the centre of rotation, Equation (2.11) can be simplified to give

$$V_r(t) = \sum_{n=0}^{N-1} A_n \frac{4\pi L_2}{\lambda} \frac{L_2}{2} \cos(\theta) \sin(\omega_r t + \frac{2\pi n}{N})$$ \hspace{1cm} (2.28)
where $A_r = A_r L_2$.

Equation (2.25) can also be simplified to give

$$V_r(f) = \sum_{k=-N_2/2}^{N_2/2} c_{Nk} \delta(f - f_r - Nkf_r)$$

(2.29)

where

$$c_{Nk} = \sum_{l=0}^{N_2/4} \frac{2(-1)^{Nk+1} A_r N_1}{4\pi \lambda \cos(\theta)} J_{Nk+2l-1} \left(\frac{4\pi}{\lambda} L_2 \cos(\theta)\right) e^{-j\frac{4\pi}{\lambda} \cos(\theta)}.$$ 

When $L_1 = 0$, Equations (2.28) and (2.29) result in a PM signal with $N_2$ pairs of sidebands about the centre frequency of the aircraft, each separated by $Nf_r$. When $L_1 \neq 0$, Equations (2.11) and (2.25) also result in a PM signal. However, depending on $L_1$, the sidebands which are nearest to the centre frequency of the aircraft will be approximately zero, resulting in a frequency notch about the centre frequency $f_r$. The greater the value of $L_1$, the greater the notch will be. In general:

When $L_1 = 0$:

$$N_1 = 0$$

(2.30)

$$N_2 = \left\lfloor \frac{8\pi L_2 \cos(\theta)}{N\lambda} \right\rfloor + 1$$

(2.31)

$$\Delta f = Nf_r$$

(2.32)

$$B_1 = N_1 \Delta f = 0$$

(2.33)

$$B_2 = N_2 \Delta f = \frac{8\pi f_r L_2 \cos(\theta)}{\lambda}$$

(2.34)

When $L_1 \neq 0$:

$$N_1 = \left\lfloor \frac{8\pi L_1 \cos(\theta)}{N\lambda} \right\rfloor + 1$$

(2.35)

$$N_2 = \left\lfloor \frac{8\pi (L_2 - L_1) \cos(\theta)}{N\lambda} \right\rfloor + 1$$

(2.36)
\[ B_1 = N_1 \Delta f = \frac{8\pi f_r L_1 \cos(\theta)}{\lambda} \]  
\[ B_2 = N_2 \Delta f = \frac{8\pi f_r (L_2 - L_1) \cos(\theta)}{\lambda} \]

where \( N_1 \) = number of notched sidebands,
\( N_2 \) = number of significant sidebands,
\( \Delta f \) = blade-passing frequency,
\( B_1 \) = bandwidth of the notched sidebands,
\( B_2 \) = bandwidth of the significant sidebands.

In Equation (2.31), \( N_2 \) is the number of sidebands, and is equivalent to the number of sidebands due to a PM signal, i.e.
\[ N_2 = \frac{2\alpha_m}{N} \]  
where \( \alpha_m = \frac{4\pi L_2 \cos(\theta)}{\lambda} \).

It can be seen from Equation (2.18) that the \( N \) term in Equations (2.31) and (2.39) arises because only every \( N \)th sideband is present, due to harmonic phase cancellation of the return signals from different blades.

In Equation (2.32), \( \Delta f \) is the blade-passing frequency of the rotor, and is equivalent to the frequency separation of the sidebands of a PM signal, i.e. [41]
\[ \Delta f = N f_m \]  
Again, the \( N \) term in Equations (2.32) and (2.40) arises because only every \( N \)th sideband is present.

In Equation (2.34), \( B_2 \) is the bandwidth of the sidebands, and is equivalent to the bandwidth of a PM signal, i.e.
\[ B_2 = 2\alpha_m f_m \]  
In Equations (2.34) and (2.41), the bandwidth is equal to the product of the number of sidebands and the frequency separation of each sideband.
Equations (2.36) and (2.38) correspond to Equations (2.31) and (2.34), respectively, and correspond to a PM signal in which the sidebands which are nearest to the carrier frequency have been notched.

A better understanding of Equation (2.34) can be obtained by rearranging $B_2$ as follows

$$B_2 = \frac{8\pi f_r L_2 \cos(\theta)}{\lambda} = \frac{4\omega_r L_2 \cos(\theta)}{\lambda} = \frac{4v_b \cos(\theta)}{\lambda}$$

(2.42)

where $v_b = \omega_r L_2 =$ rotational velocity of the blade tips.

It can be seen that Equation (2.42) is similar to the equation which describes the Doppler frequency shift of an aircraft which has a relative velocity, with respect to the radar. This is what we would intuitively expect: the upper sidebands have a positive Doppler frequency shift, and are due to reflections from the blades as they rotate towards the radar, and the lower sidebands have a negative Doppler frequency shift, and are due to reflections from the blades as the rotate away from the radar. Also, the sidebands which are nearest to the centre frequency have a smaller Doppler frequency shift, and are due to reflections from the blade roots, and the sidebands furthest from the centre frequency have a larger Doppler frequency shift, and are due to reflections from the blade tips.

It can be seen from Equations (2.30)-(2.38) that the modulation is dependent on six main variables: $N, L_1, L_2, f_r, \lambda$ and $\theta$, four of which are parameters of the aircraft blades, $N, L_1, L_2$ and $f_r$; one of which is a parameter of the radar, $\lambda$; and one of which is a function of the aspect angle, $\theta$.

### 2.3 Simulation Results

This section presents simulation results for rotating aircraft blades, using the equations derived in Section 2.2. In this section we will only be concerned with analysing the return signal and frequency spectrum of the rotating blades. Therefore, we will ignore the return signal from the airframe, and the tail rotor, in the
case of rotor aircraft, as well as noise and clutter. We will also ignore the predictable effects of range and velocity on the return signal.

2.3.1 Rotor Aircraft

Figure 2.4 shows the normalised simulated return signal from the rotor of an aircraft in which $N = 4$, $L_2 = 5$ m, $f_r = 6$ Hz, $\theta = 0$ rad., and $\lambda = 1$ m. It can be seen that $v_r(t)$ is real, even and periodic, with period $\Delta t = 0.04$ s, where $\Delta t = 1/Nf_r$ is the blade-passing interval of the rotor. The maxima correspond to points at which $\omega_r t + 2\pi n/N = \pm \pi k$, i.e. to points at which two blades are orthogonal to the line of sight from the radar to the centre of rotation, and therefore reflect the greatest amount of energy. Note that the apparent variation of the magnitudes of the blade flashes is due to the effects of sampling.

Figure 2.5 shows the the frequency spectrum of $v_r(t)$. It can be seen that $V_r(f)$ is real and even, with $N_2 = 30$, $\Delta f = 24$ Hz, and $B_2 = 720$ Hz.

Figures 2.6 and 2.7 show the real and imaginary components, respectively, of the return signal from the rotor of an aircraft which is identical to that of Figure 2.4, except that in this case $N = 5$. It can be seen that $v_r(t)$ is complex, the real component being even and periodic, with $\Delta t = 0.02$ s, and the imaginary component being odd and periodic, with $\Delta t = 0.03$ s. In this case, the maxima correspond to points at which one blade is orthogonal to the line of sight from the radar to the centre of rotation, and therefore reflects the greatest amount of energy.

Figure 2.8 shows the frequency spectrum of $v_r(t)$. It can be seen that $V_r(f)$ is real and unsymmetrical, with $N_2 = 24$, $\Delta f = 30$ Hz, and $B_2 = 720$ Hz. Although $V_r(f)$ is real and unsymmetrical, Figures 2.9, 2.10 and 2.11, respectively, show that $F(\text{Re}(v_r(t)))$ is real and even, and that $F(\text{Im}(v_r(t)))$ is real and odd, therefore $|V_r(f)|$ is real and even.

2.3.2 Propeller Aircraft

Figure 2.12 shows the return signal from the propeller of an aircraft in which $N = 2$, $L_1 = 0.25$ m, $L_2 = 2$ m, $f_r = 16$ Hz, $\theta = 0$ rad., and $\lambda = 1$ m. It can be seen
Return signal from the rotor of an aircraft in which $N = 4$, $L_2 = 5$ m, $f_r = 6$ Hz, $\theta = 0$ rad., and $\lambda = 1$ m.
Figure 2.5.
Frequency spectrum of the rotor of an aircraft in which $N=4$, $L_2=5$ m, $f_r=6$ Hz, $\theta=0$ rad., and $\lambda=1$ m.
Figure 2.6.
Real component of the return signal from the rotor of an aircraft in which $N = 5$, $L_2 = 5 \text{ m}$, $f_r = 6 \text{ Hz}$, $\theta = 0 \text{ rad.}$, and $\lambda = 1 \text{ m}$. 
Figure 2.7.
Imaginary component of the return signal from the rotor of an aircraft in which 
$N=5$, $L_2=5$ m, $f_r=6$ Hz, $\theta=0$ rad., and $\lambda=1$ m.
Figure 2.8.
Frequency spectrum of the rotor of an aircraft in which $N = 5$, $L_2 = 5$ m, $f_r = 6$ Hz, $\theta = 0$ rad., and $\lambda = 1$ m.
Figure 2.9.
Frequency spectrum of the real component of the return signal from the rotor of an aircraft in which $N=5$, $L_2=5$ m, $f_r=6$ Hz, $\theta=0$ rad., and $\lambda=1$ m.
Figure 2.10.
Frequency spectrum of the imaginary component of the return signal from the rotor of an aircraft in which \( N=5, L_2=5 \) m, \( f_r=6 \) Hz, \( \theta=0 \) rad., and \( \lambda=1 \) m.
Figure 2.11.
Frequency spectrum of the rotor of an aircraft in which $N=5$, $L_2=5$ m, $f_r=6$ Hz, $	heta=0$ rad., and $\lambda=1$ m.
Return signal from the propeller of an aircraft in which $N=2$, $L_1=0.25$ m, $L_2=2$ m, $f_r=16$ Hz, $\theta=0$ rad., and $\lambda=1$ m.
that \(v_r(t)\) is real, even and periodic, with \(\Delta t = 0.03\) s.

Figure 2.13 shows the frequency spectrum of the aircraft of Figure 2.12. It can be seen that \(V_r(f)\) is real and symmetrical, with \(N_1 = 2\), \(N_2 = 22\), \(\Delta f = 32\) Hz, \(B_1 = 64\) Hz, and \(B_2 = 640\) Hz.

Note that there is a frequency notch about the centre frequency. This is caused by the propeller spinner. A propeller spinner is a streamlined fairing which fits coaxially over the end of the propeller shaft in front of the propeller, and which rotates with the propeller. The spinner is fitted to the propeller in order to reduce the drag of the propeller shaft, propeller hub, blade shanks, etc., and its size, and therefore the value of \(L_1\), is determined mainly by the size of the propeller hub. The spinner is of approximately conical or paraboloidal shape, and is a body of revolution about its axis of rotation. Most propellers have spinners, and in most cases this will tend to cause a frequency notch about the centre frequency of the aircraft.

Note that the transitions at the inner and outer cut-off frequencies for \(B_1\) and \(B_2\), respectively, are not perfect, i.e. there is some sideband energy outside the bandpass region. This is what we would intuitively expect: if the transitions were perfect, with no sideband energy outside the bandpass region, then this would be analogous to creating a perfect bandpass filter, which is unrealisable [42]. Another way of explaining the same phenomenon is that there is some sideband energy lower than the inner cut-off frequencies, since, referring to Equation (2.25)

\[
J_k(x_2) - J_k(x_1) \neq 0 \quad \text{if} \quad k < x_1, \quad \text{and} \quad x_1 < x_2
\]  

(2.43)

Also, there is some sideband energy higher than the outer cut-off frequencies, since, referring to Equation (2.25), again

\[
J_k(x) \neq 0 \quad \text{if} \quad k > x, \quad \text{and} \quad x >> 1
\]  

(2.44)

Note that care must be taken with the terminology here, since we are referring to a double-sided spectrum in which there is a frequency notch about the centre fre-
Figure 2.13.
Frequency spectrum from the propeller of an aircraft in which \( N = 2, \quad L_1 = 0.25 \text{ m}, \quad L_2 = 2 \text{ m}, \quad f_r = 16 \text{ Hz}, \quad \theta = 0 \text{ rad.}, \) and \( \lambda = 1 \text{ m}. \)
quency, i.e. we are referring to a spectrum in which there are four transitions, hence the use of the terms "inner" and "outer", instead of "lower" and "upper", respectively.

2.3.3 General Results

From the results above we can make some general observations, regarding the return signal and frequency spectrum of rotating aircraft blades. In general:

When $N$ is even:

$v_r(t)$ is even and periodic, with $\Delta t = 1/N_\nu$.

$V_r(f)$ is real and even.

When $N$ is odd:

$v_r(t)$ is complex, the real component being even and periodic, with $\Delta t = 1/2N_\nu$, and the imaginary component being odd and periodic, with $\Delta t = 1/N_\nu$.

$V_r(f)$ is real and unsymmetrical, although $|V_r(f)|$ is even.

Note that if the observation interval does not begin at a blade flash, but instead begins when the rotor has a random rotation angle, with respect to the radar, then Equation (2.11) will be modified to give

$$v_r(t) = \sum_{n=0}^{N-1} A_n \sin\left(\frac{4\pi}{\lambda} \frac{(L_2-L_1)}{2}\cos(\theta)\sin(\omega, t + \frac{2\pi n}{N} + \phi_r)\right)$$

where $\phi_r$ = random rotation angle of the rotor.

Similarly, Equation (2.25) will be modified to give

$$V_r(f) = \sum_{k=-N/2}^{N/2} c_{nk} \delta(f - f_c - f_d - Nkf_r)$$

where $c_{nk} = \frac{2(-1)^{N/2} A_n N}{\lambda \cos(\theta)}$.
It can be seen from Equations (2.45) and (2.46) that if the rotor has a random rotation angle, then this will cause a time shift of the return signal, and a phase rotation of the frequency spectrum. Note that if there is any phase rotation of the frequency spectrum, then this will cause $V_r(f)$ to be complex, not real.

The results above apply in the case where $v_r(t)$ and $V_r(f)$ are continuous signals. In the case where $v_r(t)$ and $V_r(f)$ are discrete, the results will be modified slightly, in that, in most cases, as with phase rotation, $V_r(f)$ will be complex, not real. This is due to the discrete nature of the discrete Fourier transform (DFT) algorithm, sampling, windowing, etc.

Figure 2.14 shows the frequency spectrum of the rotor of an aircraft in which $N=5$, $L_2=6.25$ m, $f_r=4.8$ Hz, $\theta=0$ rad., and $\lambda=1$ m. It can be seen that $N_2=30$, $\Delta f=24$ Hz, and $B_2=720$ Hz. Note that this rotor has exactly the same values of $N_2$, $\Delta f$ and $B_2$ as the rotor of Figure 2.4, despite having completely different blade parameters - only the magnitudes of the sidebands of the two rotors are different. This is because $N_2$, $\Delta f$ and $B_2$ are inherently ambiguous, since it can be seen from Equations (2.30)-(2.34) that there are three unknown parameters (assuming that $\theta$ and $\lambda$ are known), but only two linearly independent equations. This means that it is not possible to determine the aircraft parameters by using these equations. In theory, there will be a countable number of sets of aircraft parameters, $N$, $L_1$, $L_2$, and $f_r$, which provide a solution set of the aircraft features, $N_1$, $N_2$, $\Delta f$, $B_1$, and $B_2$. (Note that the number of solution sets is countable because fixing $N$ also fixes other parameters.) In practice, the number of solution sets will be limited, because the range of values of the blade parameters for practical aircraft will be limited. This is discussed further in Chapter 5.

Initially, it may seem that there is no particular pattern to the magnitudes of the sidebands. However, this is not the case. In order to illustrate this, consider the case of a hypothetical single-bladed rotor.
Figure 2.14.
Frequency spectrum of the rotor of an aircraft in which $N = 5$, $L_2 = 6.25$ m, $f_r = 4.8$ Hz, $\theta = 0$ rad., and $\lambda = 1$ m.
Figure 2.15 shows the frequency spectrum of the rotor of an aircraft in which \( N = 1, L_2 = 5 \text{ m}, f_\text{r} = 6 \text{ Hz}, \theta = 0 \text{ rad.}, \) and \( \lambda = 1 \text{ m}. \) It can be seen that \( N_2 = 126, \) \( \Delta f = 6 \text{ Hz}, \) and \( B_2 = 756 \text{ Hz}. \) It can also be seen that the sidebands have a lobe structure, in which each lobe contains a number of sidebands. It can further be seen that the magnitude and frequency separation of the lobes increases with frequency, therefore the number of sidebands in each lobe increases with frequency.

By analysing the frequency spectra of rotating aircraft blades with different parameters, we can make some general observations, regarding the number of lobes in the frequency spectrum of rotating aircraft blades. In general:

When \( L_1 = 0: \)

\[ N_{i_1} = 0 \]  

\[ N_{i_2} = \left[ \frac{4L_2 \cos(\theta)}{\lambda} + 1 \right] \]  

\[ \Delta f_1 = 2\pi f_r \]  

\[ B_1 = 0 \]  

\[ B_2 = N_{i_2} \Delta f_1 = \frac{8\pi f_r L_2 \cos(\theta)}{\lambda} \]

When \( L_1 \neq 0: \)

\[ N_{i_1} = \left[ \frac{4L_1 \cos(\theta)}{\lambda} + 1 \right] \]  

\[ N_{i_2} = \left[ \frac{4(L_2 - L_1) \cos(\theta)}{\lambda} + 1 \right] \]  

\[ B_1 = N_{i_1} \Delta f_1 = \frac{8\pi f_r L_1 \cos(\theta)}{\lambda} \]  

\[ B_2 = N_{i_2} \Delta f_1 = \frac{8\pi f_r (L_2 - L_1) \cos(\theta)}{\lambda} \]

where \( N_{i_1} = \text{number of notched lobes}, \)
Figure 2.15.
Frequency spectrum of the rotor of an aircraft in which $N=1$, $L_2=5$ m, $f_r=6$ Hz, $\theta=0$ rad., and $\lambda=1$ m.
\( N_t = \) number of significant lobes,

\( \Delta f_i = \) frequency separation of each lobe.

It can be seen from Equations (2.47)-(2.55) that, as with the modulation, the lobes are dependent on six main variables: \( N, L_1, L_2, f_r, \lambda \) and \( \theta \), four of which are parameters of the aircraft blades, \( N, L_1, L_2 \) and \( f_r \); one of which is a parameter of the radar, \( \lambda \); and one of which is a function of the aspect angle, \( \theta \).

Note that in the case where \( L_1 \neq 0 \), Equations (2.52) and (2.53) will only be approximate, since the frequency separation of the lobes increases with frequency. This means that Equation (2.52) will tend to underestimate the number of lobes, and Equation (2.53) will tend to overestimate the number of lobes.

Figures 2.16-2.18 show the frequency spectrum of the rotor of an aircraft which is identical to that of Figure 2.15, as \( N \) is increased from 2 to 4. It can be seen that as the number of blades is increased, the number of sidebands is reduced. This will make it increasingly difficult to determine the number of lobes, until eventually it will no longer be possible to do so.

Note the significance of the results above: if it were possible to determine the number of lobes in the frequency spectrum of the rotating aircraft blades, then it would be possible to determine the values of the blade parameters, \( N, L_1, L_2 \) and \( f_r \), instead of the blade features, \( N_1, N_2, \Delta f, B_1 \) and \( B_2 \) (assuming that \( \theta \) and \( \lambda \) are known). Therefore, it would be possible to uniquely classify aircraft which have otherwise ambiguous blade features (assuming that different types of aircraft do not have identical blade parameters).

2.4 Blade Pitch

When in flight, the aircraft blades will have a positive blade pitch angle, in order to generate thrust. At some aspect angles, this will cause a periodic variation of the cross section of each blade [43]. This will cause AM of the return signal from each blade, and will affect some of the previous results. The main effect of
Figure 2.16.
Frequency spectrum of the rotor of an aircraft in which $N = 2$, $L_2 = 5$ m, $f_r = 6$ Hz, $	heta = 0$ rad., and $\lambda = 1$ m.
Figure 2.17.
Frequency spectrum of the rotor of an aircraft in which \( N = 3, L_2 = 5 \text{ m}, f_r = 6 \text{ Hz}, \theta = 0 \text{ rad.}, \) and \( \lambda = 1 \text{ m}. \)
Figure 2.18.
Frequency spectrum of the rotor of an aircraft in which $N = 4$, $L_2 = 5$ m, $f = 6$ Hz, $	heta = 0$ rad., and $\lambda = 1$ m.
AM of the return signal is that when $\theta$ is positive, the cross section of each blade will be smaller when the blade is rotating towards the radar, and will be greater when the blade is rotating away from the radar. This will tend to reduce the magnitudes of the blade flashes and the upper sidebands when the blades are rotating towards the radar, and increase the magnitudes of the blade flashes and the lower sidebands when the blades are rotating away from the radar. Conversely, when $\theta$ is negative, the cross section of each blade will be greater when the blade is rotating towards the radar, and will be smaller when the blade is rotating away from the radar. This will tend to reduce the magnitudes of the blade flashes and the lower sidebands when the blades are rotating away from the radar, and increase the magnitudes of the blade flashes and the upper sidebands when the blades are rotating towards the radar.

In practice, rotor blades will have a blade pitch of about 10° for all flight conditions, and propeller blades will have a blade pitch which will vary from about 30 to 60°, depending on the flight conditions. In the case of propeller blades, a small blade pitch is used at low forward speeds, and a high blade pitch is used at high forward speeds.

To some extent, there will also be a second mechanism which will cause AM of the return signal from each blade. In this case, the AM occurs because the cross section of the leading edge of each blade is greater than that of the trailing edge [44]. This is due to the "bluntness" and "sharpness" of the leading and trailing edges, respectively, and will tend to increase the magnitudes of the blade flashes and the upper sidebands when the blades are rotating towards the radar, and reduce the magnitudes of the blade flashes and the lower sidebands when the blades are rotating away from the radar.

As well as having blade pitch, most aircraft blades will also have blade twist, in which the blade pitch varies with distance from the centre of rotation. Blade twist is applied in order that each section of blade generates the same amount of thrust. This reduces the stresses on the blades, and also increases the efficiency of
the blades. In most cases, the blade twist will be linear, in which case the blade twist of each section of blade will be given by [45]

$$\phi_p = \tan^{-1}(\frac{p}{2\pi r})$$

(2.56)

where \(p\) = geometric pitch,

\(\phi_p\) = blade pitch angle.

In practice, rotor blades will have a blade twist of about 5°, and propeller blades will have a blade twist of about 30°.

In the analysis below, we will only consider the case of propeller blades. This is partly because rotor blades will also have cyclic pitch, in which the blade pitch is reduced when the blades are advancing, and increased when the blades are retreating, in order to reduce the roll caused by the asymmetry of lift between the advancing and retreating blades, and partly because the rotor blades are so large that the effect of any AM on the return signal from rotor blades will be small in comparison with that on the return signal from propeller blades.

The effect of any blade twist on the return signal will be small in comparison with that of the blade pitch. Therefore, if we assume that the blade twist can be ignored, then the return signal from the rotating aircraft blades when there is AM of the return signal is given by modifying Equation (2.11).

Equation (2.11) can be modified to give

$$v_r(t) = \sum_{n=0}^{N-1} A_r'(\alpha + \beta \cos(\omega_c t + \frac{2\pi n}{N}))$$

$$\cdot \text{sinc}\left(\frac{4\pi}{\lambda} \frac{(L_2 - L_1)}{2} \cos(\theta) \sin(\omega_c t + \frac{2\pi n}{N})\right)$$

$$\cdot j^{N-n-1} e^{j\frac{4\pi}{\lambda}(\sigma + \frac{L_1 + L_2}{2} \cos(\theta) \sin(\omega_c t + \frac{2\pi n}{N}))}$$

(2.57)

where \(A_r' = A_r(L_2 - L_1)\).

\(\alpha = \sin(|\theta| + \phi_p) + \sin(|\theta| - \phi_p)\),

\(\beta = \text{sign}(\theta)(\sin(|\theta| + \phi_p) - \sin(|\theta| - \phi_p))\).
The Fourier transform of Equation (2.57) is difficult to derive and is more easily determined by modifying Equation (2.10).

Equation (2.10) can be modified to give

\[ v_r(t) = \sum_{n=0}^{N-1} \int \bar{A}_r(\alpha + \beta \cos(\omega_r t + \frac{2\pi n}{N})) e^{-\frac{4\pi}{\lambda} r \cos(\theta) \sin(\omega_r t + \frac{2\pi n}{N})} \, dr \]  

(2.58)

where \( A_r = A_r e^{j(\omega_r t + \omega_0 t - \frac{4\pi n}{\lambda})} \).

Equation (2.58) can be expressed as a Bessel series to give

\[ v_r(t) = \sum_{k=-\infty}^{\infty} \sum_{n=0}^{N-1} \int \bar{A}_r J_k\left(-\frac{4\pi}{\lambda} r \cos(\theta)\right) \]

\[ \cdot \left(\alpha + \frac{\beta}{2} \left(e^{j(\omega_r t + \frac{2\pi n}{N})} + e^{-j(\omega_r t + \frac{2\pi n}{N})}\right)e^{j(k-1)(\omega_r t + \frac{2\pi n}{N})} \right) \, dr \]  

(2.59)

Equation (2.59) can be rearranged to give

\[ v_r(t) = \sum_{k=-\infty}^{\infty} \sum_{n=0}^{N-1} \int \bar{A}_r J_k\left(-\frac{4\pi}{\lambda} r \cos(\theta)\right) \]

\[ \cdot \left(\alpha J_{k+1} - \frac{\beta}{2} \left(e^{j(k+1)(\omega_r t + \frac{2\pi n}{N})} + e^{-j(k+1)(\omega_r t + \frac{2\pi n}{N})}\right) \right) \, dr \]  

(2.60)

Equation (2.60) can be rearranged to give

\[ v_r(t) = \sum_{k=-\infty}^{\infty} \int \bar{A}_r J_k\left(-\frac{4\pi}{\lambda} r \cos(\theta)\right) \]

\[ \cdot \left(\alpha J_{k+1} - \frac{\beta}{2} \left(e^{j(k+1)(\omega_r t + \frac{2\pi n}{N})} + e^{-j(k+1)(\omega_r t + \frac{2\pi n}{N})}\right) \right) \, dr \]  

(2.61)

Equation (2.61) can be simplified to give

\[ v_r(t) = \sum_{k=-\infty}^{\infty} \int \bar{A}_r N \]

(2.62)
Equation (2.62) can be solved to give

\[ \nu_r (t) = \sum_{k=-\infty}^{\infty} \sum_{l=0}^{\infty} \frac{2(-1)^{nk+l}(\alpha(J_{N|k|} + (c_{N|k+1|} + c_{N|k-1|})))}{4\pi \lambda \cos(\theta)} e^{j(\omega_r \nu - \omega_d + Nk f_r)} \]  

Equation (2.63) can be rearranged to give

\[ \nu_r (t) = \sum_{k=-\infty}^{\infty} \alpha c_{N|k|} \left( \frac{\beta}{2} (c_{N|k+1|} + c_{N|k-1|}) \right) e^{j(\omega_r \nu - \omega_d + Nk f_r)} \]  

where

\[ c_{N|k+1|} = \sum_{l=0}^{\infty} \frac{2(-1)^{nk+l}(\alpha(J_{N|k|} + (c_{N|k+1|} + c_{N|k-1|})))}{4\pi \lambda \cos(\theta)} \]

The Fourier transform of Equation (2.64) is given by

\[ V_r (f) = \sum_{k=-\infty}^{\infty} \left( \alpha c_{N|k|} + \frac{(-1)^{nk} \beta}{2} (c_{N|k+1|} + c_{N|k-1|}) \right) \delta (f - f_e - f_d - Nk f_r) \]  

Again, it is possible to make some simplifying assumptions. In practice, it is found that \( J_k (x) \approx 0 \) if \( k > x \), and \( x \gg 1 \). Therefore, if we assume that \( x \gg 1 \), then Equation (2.65) can be modified to give

\[ V_r (f) = \sum_{k=-\frac{N_2}{2}}^{\frac{N_2}{2}} \left( \alpha c_{N|k|} + \frac{(-1)^{nk} \beta}{2} (c_{N|k+1|} + c_{N|k-1|}) \right) \delta (f - f_e - f_d - Nk f_r) \]  

53
where

\[
c_n^{k+1} = \sum_{l=0}^{N_3} \frac{2(-1)^{n+k}(0)\lambda}{4\pi N_2 \cos(\theta)} \left( J_{N|k|+2l+2} \left( \frac{4\pi \lambda}{L_2 \cos(\theta)} \right) - J_{N|k|+2l+2} \left( \frac{4\pi \lambda}{L_1 \cos(\theta)} \right) \right) e^{-j\frac{4\pi R}{\lambda}},
\]

\[
c_n^{k-1} = \sum_{l=0}^{N_3} \frac{2(-1)^{n+k}(0)\lambda}{4\pi N_2 \cos(\theta)} \left( J_{N|k|+2l} \left( \frac{4\pi \lambda}{L_2 \cos(\theta)} \right) - J_{N|k|+2l} \left( \frac{4\pi \lambda}{L_1 \cos(\theta)} \right) \right) e^{-j\frac{4\pi R}{\lambda}},
\]

\[
N_2 = \frac{-N|k| - 2}{2} + 1,
\]

\[
N_3 = \left\lfloor \frac{-N|k|}{2} + 1 \right\rfloor.
\]

It can be seen that Equations (2.57) and (2.66) are similar to the equations which describe AM, i.e. [46]

\[
g(t) = A \left( 1 + \alpha_m \cos(\omega_m t) \right) \cos(\omega_c t)
\]  

(2.67)

and

\[
G(f) = A' \left( \delta(f + f_c) + \frac{\alpha_m}{2} (\delta(f + f_c + f_m) + \delta(f + f_c - f_m)) \right)
\]  

\[
+ \delta(f - f_c) + \frac{\alpha_m}{2} (\delta(f - f_c + f_m) + \delta(f - f_c - f_m))
\]  

(2.68)

where \( A' = \frac{A}{2} \).

In fact, the periodic variation of the cross section of each scatterer on each blade will cause pure AM of the return signal, therefore this is an electromechanical method of generating AM (with PM superimposed, due to the rotation of the scatterer). Also, as with PM, since there is more than one blade on the rotor, this will cause a phase angle between the scatterers on different blades. The result of this is that Equations (2.11) and (2.25) will be modified to give Equations (2.57) and (2.66), respectively.

When \( |\beta| = 0 \), this corresponds to the case in which \( \phi_p = 0 \), and is equivalent
to there being no AM. In this case, Equations (2.57) and (2.66) can be reduced to give Equations (2.11) and (2.25), respectively. When \(0 < |\beta| < \alpha\), this corresponds to the case in which \(||\theta| - \phi_p|| > 0\), and is equivalent to there being AM with a modulation index of less than 100%. When \(|\beta| = \alpha\), this corresponds to the case in which \(|\theta| = \phi_p\), and is equivalent to there being AM with a modulation index of 100%. In this case, Equations (2.57) and (2.66) can be modified to give

\[
v_c(t) = \sum_{n=0}^{N-1} A_n \alpha(1 + \text{sign}(\theta)\cos(\omega, t + \frac{2\pi n}{N}))
\]

\[
\cdot \sin\left(\frac{4\pi}{\lambda} \frac{(L_2 - L_1)}{2} \cos(\theta)\sin(\omega, t + \frac{2\pi n}{N})\right)
\]

\[
\cdot e^{j(\omega, t + \frac{4\pi}{\lambda} (k \times t - \frac{L_1 - L_2}{2} \cos(\theta)\sin(\omega, t + \frac{2\pi n}{N}))}
\]

and

\[
V_r(f) = \sum_{k=-N_2/2}^{N_2/2} \alpha(c_{N_k} + \text{sign}(\theta) \frac{(-1)^k}{2} (c_{N_k-1} + c_{N_k-1})) \delta(f - f_c - f_d - Nkf)
\]

Equation (2.70) can be simplified to give

\[
V_r(f) = \sum_{k=0}^{N_2/2} 2\alpha c_{N_k} \delta(f - f_c - f_d + \text{sign}(\theta)Nkf)
\]

since

\[
\sum_{k=0}^{w} J_{2k+1}(x) = \sum_{k=0}^{w} \frac{1}{2} (J_{2k+2}(x) + J_{2k}(x))
\]

Note the significance of Equation (2.71). When the modulation index is 100%, the magnitudes of the lower sidebands will be a maximum and those of the upper sidebands will be approximately zero, or the magnitudes of the upper sidebands will be a maximum and those of the lower sidebands will be approximately zero, depending on whether \(\theta\) is positive or negative, respectively.

Figures 2.19 and 2.20 show the real and imaginary components, respectively, of the return signal from the propeller of an aircraft in which \(N=2\), \(L_1=0.25\) m,
Figure 2.19.
Real component of the return signal from the propeller of an aircraft in which $N=2$, $L_1=0.25$ m, $L_2=2$ m, $f_r=16$ Hz, $\theta=\pi/4$ rad., and $\lambda=1$ m, as $\phi_p$ is increased from 0 to $\pi/4$ rad.
Figure 2.20.
Imaginary component of the return signal from the propeller of an aircraft in which $N=2$, $L_1=0.25$ m, $L_2=2$ m, $f_r=16$ Hz, $\theta=\pi/4$ rad., and $\lambda=1$ m, as $\phi_p$ is increased from 0 to $\pi/4$ rad.
\[ L_2=2 \text{ m}, f_r=16 \text{ Hz}, \theta=\pi/4 \text{ rad.}, \text{ and } \lambda=1 \text{ m}, \text{ i.e. the aircraft is heading towards the radar, as } \phi_p \text{ is increased from 0 to } \pi/4 \text{ rad.}, \text{ i.e. as the modulation index is increased from 0 to } 100\%. \text{ It can be seen that as the modulation index is increased, blade flashes begin to appear in the imaginary component of the return signal until, when the modulation index is } 100\%, \text{ the magnitudes of the blade flashes are a maximum. It can also be seen that when the modulation index is greater than 0\%, } v_r(t) \text{ is complex, the real component being even and periodic, with } \Delta t=1/Nf_r, \text{ and the imaginary component being odd and periodic, with } \Delta t=1/Nf_r. \]

Figure 2.21 shows the frequency spectrum of the aircraft of Figure 2.19. It can be seen that as the modulation index is increased, the magnitudes of the upper sidebands are reduced until, when the modulation index is 100\%, the magnitudes of the upper sidebands are approximately zero. It can also be seen that when the modulation index is greater than 0\%, } V_r(f) \text{ is real and unsymmetrical.}

Figure 2.22 shows the frequency spectrum of an aircraft which is identical to that of Figure 2.19, except that in this case } \theta=\pi/4 \text{ rad.}, \text{ i.e. the aircraft is heading away from the radar. In this case, it can be seen that as the modulation index is increased, the magnitudes of the lower sidebands are reduced until, when the modulation index is 100\%, the magnitudes of the lower sidebands are approximately zero.}

Figures 2.23 and 2.24 show the real and imaginary components, respectively, of the return signal from the propeller of an aircraft which is identical to that of Figure 2.19, except that in this case } N=3. \text{ In each case, it can be seen that as the modulation index is increased, the magnitudes of alternate blade flashes are reduced until, when the modulation index is 100\%, the magnitudes of alternate blade flashes are approximately zero. It can also be seen that when the modulation index is greater than 0\%, } v_r(t) \text{ is complex, the real component being even and periodic, with } \Delta t=1/Nf_r, \text{ and the imaginary component being odd and periodic, with } \Delta t=1/Nf_r.
Frequency spectrum of the propeller of an aircraft in which $N = 2$, $L_1 = 0.25$ m, $L_2 = 2$ m, $f_r = 16$ Hz, $\theta = \pi/4$ rad., and $\lambda = 1$ m, as $\phi_p$ is increased from 0 to $\pi/4$ rad.
Figure 2.22.
Frequency spectrum of the propeller of an aircraft in which $N=2$, $L_1=0.25$ m, $L_2=2$ m, $f_r=16$ Hz, $\theta=-\pi/4$ rad., and $\lambda=1$ m, as $\phi_p$ is increased from 0 to $\pi/4$ rad.
Real component of the return signal from the propeller of an aircraft in which $N=3$, $L_1=0.25$ m, $L_2=2$ m, $f_r=16$ Hz, $\theta=\pi/4$ rad., and $\lambda=1$ m, as $\phi_p$ is increased from 0 to $\pi/4$ rad.
Imaginary component of the return signal from the propeller of an aircraft in which $N=3$, $L_1=0.25$ m, $L_2=2$ m, $f_r=16$ Hz, $\theta=\pi/4$ rad., and $\lambda=1$ m, as $\phi_p$ is increased from 0 to $\pi/4$ rad.

Figure 2.24.
Figure 2.25 shows the frequency spectrum of the aircraft of Figure 2.23. It can be seen that as the modulation index is increased, the magnitudes of the upper sidebands are reduced until, when the modulation index is 100%, the magnitudes of the upper sidebands are approximately zero. It can also be seen that when the modulation index is greater than 0%, $V_r(f)$ is real and unsymmetrical.

Figure 2.26 shows the frequency spectrum of an aircraft which is identical to that of Figure 2.23, except that in this case $\theta = -\pi/4$ rad. In this case, it can be seen that as the modulation index is increased, the magnitudes of the lower sidebands are reduced until, when the modulation index is 100%, the magnitudes of the lower sidebands are approximately zero.

It can be seen from Figures 2.19-2.26 that when $\theta$ is positive, any AM will tend to reduce the magnitudes of the blade flashes and the upper sidebands when the blades are rotating towards the radar, and increase the magnitudes of the blade flashes and the lower sidebands when the blades are rotating towards the radar. Conversely, when $\theta$ is negative, any AM will tend to reduce the magnitudes of the blade flashes and the lower sidebands when the blades are rotating away from the radar, and increase the magnitudes of the blade flashes and the upper sidebands when the blades are rotating towards the radar. This is what we would intuitively expect: when $\theta$ is positive, the cross section of each blade will be smaller when the blade is rotating towards the radar, and will be greater when the blade is rotating away from the radar, therefore the magnitudes of the blade flashes will be smaller when the blades are rotating towards the radar, and will be greater when the blades are rotating away from the radar, and the magnitudes of the upper sidebands will be smaller than those of the lower sidebands. Conversely, when $\theta$ is negative, the cross section of each blade will be greater when the blade is rotating towards the radar, and will be smaller when the blade is rotating away from the radar, therefore the magnitudes of the blade flashes will be greater when the blades are rotating towards the radar, and will be smaller when the blades are rotating away from the radar, and the magnitudes of the upper sidebands will be greater than those of the
Figure 2.25.
Frequency spectrum of the propeller of an aircraft in which \( N=3, L_1=0.25 \text{ m}, \ L_2=2 \text{ m}, \ f_r=16 \text{ Hz}, \ \theta=\pi/4 \text{ rad.}, \) and \( \lambda=1 \text{ m}, \) as \( \phi_p \) is increased from 0 to \( \pi/4 \) rad.
Figure 2.26.
Frequency spectrum of the propeller of an aircraft in which \( N = 3 \), \( L_1 = 0.25 \) m, \( L_2 = 2 \) m, \( f_r = 16 \) Hz, \( \theta = -\pi/4 \) rad., and \( \lambda = 1 \) m, as \( \phi_p \) is increased from 0 to \( \pi/4 \) rad.
lower sidebands.

The results above highlight an error which has appeared in the literature. Some authors have claimed that if there is any AM, then this will reduce the magnitudes of the upper sidebands [47]. However, it has been shown above that if there is any AM then either the upper or lower sidebands can be reduced, depending on whether \( \theta \) is positive or negative, respectively. It is likely that this error has occurred because real data was analysed, for the case in which the aircraft was heading toward the radar, i.e. the error occurred because a limited amount of real data was analysed without a full understanding of the underlying theory, therefore only part of the full explanation of the phenomenon was given.

Note that at some aspect angles the propeller may he shielded, to some extent, by the airframe. This may also cause a reduction of the magnitude of the upper or lower sidebands, depending on the aspect angle, the position of the propeller on the airframe, the shape of the airframe, etc.

From the results above we can make some general observations, regarding the return signal and frequency spectrum of rotating aircraft blades when there is AM of the return signal. In general:

When \( N \) is even:

\[ v_r(t) \] is complex, the real component being even and periodic, with

\[ \Delta t = \frac{1}{N f_r}, \] and the imaginary component being odd, with \( \Delta t = \frac{1}{N f_r} \).

\[ V_r(f) \] is real and unsymmetrical.

When \( N \) is odd:

\[ v_r(t) \] is complex, the real component being even and periodic, with

\[ \Delta t = \frac{1}{N f_r}, \] and the imaginary component being odd and periodic, with \( \Delta t = \frac{1}{N f_r} \).

\[ V_r(f) \] is real and unsymmetrical.

Again, the results above apply in the case where \( v_r(t) \) and \( V_r(f) \) are continu-
ous signals. In the case where $v_r(t)$ and $V_r(f)$ are discrete, the results will be modified slightly.

Note that the results above will cause some of the results of Section 2.2 to be modified. More specifically, those equations which involve the number of sidebands, or the bandwidth of the sidebands, caused by rotating aircraft blades, will be modified to account for any AM of the return signal. In the case where the modulation index is 100%, these values will be halved.

The analysis above assumes that the effect of the blade twist on the return signal will be small in comparison with that of the blade pitch, and therefore can be ignored. In this case, the cross section of each section of a given blade at a given time will be equal. When there is AM of the return signal, this will have the effect of reducing the magnitudes of the upper or lower sidebands uniformly, i.e. the magnitude of each upper or lower sideband will be reduced by the same amount. However, as well as having blade pitch, most aircraft blades will also have blade twist. In this case, the cross section of each section of a given blade at a given time will vary with the distance of the section from the centre of rotation. This will have the effect of reducing the magnitudes of the upper or lower sidebands non-uniformly, i.e. the magnitude of each sideband will be reduced by an amount which depends on the distance of the corresponding section of blade from the centre of rotation.

The results above may affect the implementation of any feature extraction and aircraft classification algorithms, since it has been shown that if there is any AM of the return signal, then this may reduce the magnitudes of the blade flashes and the upper or lower sidebands.

2.5 Practical Considerations

This section discusses some of the ways in which practical aircraft blades and return signals will differ from theoretical aircraft blades and return signals, respec-
2.5.1 Practical Aircraft Blades

In the analysis above, it was assumed that each blade acts as a homogeneous, linear, rigid antenna. In practice, aircraft blades are very complex structures, and this will affect the return signal in various ways. In the case where each blade does not act as a homogeneous, linear, rigid antenna, Equation (2.11) can be modified to give

\[ v_r(r) = \sum_{n=0}^{N-1} \int_{L_1}^{L_2} f(r, t, \theta) e^{j(\omega_r t - \frac{4\pi}{\lambda}(\theta + r \cos(\theta) \sin(\omega_r t + \frac{2\pi}{N})))} dr \]  

where \( f(r, t, \theta) \) = function of the distance of each incremental chord-wise section of blade from the centre of rotation, of the angular position of each blade, with respect to time, and of the aspect angle of the rotor or propeller.

In practice, each blade is a rotating aerofoil, which enables the blade to generate thrust in exactly the same way as any other aerofoil. As with any other aerofoil, the blades will have camber, taper, twist, etc. Many different blade shapes have been investigated - much of the early work having been done by the National Advisory Committee for Aeronautics (NACA), now called the National Aeronautics and Space Administration (NASA) - and almost no two blade designs will be exactly the same.

It was noted in Section 2.4 that most aircraft blades will have blade twist. The planform (i.e. the plan view) of most blades will also have a slight taper, in which the chord of the blade is reduced linearly with distance from the centre of rotation. The shape of most aircraft blades will also change near to the blade roots and the blade tips. At the blade roots, the mechanical cross section of the blades will be round, in most cases, since the main requirement at the blade roots is mechanical strength, not thrust. At the blade tips, the planform of the blades will be rounded,
in most cases, in order to reduce compressibility effects, etc. Some advanced blades will also have swept or paddle tips, in order to reduce these effects further when the blade tips are rotating at high subsonic speeds. Figure 2.27 shows some advanced rotor blade tip shapes. It is likely that any variations in the shape of the blades will have little effect on the return signal from the blades.

In the analysis above it was assumed that each blade extends radially from the axis of rotation and lies on the plane of rotation. Figure 2.28 shows a typical hinge arrangement for connecting the rotor blades to the rotor shaft of a rotor aircraft. (The hinges do not have to be in the order shown.) It can be seen that each blade has three degrees of movement, viz. movement in the plane containing the blade and shaft, called flapping; movement in the plane of rotation, called lagging, and movement about the axis along the blade span, called feathering. These degrees of movement will be exercised in practice, and will have the effect of adding a mechanically-induced noise component to the return signal from the blades.

When in flight, the main rotors of all rotor aircraft will have a coning angle, \( \phi_c \), in which the rotor blades are raised slightly from the plane of rotation passing through the centre of rotation, such that they form a shallow cone shape. The coning angle will be of about 5\(^\circ\), and will be weight-dependent - the greater the weight of the aircraft, the greater the angle will be. The coning angle will only vary by about \( \pm 1^\circ \) between the maximum and minimum weights of the aircraft, and for a given weight will be constant for all flight conditions. In order to account for the coning angle, all of the previous equations involving the term \( \cos(\theta) \) should be replaced by the term \( \cos(\theta)\cos(\phi_c) \). However, since the coning angle is always small, the \( \cos(\phi_c) \) term can be ignored (since \( \cos(\phi_c) \approx 1 \)).

All rotor aircraft will have a rotor hub which rotates with the rotor blades and the rotor shaft. The rotor hub is a mechanical structure which connects the rotor blades to the rotor shaft. Rotor hubs can be very complex structures, and if the number of blades is large, or if the hub incorporates blade folding, then they can also be very bulky. Figures 2.29 and 2.30 show diagrams of the fully-articulated...
Figure 2.27.
Some advanced rotor blade tip shapes.
Figure 2.28.
Typical hinge arrangement for connecting the rotor blades to the rotor shaft.
Figure 2.29.
Articulated rotor hub.
Figure 2.30.
Semi-rigid rotor hub.
rotor hub of the Westland Sea King, and the semi-rigid rotor hub of the Westland Lynx aircraft, respectively. In practice, the rotor hub will also modulate the return signal. However, in this case the modulation will be very complex, because of the complex shape of the hub, and, as with the flapping, lagging and feathering movement of the blades, will have the effect of adding a mechanically-induced noise component to the return signal from the blades. This noise will only affect the sidebands which are nearest to the centre frequency of the aircraft, since the hub is located near to the centre of rotation [48].

Little mention has been made of tail rotors. By Newton’s Third Law, when the main rotor is rotating it will tend to rotate the airframe in the opposite direction. This is called torque reaction, and the tail rotor is used mainly to counteract this reaction. Most rotor aircraft have simple tail rotors, in which the tail rotor is similar to the main rotor. Some rotor aircraft have fenestron, or fan-in-fin tail rotors. With fenestron tail rotors, the rotor is shrouded, i.e. the rotor is enclosed in a sheath built into the tail cone of the aircraft. Fenestron tail rotors have various aerodynamic advantages over simple tail rotors, and are also safer on the ground. Fenestron tail rotors usually have a large number of short blades.

At some aspect angles, the tail rotor will also be visible to the radar. This will have the effect of adding the return signal from the tail rotor to that of the airframe and main rotor. The return signal from the tail rotor will be smaller than that from the main rotor, since the tail rotor is smaller than the main rotor. In the case of fenestron tail rotors, at some aspect angles the tail rotor may be shielded from the radar by the fenestron. The shielding will be greatest when the line of sight from the radar to the centre of rotation is in the longitudinal-normal plane of the aircraft, since with fenestron tail rotors, the rotor will be mounted along the longitudinal axis of the aircraft.

Note that some rotating objects will not tend to modulate the return signal. In general, a rotating object will not tend to modulate the return signal if it is a body of revolution about its axis of rotation, e.g. a flat disc, a cylinder, a sphere, etc.
Again, this is why there is usually a frequency notch about the centre frequency of the aircraft, in the case of propeller aircraft - because the propeller spinner is a body of revolution about its axis of rotation.

2.5.2 Practical Return Signals

As a first approximation, the airframe and each section of blade can be considered as consisting of a single point scatterer, having a single cross section, range, velocity, etc., therefore, the airframe line and each sideband will consist of a single spectral line. In practice, the airframe and each section of blade will consist of many scatterers, each having random variations of its cross section, range, velocity, etc.

The random variations are due mainly to random variations of the relative position of the aircraft, which is caused by random motion, turbulence, vibration, etc. [49]. The random motion of the aircraft can be resolved into six components: random roll, pitch and yaw, i.e. rotational motion about the longitudinal, transversal and normal axes of the aircraft, respectively; and random surge, sway and heave, i.e. translational motion about these three axes, respectively.

The random variations will have three main effects on the return signal and frequency spectrum of the rotating aircraft blades:

1. They will cause random variations of the magnitudes of the blade flashes, which will affect the symmetry of the return signal.
2. They will cause random variations of the magnitudes of the sidebands, which will affect the symmetry of the frequency spectrum.
3. They will cause a Doppler frequency spread of each sideband, the variance of which will depend mainly on the variance of the relative velocity, and the radar wavelength.

The random variations of the cross section of the aircraft can be approximated by Swerling's fluctuation models [50]. Similarly, the random variations of the range and velocity of the aircraft can be approximated by a Gaussian distribution [51].
Figure 2.31 shows the frequency spectrum of an aircraft in which $N = 4$, $L_2 = 5$ m, $f_r = 6$ Hz, $v = -256$ m/s, and $\lambda = 1$ m, when the aircraft and blades consist of many scatterers, each having random variations of its cross section, range, velocity, etc. It can be seen that the frequency spectrum is asymmetrical, due to the random variations, and that both the airframe line and the sidebands have a Doppler frequency spread, due to the random variations of the velocity.

2.6 Conclusions

This chapter has presented a detailed mathematical analysis of the return signal from rotating aircraft blades. It has been shown that the modulation caused by the rotation of the blades is a form of PM, that the frequency spectrum consists of an infinite number of sidebands, and that each sideband consists of an infinite series of Bessel functions. It has also been shown that the frequency spectrum can be approximated by a finite number of sidebands, and that each sideband can be approximated by a finite series of Bessel functions. It has also been shown that when $L_1 \neq 0$, the sidebands which are nearest to the centre frequency will be approximately zero, causing a frequency notch about the centre frequency. The greater the value of $L_1$, the greater the notch will be. It has further been shown that the values of $N_1$, $N_2$, $\Delta f$, $B_1$ and $B_2$ are ambiguous, and that there are an infinite number of sets of $N$, $L_1$, $L_2$ and $f_r$ which provide a solution set of $N_1$, $N_2$, $\Delta f$, $B_1$ and $B_2$, although in practice the number of sets will be limited because the extent to which the aircraft parameters for different types of aircraft can vary will be limited.

The chapter has also presented a detailed mathematical analysis of the effects of blade pitch on the return signal. It has been shown that if there is any blade pitch, then this will tend to cause a periodic variation of the cross section of each blade, which will cause AM of the return signal from each blade. It has also been shown that when $\theta$ is positive, any AM will tend to reduce the magnitudes of the blade flashes and the upper sidebands when the blades are rotating towards the
Figure 2.31.
Frequency spectrum of a rotor aircraft in which $N=4$, $L_2=5 \text{ m}$, $f_s=6 \text{ Hz}$, $\theta=0 \text{ rad.}$, and $\lambda=1 \text{ m}$, when the airframe and blades consist of many different scatterers, each having random variations of its velocity.
radar, and increase the magnitudes of the blade flashes and the lower sidebands when the blades are rotating away from the radar. Conversely, when $\theta$ is negative, any AM will tend to reduce the magnitudes of the blade flashes and the lower sidebands when the blades are rotating away from the radar, and increase the magnitudes of the blade flashes and the upper sidebands when the blades are rotating towards the radar. It has further been shown that when the modulation index is 100%, the magnitudes of the lower sidebands will be a maximum and those of the upper sidebands will be approximately zero, or the magnitudes of the upper sidebands will be a maximum and those of the lower sidebands will be approximately zero, depending on whether $\theta$ is positive or negative, respectively. This may affect the performance of any feature extraction or aircraft classification algorithms.

The chapter has also discussed some of the ways in which practical aircraft blades and return signals differ from theoretical aircraft blades and return signals, respectively. It has been shown that practical aircraft blades are very complex structures, and that this will affect the return signal in various ways. It has also been shown that some rotating objects will not tend to modulate the return signal, and that, in general, a rotating object will not tend to modulate the return signal if it is a body of revolution about its axis of rotation. It has further been shown that the airframe and each section of blade will consist of many scatterers, each having random variations of its cross section, range, velocity, etc., and that this will cause random variations of the magnitudes of the blade flashes and the sidebands, and will also cause a Doppler frequency spread of each sideband.
CHAPTER THREE

AIRCRAFT FEATURES

3.1 Introduction

In Chapter 2 it was shown that the blade features are ambiguous, i.e. there is a many-to-one mapping of the blade parameters to the blade features, such that it is not possible to classify a given aircraft uniquely. By using information on the physical configurations and physical features of actual aircraft, it is possible to reduce these ambiguities, and in some cases may make it possible to classify a given aircraft uniquely.

This chapter presents a detailed survey of the physical configurations, physical parameters and radar features of rotor and propeller aircraft ranging in size from light to heavy aircraft. The chapter forms an essential link between Chapters 2, 4 and 5, and is of interest with respect to the implementation of feature extraction and aircraft classification algorithms.

The chapter only discusses the features of aircraft which are currently in service. These aircraft consist of those which are currently in production, and those which are no longer in production, but which are still in service. Historical and experimental aircraft are not considered, although most of these aircraft will have features which are similar to those of the aircraft which are considered, therefore making the task of classifying these aircraft similar to that of classifying those which are discussed, because of the similarity. Also, hybrid and unusual aircraft are not discussed, although most of these aircraft will have features which are dissimilar to those of the aircraft which are considered, therefore making the task of classifying these aircraft more simple than that of classifying those which are discussed,
because of the dissimilarity.

In some cases, different versions of the same aircraft will have different features. This may affect the implementation of any feature extraction or aircraft classification algorithms, since it would be desirable for any such algorithms to be able to classify different versions of the same aircraft which have different features, as being the same aircraft. Conversely, in some cases, different aircraft will have the same features. This may also affect the implementation of such algorithms, since it would be desirable for any such algorithms to be able to classify different aircraft which have the same features, as being different aircraft.

Most of the features which are discussed in the chapter are subject to change, as new aircraft are constantly being developed, using new technology and new applications of old technology. Some of these developments may change the maximum range of values of a given feature for a given class of aircraft. This too may affect the implementation of any feature extraction or aircraft classification algorithms, since it would be desirable for any such algorithms to be able to deal with any future developments in aircraft technology which change the maximum range of values of any feature for any class of aircraft.

Most of the technical information in the chapter was obtained from Jane's All the World's Aircraft, which is published yearly by Jane's Information Group Limited. This yearbook contains information on all aircraft and aircraft engines which are currently in production or under development, and is internationally considered to be the most important aircraft reference book [52]. Most of the information on each aircraft is provided by the manufacturer, and (for rotor aircraft) includes such information as: the number of main rotor blades, the length of the main rotor blades, the frequency of rotation of the main rotor, etc. The information on each aircraft is presented alphabetically in the order of country of origin, manufacturer and aircraft, and the information on each engine is presented separately in the same order.
In this chapter, we are interested not in the features of each individual aircraft, but in the maximum range of values of each feature for each class of aircraft. Unfortunately, *Jane's All the World's Aircraft* does not contain this information separately. For example, the information on each rotor aircraft includes the number of main rotor blades, but there is no information on the maximum range of the number of main rotor blades for rotor aircraft as a class. This information can only be obtained by checking the information on each individual rotor aircraft.

Various types of aircraft are discussed in the chapter, and in the case of Soviet aircraft, North Atlantic Treaty Organisation (NATO) Air Standards Co-ordinating Committee (ASCC) reporting names are used. These names are used by NATO to enable Soviet aircraft to be reported simply and unambiguously, and to sound different, even over a poor communications channel [53]. Each name consists of a one- or two-syllable word, the first letter of which indicates the type of aircraft [54]. Bomber, carrier, fighter, helicopter and miscellaneous aircraft are indicated by names beginning with "B", "C", "F", "H" and "M", respectively, e.g. Backfin, Cab, Faceplate, Halo, Madge, etc. Rotor aircraft are indicated by one- or two-syllable words, e.g. Halo, Hare, Homer, etc.; propeller aircraft are indicated by one-syllable words, e.g. Bank, Barge, Bark, etc.; and jet aircraft are indicated by two-syllable words, e.g. Backfin, Backfire, Badger, etc. Different versions of the same aircraft are indicated by one-letter suffixes, e.g. Badger-A, Badger-B, Badger-C, etc.

Because the chapter only discusses aircraft ranging in size from light to heavy aircraft, this means that microlight aircraft are not discussed. A microlight aircraft is defined as an aircraft which has a wing area not exceeding 10 m², an empty weight not exceeding 150 kg, a wing loading not exceeding 10 kg/m² at empty weight, and which is designed to carry no more than two people in or on the aircraft [55]. Because of this, microlight aircraft are smaller, lighter and less powerful than light aircraft.

It may seem natural not to discuss microlight aircraft, however, some armed forces and Palestinian liberation organisations have investigated the use of these
aircraft in recent years.

The Soviet armed forces first investigated the use of microlight aircraft in the mid-1980s as a method of carrying out special operations. The UK and USA armed forces have also investigated the use of microlight aircraft for this purpose [56].

Microlight aircraft are currently in service with the Netherlands armed forces for special operations and for air base perimeter surveillance. An average-sized air base has a perimeter of about 20 km. A microlight aircraft can patrol the perimeter of such an air base at low level in about 15 minutes, and can free a rotor aircraft from having to do this task. Microlight aircraft are also in service with the Indian armed forces for surveillance of the Indian-Pakistani border.

Palestinian liberation organisations first investigated the use of microlight aircraft in the early 1980s as a method of carrying out military attacks on Israel, and have already used these aircraft in such attacks. For example, microlight aircraft were used by the Palestinian Liberation Front (PLF) in March 1981, and by the Popular Front for the Liberation of Palestine - General Command (PFLP-GC) in November 1987, in attacks on Israel. Microlight aircraft have also been used as suicide bomber aircraft by Shi'ite Militia in Lebanon since May 1989 [57].

Microlight aircraft can be used for attack, command and control, as well as for fire correction, reconnaissance and surveillance. They also have very low cross sections, are very small, are very quiet, and are also very light, portable and inexpensive [58].

Threat analysis of microlight aircraft has shown that they are comparable in vulnerability to rotor aircraft. Also, although the aircrew are very exposed, microlight aircraft can fly very low in nap-of-the-earth (NOE) flight operations.

Microlight aircraft are simple to buy, as they are not considered to be military aircraft, therefore they do not require export licences and end-user certificates. Also, most companies which manufacture microlight aircraft sell them through
agents, and are often unaware of the identities of the buyers.

Microlight aircraft are simple to learn to fly. It has been estimated that it takes about 10 hours for pilots, and about 20 hours for non-pilots to learn to fly microlight aircraft, whereas it takes about 40 hours for non-pilots to learn to fly light aircraft.

It is obvious from the points above that in the future there may be a need to classify microlight aircraft. Although microlight aircraft are not discussed further in this chapter, all of the theory and results in the chapter can also be extended to apply to these aircraft.

Because of the nature of this chapter, i.e. the collecting and collating of information for different types of aircraft, the chapter is somewhat repetitive, since the same task is being repeated for main rotors, tail rotors and propellers.

The structure of the chapter is as follows.

Section 3.2 discusses the features of rotor aircraft. It discusses those features which relate to the aircraft in general, and those which relate to the rotating aircraft blades in particular. It also gives the maximum range of values of each of these features. Some attempt is made to indicate why certain types of configuration are used, and what the main advantages and disadvantages of each type of configuration are. Some attempt is also made to indicate how common or uncommon each type of configuration is. It is believed that this is the first time that such a detailed work on the features of rotor aircraft has appeared in the literature.

The features of propeller aircraft are discussed in Section 3.3. The structure of this section is identical to that of Section 3.2, except that, in this case, propeller aircraft are discussed. As in the case of rotor aircraft, it is believed that this is the first time that such a detailed work on the features of propeller aircraft has appeared in the literature.

Finally, Section 3.4 draws some conclusions.
3.2 Rotor Aircraft

Rotor aircraft have between one and four engines. The size of the engines is determined mainly by the vertical take-off requirements of the aircraft, and the number of engines is determined mainly by the need for redundancy, in order to meet the safety requirements of the aircraft. If there is more than one engine, then each one will be identical. This will enable each engine to generate identical output power for identical engine settings, and will also reduce noise and vibration. Most light aircraft have one engine, most heavy aircraft have two engines, and some heavy aircraft have three or four engines, although these are rare.

Rotor aircraft have either reciprocating or gas turbine engines. With reciprocating engines, the output power from each engine drives the main rotor shaft through a reduction gear. With gas turbine engines, the output power from each engine drives a power turbine, which then drives the main rotor shaft through a reduction gear [59]. When gas turbine engines are used to power rotor aircraft, they are called "turboshaft" engines.

Rotor aircraft have either main and tail, side-by-side tandem, in-line tandem, or contra-rotating rotors.

With main and tail rotors, one rotor is mounted on the fuselage, and the other is mounted on the tail, in order to prevent torque reaction. The main advantage of main and tail rotors is that the design of the aircraft is simpler. The main disadvantage of main and tail rotors is that they generate less lift.

With side-by-side tandem rotors, two rotor are mounted side-by-side on the fuselage, both rotating in opposite directions. The main advantage of side-by-side tandem rotors is that they generate more lift, because there are two main rotors. Also, they remove the need for a tail rotor, because the torque reactions from the two rotors act in opposite directions, and their effects cancel. The main disadvantage of side-by-side tandem rotors is that they must be mounted on outriggers, or mounted at an angle, in order to allow sufficient clearance between the two rotors.
This also increases the weight and drag of the aircraft [60].

With in-line tandem rotors, two rotors are mounted in-line along the fuselage, both rotating in opposite directions. As with side-by-side tandem rotors, the main advantage of in-line tandem rotors is that they generate more lift and remove the need for a tail rotor.

With contra-rotating rotors, two rotors are mounted on the rotor shaft, one above the other, and both rotating in opposite directions. The main advantage of contra-rotating rotors is that they enable more powerful engines to be used on the aircraft, because two rotors can absorb more power than one. Also, as with tandem rotors, they generate more lift and remove the need for a tail rotor.

Main rotors are fitted such that the axis of rotation is approximately parallel to the normal axis of the aircraft, therefore the plane of rotation will be approximately parallel to the longitudinal-transversal plane of the aircraft. Tail rotors are fitted such that the axis of rotation is approximately parallel to the transversal axis of the aircraft, therefore the plane of rotation will be approximately parallel to the longitudinal-normal plane of the aircraft.

Main rotors are fitted symmetrically about the longitudinal-normal plane of the aircraft, in order to generate symmetrical lift about this plane. Except in the case of side-by-side tandem rotors, this means that the main rotors will be fitted parallel to the longitudinal-normal plane of the aircraft. Tail rotors are also fitted parallel to the longitudinal-normal plane of the aircraft, although there may be a small offset, due to the tail cone. Most light aircraft have main and tail rotors, most heavy aircraft have in-line tandem or contra-rotating rotors, and some light aircraft have side-by-side tandem rotors, although these are rare. These four types of configuration, i.e. rotor aircraft with main and tail, side-by-side tandem, in-line tandem, and contra-rotating rotors are shown in Figures 3.1-3.4, respectively [61].

Simple main rotors have between two and eight blades. Most light aircraft have main rotors with between two and four blades, most heavy aircraft have main
Figure 3.1.
Rotor aircraft with main and tail rotors.
Figure 3.2.
Rotor aircraft with side-by-side tandem main rotors.
Figure 3.3.
Rotor aircraft with in-line tandem main rotors.
Figure 3.4.
Rotor aircraft with contra-rotating main rotors.
rotors with between four and six blades, and some heavy aircraft have main rotors with between five and eight blades, although these are uncommon. Tandem and contra-rotating main rotors have three blades on each rotor.

Tail rotors have between two and six, eleven or thirteen blades. Most light aircraft have tail rotors with between two and four blades, and most heavy aircraft have tail rotors with between four and six blades. Fenestron tail rotors have eleven or thirteen blades.

Values of $L_2$ for main rotors range from about 3.5 m for light aircraft to about 15 m for heavy aircraft. The largest value of $L_2$ for the main rotor of any aircraft which is currently in service is that of the Mil Mi-6 Hook and the Mil Mi-10 Harke, for which $L_2 = 17.5$ m.

Values of $L_2$ for tail rotors range from about 1 m for light aircraft to about 3 m for heavy aircraft. The largest value of $L_2$ for the tail rotor of any rotor aircraft which is currently in service is that of the Mil Mi-26 Halo, for which $L_2 = 3.8$ m. In general, the ratio of the values of $L_2$ between main and tail rotors will be about 5:1.

Values of $f_r$ for main rotors range from about 2 Hz for heavy aircraft to about 10 Hz for light aircraft.

Values of $f_r$ for tail rotors range from about 10 Hz for heavy aircraft to about 50 Hz for light aircraft. In general, the ratio of the values of $f_r$ between main and tail rotors will be about 1:5.

Most rotor aircraft have constant-speed rotors. With constant-speed rotors, the frequency of rotation of the main rotor is kept constant to within about $\pm 1\%$. This is achieved by a fuel computer which controls the engine fuel flow, in order to compensate automatically for any changes in the frequency of rotation of the main rotor. The main advantage of constant-speed rotors is that the engine always runs at its most efficient speed. Also, with constant-speed rotors, any change in output power is achieved almost instantly by changing the fuel flow to the engine, keeping the frequency of rotation of the engine constant, and changing the blade pitch of
the main rotor to absorb the correct amount of power, whereas if the change in output power were to be achieved by changing the frequency of rotation while keeping the blade pitch of the main rotor constant, it would take a significant amount of time for the frequency of rotation to change, due to the inertia of the engine, rotor hub, rotor blades, etc.

Values of $v_b$ for main rotors range from about 200 m/s to about 250 m/s.

Values of $v_b$ for tail rotors also range from about 200 m/s to about 250 m/s. In general, the blade tip speed of the tail rotor of any rotor aircraft will be approximately equal to that of the main rotor, therefore the ratio of the values of $v_b$ between main and tail rotors will be about 1:1 [62]. The tail rotor is powered by a take-off drive taken from the main rotor drive shaft, therefore the frequency of rotation of the tail rotor cannot be controlled separately from that of the main rotor, i.e. the frequency of rotation of the tail rotor is determined by that of the main rotor. Only the blade pitch of the tail rotor can be controlled separately, and this is done in order to control the amount of thrust generated by the tail rotor.

The relative blade tip speed of the main rotor will have two components: that due to the forward motion of the aircraft, and that due to the rotation of the rotor. Because the plane of rotation is approximately parallel to the direction of forward motion, the blade tips will have a cycloidal path, therefore the relative velocity of the blade tips will be given by

$$v_r = v + v_b \sin(\omega t)$$  \hspace{1cm} (3.1)

where $v_r =$ relative velocity of the blade tips.

The maximum relative velocity of the blade tips will be given by

$$v_{r_{\text{max}}} = v + v_b$$  \hspace{1cm} (3.2)

where $v_{r_{\text{max}}} =$ maximum relative velocity of the blade tips.

A similar analysis applies to the relative velocity of the blade tips of the tail rotor.
Values of $N_2$ for main rotors range from about $40\cos(\theta)/\lambda$ to about $60\cos(\theta)/\lambda$.

Values of $N_2$ for tail rotors range from about $10\cos(\theta)/\lambda$ to about $40\cos(\theta)/\lambda$.

Values of $\Delta f$ for main rotors range from about 10 Hz for heavy aircraft to about 20 Hz for light aircraft.

Values of $\Delta f$ for tail rotors range from about 50 Hz for heavy aircraft to about 100 Hz for light aircraft.

Values of $B_2$ for main rotors range from about $800\cos(\theta)/\lambda$ Hz to about $\cos(\theta)/\lambda$ kHz.

Values of $B_2$ for tail rotors range from about $800\cos(\theta)/\lambda$ Hz to about $\cos(\theta)/\lambda$ kHz.

All aircraft will have a maximum speed at which they can fly. The maximum speed of an aircraft, $v_{\text{max}}$, will be determined by the thrust, drag, lift and weight of the aircraft. Thrust and lift will tend to increase the maximum speed of the aircraft. Conversely, drag and weight will tend to reduce the maximum speed of the aircraft.

Values of $v_{\text{max}}$ range from about 50 m/s for light aircraft to about 100 m/s for heavy aircraft. The largest value of $v_{\text{max}}$ for any rotor aircraft which is currently in service is that of the Westland Lynx, for which $v_{\text{max}} = 111$ m/s.

All aircraft will have a maximum altitude, or ceiling, at which they can fly. The absolute ceiling of an aircraft, $h_{\text{max}}$, is the altitude at which the maximum rate of climb of the aircraft is 0 ft./min. [63]. At this altitude, the maximum lift of the aircraft is equal to the weight of the aircraft, therefore the aircraft cannot climb any higher. It takes a very long time for an aircraft to reach its absolute ceiling, and because of the very limited performance of the aircraft when flying at this altitude, i.e. the aircraft can only fly at a single speed, there is very little point in flying at this altitude. Because of this, the concept of the service ceiling is used. The service ceiling of an aircraft is the altitude at which the maximum rate of climb of the
aircraft is less than 100 ft./min. [64]. At this altitude the maximum lift of the aircraft is slightly greater than the weight of the aircraft, therefore the aircraft is only able to climb very slowly. Note that the absolute ceiling will always be greater than the service ceiling. Rotor aircraft can have two types of absolute and service ceiling: that when the aircraft is in forward motion, and that when the aircraft is hovering. Note that the absolute and service ceilings when the aircraft is in forward motion will be greater than those when the aircraft is hovering, since the aircraft can generate more lift when in forward motion that when hovering.

Values of $h_{\text{max}}$, range from about 2,500 m for light aircraft to about 5,000 m for heavy aircraft. The largest value of $h_{\text{max}}$ for any rotor aircraft which is currently in service is that of the Aerospatiale SA 315B Lama, for which $h_{\text{max}} = 12,442$ m.

### 3.3 Propeller Aircraft

A number of points which were made in Section 3.2 concerning rotor aircraft also apply to propeller aircraft. Where these points are obvious they will not be repeated, in order to avoid any unnecessary repetition.

Propeller aircraft have between one and four engines. As with rotor aircraft, the size of the engines is determined mainly by the thrust requirements of the aircraft, and the number of engines is determined mainly by the need for redundancy, in order to meet the safety requirements of the aircraft. If there is more than one engine, then each one will be identical. Most light aircraft have one or two engines, most heavy aircraft have two or four engines, and some aircraft have three engines, although these are rare.

Propeller aircraft have either reciprocating or gas turbine engines. With reciprocating engines, the output power from each engine drives a propeller shaft, possibly through a reduction gear. With gas turbine engines, the output power from each engine drives a power turbine, which then drives a propeller shaft through a reduction gear [65]. When gas turbine engines are used to power pro-
peller aircraft, they are called "turboprop" engines.

Propeller aircraft have between one and four propellers, one propeller for each engine. If there is more than one propeller, then each one will be identical. Propellers are fitted such that the axis of rotation is approximately parallel to the longitudinal axis of the aircraft, therefore the plane of rotation will be approximately parallel to the transversal-normal plane of the aircraft.

The propellers are fitted symmetrical about the longitudinal-normal plane of the aircraft, in order to generate symmetrical thrust about this plane. If there is one propeller, then it will be fitted in the nose, on top of the fuselage, or in the tail. If there are two or four propellers, then they will be fitted symmetrically on the wings. If there are three propellers, then two will be fitted symmetrically on the wings, and one will be fitted on the tail. Most light aircraft have one or two propellers, most heavy aircraft have two or four propellers, and some light aircraft have three propellers, although these are rare. These four types of configuration, i.e. propeller aircraft with one, two, three and four propellers are shown in Figures 3.5-3.8, respectively.

Propeller aircraft have either tractor or pusher propellers. Tractor propellers are fitted in front of the engine, and provide thrust which pulls the aircraft forward. Pusher propellers are fitted behind the engine, and provide thrust which pushes the aircraft forward. Tractor propellers have three main advantages:

1. They increased the balance and stability of the aircraft, because the weight of the engines is further forward in the aircraft.
2. They increase the efficiency of the propellers, because there are no other parts of the aircraft in front of the propellers.
3. They increase the elevator and rudder control of the aircraft, because the slipstream from the propellers passes over more of the control surfaces of the aircraft.
Figure 3.5.
Propeller aircraft with one propeller.
Figure 3.6.
Propeller aircraft with two propellers.
Figure 3.7.
Propeller aircraft with three propellers.
Figure 3.8.
Propeller aircraft with four propellers.
Pusher propellers also have three main advantages:

1. They reduce the drag of the aircraft, because the slipstream from the propellers passes over less of the aircraft.

2. They reduce the noise and vibration of the aircraft, again because the slipstream from the propellers passes over less of the aircraft.

3. They reduce the propeller-induced roll and yaw, because the slipstream from the propellers passes over less of the control surfaces of the aircraft.

Most propeller aircraft have tractor propellers, but some propeller aircraft have pusher propellers, although these are uncommon. It is interesting to note that, although most propeller aircraft have tractor propellers, all propeller-driven marine craft have pusher propellers [66].

Propeller aircraft have either simple or contra-rotating propellers. Most propeller aircraft have simple propellers, but some propeller aircraft have contra-rotating propellers, although these are uncommon.

Multi-engine propeller aircraft have either co-rotating or anti-rotating propellers. With co-rotating propellers, each propeller rotates in the same direction. With anti-rotating propellers, symmetrically-fitted propellers rotate in opposite directions. The main advantage of co-rotating propellers is that they reduce the production and maintenance costs of the aircraft, because each engine and propeller is identical. The main advantage of anti-rotating propellers is that they reduce the propeller-induced roll and yaw of the aircraft, because the slipstreams from symmetrically-fitted propellers rotate in opposite directions, and their effects cancel. Most propeller aircraft have co-rotating propellers, but some propeller aircraft have anti-rotating propellers, although these are uncommon.

Simple propellers have between two and six blades. Most light aircraft have propellers with two or three blades, most heavy aircraft have propellers with three or four blades, and some heavy propeller aircraft have five or six blades, although these are uncommon. Contra-rotating propellers have four blades on each
propeller, and these are also uncommon.

Values of $L_1$ range from about $L/8$ for light aircraft to about $L/6$ for heavy aircraft, therefore values of $L_1$ range from about 0.1 m for light aircraft to about 0.5 m for heavy aircraft. The largest value of $L_1$ for any propeller aircraft which is currently in service is probably that of the Antonov An-22 Cock, for which $L_1 = 0.5$ m.

Values of $L_2$ range from about 1 m for light aircraft to about 3 m for heavy aircraft. The largest value of $L_2$ for any propeller aircraft which is currently in service is that of the Antonov An-22 Cock, for which $L_2 = 3.1$ m.

Values of $f_r$ range from about 10 Hz for heavy aircraft to about 50 Hz for light aircraft.

Most propeller aircraft have constant-speed propellers. With multi-engine aircraft, each propeller will have approximately the same frequency of rotation, to within about ±1%. This will enable each propeller to generate identical thrust for identical throttle settings, and will also reduce noise and vibration.

Some multi-engine propeller aircraft have synchronised or synchrophased propellers. With synchronised propellers, each propeller will have the same frequency of rotation to a very high degree of accuracy. This is achieved by mounting an electrical synchronising generator on each engine to generate a signal which indicates the frequency of rotation of the engine. These signals are then compared, using one of the signals as a master signal, and any signal which is not equal to the master signal is automatically corrected by adjusting the frequency of rotation of the corresponding engine. With synchrophased propellers, not only will each propeller have the same frequency of rotation, but each propeller will also have the same angular position as the master propeller, i.e. each blade of each propeller will have the same angular position as the corresponding blade of the master propeller. This is achieved by mounting an electrical synchrophasing generator on each engine to generate a signal which indicates the phase of the engine. As with synchronised
propellers, these signals are then compared, using one of the signals as a master signal, and any signal which is not equal to the master signal is automatically corrected by adjusting the frequency of rotation of the corresponding engine [67].

Values of \(v_b\) range from about 200 m/s to about 300 m/s.

As with rotor aircraft, the relative blade tip speed will have two components: that due to the forward motion of the aircraft, and that due to the rotational motion of the blades. In this case, however, because the plane of rotation of the propellers is approximately orthogonal to the direction of forward motion, the blade tips will have a helical path, therefore the relative blade tip speed will be given by

\[
v_c = \left( v^2 + (v_p \cos(\theta))^2 \right)^{1/2} \tag{3.3}
\]

The maximum relative blade tip speed will be given by

\[
v_{c_{\text{max}}} = \left( v^2 + v_b^2 \right)^{1/2} \tag{3.4}
\]

Values of \(N_1\) range from about \(\cos(\theta)/\lambda\) to about \(5\cos(\theta)/\lambda\), and values of \(N_2\) range from about \(10\cos(\theta)/\lambda\) to about \(30\cos(\theta)/\lambda\).

Values of \(\Delta f\) range from about 50 Hz for heavy aircraft to about 100 Hz for light aircraft.

Values of \(B_1\) range from about \(100\cos(\theta)/\lambda\) Hz to about \(200\cos(\theta)/\lambda\) Hz, and values of \(B_2\) range from about \(\cos(\theta)/\lambda\) kHz to about \(1.2\cos(\theta)/\lambda\) kHz.

All propeller aircraft will have a minimum speed at which they can fly. The minimum stalling speed of an aircraft, \(v_{\text{min}}\), is the minimum speed at which the aircraft can fly straight and level. Below this speed, the flow of air over the aircraft will become turbulent, instead of streamlined, and the aircraft will stall. This will cause a sudden reduction in lift and increase in drag [68]. When this happens, the aircraft will not be able to fly straight and level. As with the maximum speed of the aircraft, the minimum stalling speed of the aircraft will be determined by the thrust, drag, lift and weight of the aircraft. Thrust and lift will tend to increase the
minimum stalling speed of the aircraft. Conversely, drag and weight will tend to reduce the minimum speed of the aircraft. In practice, aircraft rarely fly at their minimum stalling speeds - even when landing - since the landing speeds of most aircraft are greater than their minimum stalling speeds, for safety reasons.

Values of $v_{\text{min}}$ range from about 25 m/s for light aircraft to about 50 m/s for heavy aircraft.

Values of $v_{\text{max}}$ range from about 50 m/s for light aircraft to about 200 m/s for heavy aircraft. The largest value of $v_{\text{max}}$ for any propeller aircraft which is currently in service is that of the Tupolev Tu-114 Cleat, for which $v_{\text{max}} = 243$ m/s.

Values of $h_{\text{max}}$ range from about 5,000 m for light aircraft to about 10,000 m for heavy aircraft. The largest value of $h_{\text{max}}$ for any propeller aircraft which is currently in service is that of the Caproni Ca 161bis, for which $h_{\text{max}} = 17,083$ m.

3.4 Conclusions

This chapter has presented a detailed survey of the physical configurations, physical parameters, and radar features of rotor and propeller aircraft ranging in size from light to heavy aircraft. It has discussed those features which relate to the aircraft in general, and those which relate to the rotating aircraft blades in particular. It has also given the maximum range of values of each of these features. Some attempt has been made to indicate why certain types of configuration are used, and what the main advantages and disadvantages of each type of configuration are. Some attempt has also been made to indicate how common or uncommon each type of configuration is.
CHAPTER FOUR

FEATURE EXTRACTION

4.1 Introduction

This chapter discusses the extraction of aircraft features from the return signal. The purpose of feature extraction is to transform an $M$-dimensional sample space, $X$, into an $N$-dimensional feature space, $Y$, where $0 < N < M$ [69]. This process reduces the computational complexity and memory requirements of the classification process, and therefore simplifies the classification process [70]. During the feature extraction process, most of the information which is required to classify the object should be extracted from the input signal, and most of the remaining information should be discarded [71]. This will reduce the amount of unnecessary information which is processed by the classification process. The features which are extracted should maximise the similarity of objects which are in the same class, and should minimise the similarity of objects which are in different classes [72]. This will minimise the distance between objects which are in the same class, and will maximise the distance between objects which are in different classes, and therefore will increase the probability of correct classification. There are many different methods of feature extraction, ranging from biological to mathematical [73]. Similarly, there are many different types of feature extraction algorithm, ranging from heuristic to deterministic.

The chapter presents an initial attempt to develop algorithms which can extract features which relate to the rotating aircraft blades, as identified in Chapter 2 and discussed in Chapter 3. It is believed that this is the first time that such algorithms have appeared in the literature.
The chapter concentrates mainly on the extraction of features which relate to the rotating aircraft blades, viz. $f_r, L_2, \Delta f$, and the blade symmetry, i.e. whether $N$ is even or odd. As well as these features, the chapter also discusses the extraction of other features from the return signal. The chapter is of interest with respect to the implementation of aircraft classification algorithms, since the extraction of these features is a first step towards classifying aircraft, based on the return signal from the rotating aircraft blades.

The structure of the chapter is as follows.

Section 2 discusses the extraction of $f_r, L_2$ from the return signal. The method which will be used is to extract the bandwidth of the significant sidebands, $B_2$, from the frequency spectrum of the aircraft, and then to extract $f_r, L_2$ from $B_2$, using tracking data and a knowledge of flight geometry.

The extraction of the blade passing frequency, $\Delta f$, from the return signal is discussed in Section 3. In this case, the method which is used is to extract $\Delta f$ from the power cepstrum of the aircraft. Very little work has been published on the cepstrum of periodic signals, mainly because cepstrum analysis was originally developed to detect echoes in non-periodic signals. Also, very little work has been published on the cepstrum of radar signals, much less the cepstrum of periodic radar signals, therefore this section discusses the basic theory of the power cepstrum, extends this theory to include periodic signals, and presents some new mathematical results.

Section 4 discusses the extraction of the blade symmetry from the return signal. As with the extraction of $\Delta f$, the method which will be used is to extract the blade symmetry from the power cepstrum of the aircraft.

The extraction of other features from the return signal is discussed in Section 5. These features can be divided into two different types: those which relate to the rotating aircraft blades in particular, e.g. $f_r, L_1$, the number of rotors or propellers, etc., and those which relate to the aircraft in general, e.g. altitude, velocity, etc.

Finally, Section 6 draws some conclusions.
4.2 Extraction of $f, L_2$

The method which will be used here to extract $f, L_2$ is to extract $B_2$ from the frequency spectrum of the aircraft, and then to extract $f, L_2$ from $B_2$, using tracking data and a knowledge of flight geometry.

2.1 Extraction of $B_2$

In theory, $B_2$ can be extracted from the return signal, since it can be shown that when $N$ is even, the time interval between the first two nulls of the blade flashes of $|v_2(t)|$ is given by

$$t_n = \frac{\lambda}{4\pi f_r (L_2 - L_1) \cos(\theta)}$$

(4.1)

where $t_n =$ time interval between the first two nulls of the blade flashes.

Similarly, it can be shown that when $N$ is odd, the time interval between the first two nulls of the blade flashes of $|v_2(t)|$ is given by

$$t_n = \frac{\lambda}{2\pi f_r (L_2 - L_1) \cos(\theta)}$$

(4.2)

However, the main disadvantage of this method is that the blade flashes may be hidden by the return signal from the airframe, or by noise or clutter.

Because of the disadvantages of extracting $B_2$ from the return signal, the method which will be used here is to extract $B_2$ from the frequency spectrum of the aircraft. $B_2$ can be extracted from the return signal by using the following algorithm:

1. Calculate the frequency spectrum of the aircraft.
2. Determine which frequency sample has the largest magnitude, i.e. determine the frequency of the airframe line.
3. Shift the frequency spectrum such that the airframe line is at 0 Hz.
4. Determine which sample has the second largest magnitude, and is greater than or equal to $\Delta f/2$ away from the airframe line (Section 4.3 discusses the extraction of $\Delta f$), i.e. determine the frequency of the largest sideband.
5. Determine which sample is furthest away from the airframe line, and has a power greater than or equal to half the power of the largest sideband, i.e. determine the frequency of the half-power sideband.

6. Select twice the difference between the half-power sideband and the airframe line as being the bandwidth of the frequency spectrum.

This algorithm makes three main assumptions:

1. The sample which has the largest magnitude is due to the airframe.

2. The sample which has the second largest magnitude, and is greater than or equal to $\Delta f/2$ away from the airframe line, is due to the largest sideband. (The requirement that this sample be greater than or equal to $\Delta f/2$ away from the airframe line is to prevent any incorrect results in selecting the frequency of the largest sideband, due to any frequency spread of the airframe line.)

3. The sample which is furthest away from the airframe line, and has a power greater than or equal to half the power of the largest sideband, is due to the half-power sideband.

The time taken to perform the algorithm could be reduced by reducing the number of samples taken when calculating the frequency spectrum. However, this will reduce the frequency resolution of the spectrum, and therefore will reduce the accuracy with which $B_2$ can be determined.

The time taken to perform the algorithm could also be reduced by adding another step between Steps 3 and 4, in which the frequency spectrum is folded about the airframe line, and then added. By doing this, only half of the spectrum would have to be examined, in order to determine the frequency of the largest and half-power sidebands. However, this may cause incorrect results in determining the frequency of these sidebands, because any asymmetry between the upper and lower parts of the frequency spectrum, due to AM of the return signal from the rotating aircraft blades, noise, clutter, etc., will affect the shape of the folded spectrum, and therefore may affect the determination of the frequency of these sidebands.
The algorithm will perform best when the peak signal-to-noise ratio (PSNR) of the frequency spectrum is high, i.e. when the ratio of peak signal power to average noise power of the spectrum is high. The PSNR of the spectrum is a better measure of the effect of noise on the performance of the algorithm than the signal-to-noise ratio (SNR), since the sidebands are present only at discrete frequencies, whereas the noise is present at all frequencies. In theory, this means that the algorithm can perform well when the SNR (but not the PSNR) is low. For example, if the SNR and $B_2$ are kept constant, and $N_2$ is reduced, i.e. $\Delta f$ is increased, then the PSNR will be increased, and the algorithm will perform better for the same SNR. Similarly, if the SNR and $\Delta f$ are kept constant, and $B_2$ is reduced, i.e. $N_2$ is reduced, then the PSNR will be increased and the algorithm will again perform better for the same SNR.

4.2.2 Extraction of $f_r L_2$

In Chapter 2 it was shown that the bandwidth of the frequency spectrum is given by

$$B_2 = \frac{8\pi f_r L_2 \cos(\theta)}{\lambda}$$

Equation (2.34) can also be written in the form

$$f_r L_2 = \frac{B_2 \lambda}{8\pi \cos(\theta)}$$ (4.3)

$f_r L_2$ can be extracted from the return signal by using the following algorithm:

1. Extract $B_2$ from the frequency spectrum of the aircraft.
2. Calculate $\theta$, using tracking data and a knowledge of flight geometry.
3. Extract $f_r L_2$ from $B_2$, using Equation (4.3).

The calculation of $\theta$ will be different for main rotors, tail rotors and propellers.

For the main rotor of a rotor aircraft, $\theta$ is given by

$$\theta = \theta_m$$ (4.4)
where $\theta_m = $ elevation angle of the aircraft.

For the tail rotor of a rotor aircraft, $\theta$ is given by

$$\theta = \sin^{-1}(\sin(\theta_{\phi}) \cos(\theta_{\phi})) \quad (4.5)$$

where $\theta_{\phi}$ = azimuth component of the angle between the line of sight from the radar to the aircraft, and the horizontal component of the direction of motion of the aircraft.

$\theta_{e\phi}$ = elevation component of the angle between the line of sight from the radar to the aircraft, and the horizontal component of the direction of motion of the aircraft.

Equation (4.5) can be simplified to give

$$\theta = \theta_{e\phi} \quad (4.6)$$

where $\theta_{e\phi}$ = angle between the line of sight from the radar to the aircraft, and the longitudinal-normal plane of the aircraft.

Note that if the direction of motion of the aircraft does not have a horizontal component, i.e. if the aircraft is moving vertically or is hovering, then it will not be possible to determine $\theta$ for the tail rotor, since it will not be possible to determine $\theta_{e\phi}$, $\theta_{\phi}$ or $\theta_{e\phi}$.

For a propeller aircraft, $\theta$ is given by

$$\theta = \sin^{-1}(\cos(\theta_{\phi}) \cos(\theta_{e})) \quad (4.7)$$

where $\theta_{\phi}$ = azimuth component of the angle between the line of sight from the radar to the aircraft, and the direction of motion of the aircraft.

$\theta_{e}$ = elevation component of the angle between the line of sight from the radar to the aircraft, and the direction of motion of the aircraft.

Equation (4.7) can be simplified to give

$$\theta = \frac{\pi}{2} - \theta_{e} \quad (4.8)$$
where $\theta_p$ = angle between the line of sight from the radar to the aircraft, and the direction of motion of the aircraft.

The analysis above assumes that, in the case of rotor aircraft, the plane of rotation of the main rotor is parallel to the ground, and that of the tail rotor is orthogonal to the ground, whereas in the case of propeller aircraft, the analysis assumes that the plane of rotation of the propeller can be at any angle, with respect to the ground. Note also that the analysis assumes that the radar can be stationary or moving.

4.3 Extraction of $\Delta f$

In theory, $\Delta f$ can be extracted from the return signal or frequency spectrum of the aircraft. However, both of these methods have disadvantages.

In order to extract $\Delta f$ from the return signal, it is necessary to measure the time interval between two blade flashes. However, as with the extraction of $B_2$, the main disadvantage of this method is that the blade flashes may be hidden by the return signal from the airframe, or by noise or clutter.

In order to extract $\Delta f$ from the frequency spectrum, it is necessary to measure the frequency interval between two sidebands. However, the main disadvantage of this method is that some of the sidebands may be missing, due to AM of the return signal from the rotating aircraft blades, or may be notched, due to $L_1$ being non-zero. Some of the sidebands may also be hidden by noise or clutter.

Because of the disadvantages of extracting $\Delta f$ from the return signal or frequency spectrum of the aircraft, the method which will be used here is to extract $\Delta f$ from the power cepstrum of the aircraft.

4.3.1 Cepstrum Analysis

Cepstrum analysis is a form of time series analysis which was originally developed in order to detect echoes in seismic signals, so that the depth of the seismic source could be determined from the echo time delay [74]. It is now used
in many areas of signal processing, e.g. radar, sonar, speech, etc. [75].

Cepstrum analysis is based on the fact that the convolution of two signals in the time domain is equal to the multiplication of the two signals in the frequency domain, and on the fact that the logarithm of two multiplicative signals is equal to the sum of the logarithms of the two signals [76]. This means that two signals which are convolved in the time domain can be separated by calculating the frequency spectrum of the convolved signal, and then calculating the logarithm of the frequency spectrum. It can be shown from this that the logarithmic frequency spectrum of an input signal consisting of a fundamental signal and a scaled, delayed echo of the signal will have a fundamental component, due to the fundamental signal, and a ripple component, due to the echo signal. The "frequency" of the ripple component can be determined by calculating the inverse spectrum of the logarithmic spectrum of the input signal, i.e. by calculating the cepstrum of the input signal.

There are three different types of cepstrum: the complex, power and phase. These will now be described briefly.

The complex cepstrum of an input signal is defined as the inverse Fourier transform of the natural logarithm of the Fourier transform of the signal, i.e. is defined as the inverse complex spectrum of the natural logarithm of the complex spectrum of the signal, and is given by

$$G_{co}(\tau_q) = F^{-1}(\log_e(F(f(t))))$$

(4.9)

where $f(t)$=input signal,

$F(f(t))$=Fourier transform of $f(t)$,

$F^{-1}(F(f))$=inverse Fourier transform of $F(f)$,

$G_{co}(\tau_q)$=complex cepstrum of $f(t)$,

$\tau_q$=quefrency.

Figure 4.1 shows a block diagram of the complex cepstrum of an input signal. Because the complex cepstrum is determined using the magnitude and phase components of the Fourier transform of the input signal, it can be used to extract the
Figure 4.1.
Block diagram of the complex cepstrum of an input signal.
fundamental signal and the echo time delay from the input signal.

The calculation of the complex cepstrum is complicated by the fact that the imaginary component is given by its principle value, i.e. is given by its true value modulo $2\pi$ rad. [77]. This causes the imaginary component to be discontinuous, odd and periodic, whereas it should be continuous, odd and periodic, in order to ensure that the complex cepstrum of a causal signal will be causal [78]. This can be corrected by unwrapping the imaginary component, which involves adding a correction signal to the imaginary component [79]. If the imaginary component is sampled at a rate such that it never changes by more than $\pi$ between samples, then the correction signal is given by

$$G_c(0)=0$$

$$G_c(n)=\begin{cases} G_c(n-1)-2\pi & \text{if } \text{Im}(G_{co}(n)) - \text{Im}(G_{co}(n-1)) > \pi \\ G_c(n-1) & \text{if } -\pi \leq \text{Im}(G_{co}(n)) - \text{Im}(G_{co}(n-1)) \leq \pi \\ G_c(n-1)+2\pi & \text{if } \text{Im}(G_{co}(n)) - \text{Im}(G_{co}(n-1)) < -\pi \end{cases}$$

where $G_c(n)$= correction signal.

The power cepstrum of an input signal is defined as the square of the magnitude of the inverse Fourier transform of the natural logarithm of the square of the magnitude of the Fourier transform of the signal, i.e. is defined as the inverse power spectrum of the natural logarithm of the power spectrum of the signal, and is given by

$$G_{pa}(\tau_q) = |F^{-1}(\log_e(|F(f(t))|^2))|^2$$

where $G_{pa}(\tau_q)$= power cepstrum of $f(t)$.

Figure 4.2 shows a block diagram of the power cepstrum of an input signal. Because the power cepstrum is determined using only the magnitude component of the Fourier transform of the input signal, it cannot be used to extract the fundamental signal from the input signal, but it can be used to extract the echo delay.

The phase cepstrum of an input signal is defined as the square of the magnitude of the inverse Fourier transform of the natural logarithm of the square of the
Figure 4.2.
Block diagram of the power cepstrum of an input signal.
phase of the Fourier transform of the signal, i.e. is defined as the inverse power spectrum of the natural logarithm of the square of the phase spectrum of the signal, and is given by

$$G_{ph}(\tau_q) = \left| F^{-1} \left( \log_e (|F(f(t))|^2) \right) \right|^2$$  \hspace{1cm} (4.12)

where \( G_{ph}(\tau_q) = \) phase cepstrum of \( f(t) \).

Figure 4.3 shows a block diagram of the phase cepstrum of an input signal. As with the power cepstrum, because the phase cepstrum is determined using only the phase component of the Fourier transform of the input signal, it cannot be used to extract the fundamental signal from the input signal, but it can be used to extract the echo delay. Also, as with the complex cepstrum, the phase cepstrum is given by its principle value, and therefore has to be unwrapped.

Note that the complex, power and phase cepstra do not have universal definitions, i.e. different authors may use different definitions [80]. For example, some authors use the forward Fourier transform instead of the inverse Fourier transform as the second transform in the power and phase cepstra.

Because we are only interested in extracting \( \Delta f \) from the return signal, the power cepstrum will be used, since the complex and phase cepstra both have to be unwrapped.

If we assume that an input signal consists of a fundamental signal and a scaled, delayed echo of the fundamental signal, then the input signal can be written in the form

$$g(t) = f(t) + a_1 f(t - \tau)$$ \hspace{1cm} (4.13)

where \( a_1 = \) scale factor.

The Fourier transform of Equation (4.13) is given by

$$G(f) = F(f) \left( 1 + a_1 e^{-j\omega \tau} \right)$$ \hspace{1cm} (4.14)

The square of the magnitude of Equation (4.14) is given by
Figure 4.3.
Block diagram of the phase cepstrum of an input signal.
\[ |G(f)|^2 = b_0 |F(f)|^2 (1 + b_1 \cos(\omega \tau)) \]  

(4.15)

where \( b_0, b_1 = \text{scale factor}. \)

The natural logarithm of Equation (4.15) is given by

\[
\log_e (|G(f)|^2) = \log_e (b_0 |F(f)|^2) + \log_e (1 + b_1 \cos(\omega \tau))
\]

(4.16)

If \(-1 < b_1 < 1\), then Equation (4.16) can also be written in the form

\[
\log_e (|G(f)|^2) = \log_e (b_0 |F(f)|^2) + \sum_{n=1}^{\infty} \frac{(-1)^{n-1}(b_1 \cos(\omega \tau))^n}{m}
\]

(4.17)

since [81]

\[
\log_e (1 + x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}x^n}{m} \text{ if } -1 < x < 1
\]

(4.18)

Equation (4.17) can also be written in the form

\[
\log_e (|G(f)|^2) = \log_e (c_0 |F(f)|^2) + \sum_{n=1}^{\infty} (-1)^{n-1} c_m \cos(m \omega \tau)
\]

(4.19)

where \( c_0, c_m = \text{scale factor}. \)

The inverse Fourier transform of Equation (4.19) is given by

\[
F^{-1}(\log_e (|G(f)|^2)) = F^{-1}(\log_e (d_0 |F(f)|^2)) + \sum_{n=-\infty}^{\infty} (-1)^{n-1} d_m \delta(\tau_q + m \tau)
\]

(4.20)

where \( d_0, d_m = \text{scale factor}. \)

The square of the magnitude of Equation (4.20) is given by

\[
G_{po}(\tau) = |F^{-1}(\log_e (d_0 |F(f)|^2))|^2 + \sum_{m=-\infty}^{\infty} d_m^2 \delta(\tau_q + m \tau)
\]

(4.21)

where \( G_{po}(\tau_q) = |F^{-1}(\log_e (|G(f)|^2))|^2 = \text{power cepstrum of } g(t). \)

It can be seen that \( G_{po}(\tau) \) has two components: a fundamental component, due to the fundamental signal, and an echo component, due to the echo signal. It can also be seen that the echo component consists of an infinite series of impulses, which are separated from each other by \( \tau \). In practice, the impulses will decay with
the rate of decay depending on the value of \( a_1 \), the scale factor of the echo.

The analysis above applies in the case where \(-1 < b_1 < 1\). By rearranging Equation (4.15), the analysis will also apply in the case where \( b_1 < -1 \), or \( b_1 > 1 \). In the case where \( a_1 = 1 \), i.e. in the case where the fundamental and echo signals have the same magnitude, Equation (4.15) can be modified to give

\[
|G(\omega)|^2 = b_0 |F(\omega)|^2 (1 + \cos(\omega \tau))
\]  

(4.22)

Equation (4.21) can modified and approximated to give

\[
G_{po}(\tau) = |F^{-1}(\log|F(\omega)|^2)|^2 + d^2_1 (\delta(\tau + \tau) + \delta(\tau - \tau))
\]  

(4.23)

where \( d_1 = \) scale factor.

In this case, it can be seen that the echo component of \( G_{po}(\tau) \) consists of two impulses, which have the same magnitude, and which are separated from 0 s by \( \tau \).

If we now assume that the input signal consists of a fundamental signal and a number of scaled, delayed echoes of the fundamental signal, where the time delay between echoes is constant, then the input signal can be written in the form

\[
g(t) = f(t) + a_1 f(t - \tau) + a_2 f(t - 2\tau) + \cdots + a_N f(t - N_\tau)\]

(4.24)

where \( a_1, a_2, a_3, \ldots, a_N = \) scale factor,

\( N_\tau = \) number of echoes.

The Fourier transform of Equation (4.24) is given by

\[
G(\omega) = F(\omega)(1 + \sum_{n=1}^{N_\tau} a_n e^{-i n \omega \tau})
\]

(4.25)

The square of the magnitude of Equation (4.25) is given by

\[
|G(\omega)|^2 = b_0 |F(\omega)|^2 (1 + \sum_{n=1}^{N_\tau} b_n \cos(n \omega \tau))
\]

(4.26)

where \( b_n = \) scale factor.

The natural logarithm of Equation (4.26) is given by

\[
\log_e (|G(\omega)|^2) = \log_e (b_0 |F(\omega)|^2) + \log_e (1 + \sum_{n=1}^{N_\tau} b_n \cos(n \omega \tau))
\]

(4.27)
If $-1 < \sum_{n=1}^{N_e} b_n \cos(n \omega T) < 1$, then Equation (4.27) can also be written in the form

$$\log_e (|G(f)|^2) = \log_e (b_0 |F(f)|^2) + \sum_{m=1}^\infty \frac{(-1)^{m+1}(\sum_{n=1}^{N_e} b_n \cos(n \omega T))^m}{m}$$  \hspace{1cm} (4.28)

Equation (4.28) can also be written in the form

$$\log_e (|G(f)|^2) = \log_e (c_0 |F(f)|^2) + \sum_{m=1}^\infty (-1)^{m+1} c_m \cos(m \omega T)$$  \hspace{1cm} (4.29)

The inverse Fourier transform of Equation (4.29) is given by

$$F^{-1}(\log_e (|G(f)|^2)) = F^{-1}(\log_e (d_0 |F(f)|^2))$$
$$+ \sum_{m=\infty}^\infty (-1)^{m+1} d_m \delta(\tau_q + m \tau)$$  \hspace{1cm} (4.30)

The square of the magnitude of Equation (4.30) is given by

$$G(\tau_q) = |F(\log_e (d_0 |F(f)|^2))|^2 + \sum_{m=\infty}^\infty d_m^2 \delta(\tau_q + m \tau)$$  \hspace{1cm} (4.31)

As with Equation (4.21), it can be seen that $G_{p_0}(\tau)$ has two components: a fundamental component, due to the fundamental signal, and an echo component, due to the echo signals. It can also be seen that the echo component consists of an infinite series of impulses, which are separated from each other by $\tau$. In practice, the impulses will decay with $m$, the rate of decay depending on the values of $a_n$, the scale factors of the echoes.

As with the single echo case, the analysis above applies in the case where

$-1 < \sum_{n=1}^{N_e} b_n \cos(n \omega T) < 1$. By rearranging Equation (4.27), the analysis will also apply in the case where $\sum_{n=1}^{N_e} b_n \cos(n \omega T) < -1$, or $\sum_{n=1}^{N_e} b_n \cos(n \omega T) > 1$. In the case where $a_1 = a_2 = a_3 = \cdots = a_{N_e} = 1$, and $N_e \to \infty$, i.e. in the case where $g(t)$ is periodic, Equation (4.26) can be modified to give

$$|G(f)|^2 = b_0 |F(f)|^2 \sum_{m=-\infty}^\infty \delta(f + \frac{m}{\tau})$$  \hspace{1cm} (4.32)
Equation (4.31) can modified and approximated to give

\[ G_{po}(\tau_q) = |F^{-1}(|d_0|^2)|^2 + \sum_{m=-\infty}^{\infty} d_m^2 \delta(\tau_q + m \tau) \]  

(4.33)

In this case, it can be seen that the echo component of \( G_{po}(\tau) \) consists of an infinite series of impulses, which have the same magnitude, and which are separated from each other by \( \tau \), i.e. the echo component of the power cepstrum of a periodic signal is also periodic.

The return signal from the rotating aircraft blades can be considered as consisting of a series of blade flashes, which in turn can be considered as multiple echoes of a single blade flash. This interpretation leads to the extension of cepstrum analysis above, which in turn leads to a theoretical basis for using the power cepstrum to extract \( \Delta f \).

In theory, the power cepstrum of the rotating aircraft blades will be given by Equation (4.33). In practice, it will be given by an equation which is similar to Equation (4.31), i.e. the impulses will decay with \( m \), due to random variations of the return signal, noise, clutter, etc.

\( \Delta f \) can be extracted from the return signal by using the following algorithm:

1. Calculate the power cepstrum of the aircraft.
2. High-pass filter the power cepstrum to remove the lowest 5 ms from the cepstrum, i.e. remove the fundamental component from the cepstrum.
3. Determine which cepstral line has the largest magnitude, i.e. determine the quefrency of the largest echo.
4. Determine which cepstral line is nearest to 0 s, and has a power greater than or equal to half the power of the largest cepstral line, i.e. determine the quefrency of the fundamental echo.
5. Select this cepstral line as being the blade-passing interval of the aircraft.

This algorithm makes three main assumptions:
1. The lowest 5 ms of the power cepstrum is due to the fundamental component of the power cepstrum. We assume that the fundamental echo is greater than 5 ms from the fundamental component, since it was shown in Chapter 3 that the highest blade-passing frequency of any rotor or propeller aircraft is about 100 Hz, corresponding to a blade-passing interval of about 10 ms.

2. The sample which has the largest magnitude is due to the largest echo of the echo component of the power cepstrum.

3. The cepstral line which is nearest to 0 s, and has a power greater than or equal to half the power of the largest cepstral line, is due to the fundamental echo. (Note that we declare than this line must be greater than or equal to half the power of the largest cepstral line in order to ensure that only significant cepstral lines are considered.)

Note that it is important that the feature extraction algorithm is performed on the complex return signal, i.e. on both the real and imaginary components of the return signal, otherwise the algorithm may produce incorrect results. (In Section 4.4 it is shown that the blade symmetry can be extracted from the return signal by performing the feature extraction algorithm on the real, imaginary or magnitude component of the return signal.)

As with the algorithm which extracts $B_2$, the time taken to perform the algorithm could be reduced by reducing the number of samples taken when calculating the power cepstrum. The time taken to perform the algorithm could also be reduced by adding another step between Steps 1 and 2, in which the power cepstrum is folded, and then added. However, as with the algorithm which extracts $B_2$, this may cause incorrect results in selecting the largest and fundamental echoes, due to AM of the return signal from the rotating aircraft blades, noise, clutter, etc.

4.3.2 Simulation Results

Figure 4.4 shows the power cepstrum of a rotor aircraft in which $N=4$, $L_2=5$ m, $f_r=6$ Hz, $\theta=0$ rad., and $\lambda=1$ m. The power cepstrum is dominated by an
Figure 4.4.

Power cepstrum of the return signal from a rotor aircraft in which $N=4$, $L_2=5$ m, $f_r=6$ Hz, $\theta=0$ rad., and $\lambda=1$ m, before high-pass filtering to remove the fundamental component.
impulse at 0 s. This component is due to the magnitude component of the power spectrum [82]. When the inverse Fourier transform is taken of the natural logarithm of the power spectrum, this component is transformed into an impulse at 0 s in the power cepstrum.

Figure 4.5 shows the power cepstrum of the aircraft of Figure 4.4 after it has been high-pass filtered to remove the fundamental component, and then renormalised. It can be seen that the power cepstrum now consists of a number of decaying impulses, where $\Delta f = 24$ Hz. The rate of decay of these impulses will depend on the magnitudes of the blade flashes.

Note that the power cepstrum is very data dependent, i.e. is very dependent on the input signal, and is also very dependent on implementation, e.g. data sampling, data padding, data windowing, etc. [83]. The remainder of this section discusses some of the implementation problems.

If the duration of the input signal is short, with respect to the blade-passing interval of the aircraft, then it is possible that the cepstrum will be aliased. This aliasing can be removed by zero-padding the return signal before taking the first Fourier transform [84]. In general, the greater the aliasing, the greater the padding must be.

Ideally, after the power cepstrum has been high-pass filtered to remove the fundamental component, the cepstral line which has the largest magnitude would be that due to the fundamental echo. In this case, it would only be necessary to determine which cepstral line has the largest magnitude, in order to determine the quefrency of the fundamental echo, and therefore the blade-passing frequency of the aircraft. In practice, the cepstral line which has the largest magnitude may be due to the fundamental echo or one of its harmonics. However, it is found that zero-padding the return signal increases the likelihood that the cepstral line which has the largest magnitude will be that due to the the fundamental echo - the greater the padding, the greater the likelihood will be.
Figure 4.5.
Power cepstrum of the return signal from a rotor aircraft in which \( N = 4, \ L_2 = 5 \text{ m}, \ f_r = 6 \text{ Hz}, \ \theta = 0 \text{ rad.}, \) and \( \lambda = 1 \text{ m}, \) after high-pass filtering to remove the fundamental component.
Although a window function is applied to the first Fourier transform of the power cepstrum, a window function should not be applied to the second Fourier transform, since applying a window function to the second Fourier transform is equivalent to smoothing the power cepstrum of the aircraft. This will reduce the quefrency resolution of the power cepstrum, and will also reduce the magnitudes of the cepstral lines due to the echo component of the cepstrum, i.e. this will reduce the magnitudes of the cepstral lines which the feature extraction algorithm is trying to extract. This will reduce the PSNR of the power cepstrum, which will reduce the performance of the algorithm.

Another problem with the power cepstrum is that it is possible to oversample the input signal. Outside the signal bandwidth, the frequency spectrum is dominated by noise. This is not usually a problem in spectrum analysis, since the power of the noise component outside the signal bandwidth is often small. However, this may not be the case with cepstrum analysis. Because of the non-linear logarithmic operation in the power cepstrum, it is possible that the noise component of the frequency spectrum outside the signal bandwidth may contribute as much or more to the cepstrum as the signal and noise components of the spectrum inside the signal bandwidth. This will reduce the PSNR of the power cepstrum, which will again reduce the performance of the algorithm. Oversampling may also increase the aliasing of the power cepstrum, since it will reduce the duration of the input signal, for a given number of samples. This will then increase the number of samples between the cepstral lines of the echo component of the return signal, and therefore may increase the aliasing of the power cepstrum [85].

Note that if there is more than one rotor or propeller on the aircraft, then the algorithm may produce incorrect results, since if there is any phase angle between the different rotors or propellers, then this will affect the return signal, frequency spectrum and power spectrum of the aircraft.
4.4 Extraction of Blade Symmetry

In theory, as with the extraction of $\Delta f$, the blade symmetry can be extracted from the return signal or frequency spectrum of the aircraft. However, as was shown in Section 4.3, regarding the extraction of $\Delta f$, both of these methods have disadvantages. The method which will be used here is to extract the blade symmetry from the power cepstrum of the aircraft. Because of this, most of the results of Section 4.3, will also apply to this section.

The blade symmetry can be extracted from the return signal by using the following algorithm:

1. Extract $\Delta f_1$ from the power cepstrum of the aircraft.
2. Extract $\Delta f_2$ from the power cepstrum of the magnitude component of the return signal.
3. If $\Delta f_2 = \Delta f_1$, then select $N$ as being even, or if $\Delta f_2 = 2\Delta f_1$, then select $N$ as being odd.

The operation of this algorithm is based on the fact that, as was shown in Chapter 2, $v_r(t)$ is periodic with $\Delta t = 1/Nf_r$, whereas when $N$ is even, $|v_r(t)|$ is periodic with $\Delta t = 1/Nf_r$, and when $N$ is odd, $|v_r(t)|$ is periodic with $\Delta t = 1/2Nf_r$. Therefore, by extracting $\Delta f_1$ from the power cepstrum of the aircraft, extracting $\Delta f_2$ from the power cepstrum of the magnitude component of the return signal, and then comparing $\Delta f_2$ with $\Delta f_1$, it is possible to determine whether $N$ is even or odd.

In the case where there is no relative velocity, or where the effects of relative velocity can be removed, the algorithm will also work if, in Step 2, the magnitude component of the return signal is replaced by the real or imaginary component of the return signal, i.e. if $\Delta f_2$ is extracted from the power cepstrum of the real or imaginary component of the return signal, instead of from the power cepstrum of the magnitude component of the return signal. This is because when $N$ is even, $\text{Re}(v_r(t))$ and $\text{Im}(v_r(t))$ are periodic with $\Delta t = 1/Nf_r$, whereas when $N$ is odd, $\text{Re}(v_r(t))$ and $\text{Im}(v_r(t))$ are half-wave periodic with $\Delta t = 1/2Nf_r$.  

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As with the algorithms which extract $B_2$ and $\Delta f$, the time taken to perform the algorithm could be reduced by reducing the number of samples taken when calculating the power cepstrum. The time taken to perform the algorithm could also be reduced by adding another step between Steps 1 and 2 of the algorithm which extracts $\Delta f$, in which the power cepstrum is folded, and then added. However, as with the algorithms which extract $B_2$ and $\Delta f$, this may cause incorrect results in selecting the largest and fundamental echoes, due to AM of the return signal from the rotating aircraft blades, noise, clutter, etc.

4.4.1 Simulation Results

Figure 4.6 shows the power cepstrum of the magnitude component of the return signal from the aircraft of Figure 4.5. It can be seen that $\Delta f = 24$ Hz, i.e. $\Delta f_2 = \Delta f_1$, therefore $N$ is even. Figure 4.7 shows the power cepstrum of an aircraft which is identical to that of Figure 4.5, except that in this case, $N = 5$. It can be seen that $\Delta f = 30$ Hz. Figure 4.8 shows the power cepstrum of the magnitude component of the return signal from the aircraft of Figure 4.7. It can be seen that $\Delta f = 60$ Hz, i.e. $\Delta f_2 = 2\Delta f_1$, therefore $N$ is odd.

If there is any AM of the return signal from the rotating aircraft blades, then this will reduce the performance of the algorithm since, as was shown in Chapter 2, when $N$ is odd, any AM will reduce the magnitudes of alternate blade flashes. This will adversely affect the extraction of $\Delta f_2$ from the power cepstrum of the magnitude component of the return signal, and therefore will reduce the performance of the algorithm.

Figure 4.9 shows the power cepstrum of the magnitude component of the return signal from the aircraft of Figure 4.7, as the AM of the return signal from the rotating aircraft blades is increased from 0% to 100%. It can be seen that as the modulation is increased, the magnitudes of alternate cepstral lines are reduced. When the modulation is 0%, the power of the first cepstral line is approximately double that of the second. When the modulation is 20%, the power of the first line
Figure 4.6.
Power cepstrum of the magnitude component of the return signal from a rotor aircraft in which $N = 4$, $L_2 = 5$ m, $f_r = 6$ Hz, $\theta = 0$ rad., and $\lambda = 1$ m.
Figure 4.7.
Power cepstrum of the return signal from a rotor aircraft in which $N=5$, $L_2=5$ m, $f_x=6$ Hz, $\theta=0$ rad., and $\lambda=1$ m.
Figure 4.8.
Power cepstrum of the magnitude component of the return signal from a rotor aircraft in which $N=5$, $L_2=5$ m, $f_r=6$ Hz, $\theta=0$ rad., and $\lambda=1$ m.
Figure 4.9.
Power cepstrum of the magnitude component of the return signal from a propeller aircraft in which \( N = 2, L_1 = 0.25 \, \text{m}, L_2 = 2 \, \text{m}, f_r = 16 \, \text{Hz}, \theta = \pi/4 \, \text{rad.}, \) and \( \lambda = 1 \, \text{m}, \) as \( \phi_r \) is increased from 0 to \( \pi/4 \, \text{rad}. \)
is approximately equal to that of the second. When the modulation is 30%, the power of the first line is approximately half that of the second. This process continues until, when the modulation is 100%, the power of the first line is at a minimum.

Note that, as with the extraction of $\Delta f$, if there is more than one rotor or propeller on the aircraft, then the algorithm may produce incorrect results.

4.5 Extraction of Other Features

As well as the features above, there are other features which can be extracted from the return signal. These features can be divided into two different types: those which relate to the rotating aircraft blades, and those which relate to the aircraft in general.

By extending the algorithm which extracts $f, L_2$ from the return signal, it may also be possible to extract $f, L_1$ from the return signal. This algorithm would be similar to that which extracts $f, L_2$ from the return signal, except that, in this case, instead of determining which sideband is furthest away from the airframe line, has a power greater than or equal to half the power of the largest sideband, and is a local maximum, Step 5 would determine which sideband is nearest to the airframe line, has a power greater than or equal to half the power of the largest sideband, and is a local maximum.

For aircraft which have more than one rotor or propeller, it may also be possible to extract the number of rotors or propellers from the return signal. There are two ways in which this could be done.

First, it could be done on the basis that there will be slight differences in the frequency of rotation of different rotors or propellers on the same aircraft. In this case, the extraction algorithm could be based on the fact that there will be slight differences in the time between blade flashes for different rotors or propellers, or it could be based on the fact that there will be slight differences in the frequency
between sidebands for different rotors or propellers. Note that if the rotors or propellers are synchronised, then the second method will not work, since there will then be no differences in the frequency of rotation of different rotors or propellers on the same aircraft.

Second, it could be done on the basis that there will be a phase angle between different rotors or propellers on the same aircraft. In this case, the extraction algorithm could be based on the fact that there will be differences in the time between blade flashes for different rotors or propellers. Note that if the rotors or propellers are synchrophased, then this method will not work, since there will then be no phase angle between different rotors or propellers. Note also that if the phase angle between two rotors or propellers is approximately $\pi/N$ rad., then this will cause incorrect results, since the return signal from the rotating aircraft blades will then correspond to that of an aircraft which has $2N$ blades on one rotor or propeller, instead of $N$ blades on two rotors or propellers. For example, if a rotor aircraft has two rotors in which $N = 4$, and the phase angle between the rotors is $\pi/4$ rad., then the return signal from the rotating aircraft blades will correspond to that of a rotor aircraft which has one rotor in which $N = 8$. The same theory can be extended to apply to aircraft which have a larger number of rotors or propellers. It is unlikely that it will be possible to extract the number of rotors or propellers from the frequency spectrum of the aircraft on the basis that there will be differences in the phase angle between different rotors or propellers, since any differences in the phase angle between different rotors or propellers will only affect the magnitudes of the spectral lines, which will be more difficult to analyse.

With rotor aircraft, both the main and tail rotors of rotor aircraft may be visible to the radar at some aspect angles. In this case, it may be possible to extract the features of both the main and tail rotors. However, it is likely that the magnitude of the return signal from the main rotor will be much larger than that from the tail rotor, therefore it is possible that the features of the tail rotor may be hidden by those of the main rotor.
With respect to those features which relate to the aircraft in general, these features can also be used to classify aircraft. These features can be divided into two different types: those which are usually determined by the radar, e.g. altitude, velocity, etc., and those which are not usually determined by the radar, e.g. acceleration, rate of climb, etc. As with the extraction of \( f, L_2 \), these latter features can be extracted from the return signal, using tracking data and a knowledge of flight geometry, etc.

4.6 Conclusions

This chapter has discussed the extraction of aircraft features from the return signal, and has presented an initial attempt to develop algorithms which can extract these features. Algorithms have been presented which extract the blade features from the return signal: \( f, L_2 \) was extracted from the frequency spectrum using tracking data and a knowledge of flight geometry, and \( \Delta f \) and the blade symmetry were extracted from the power cepstrum of the aircraft. As well as discussing the features above, the chapter has also discussed the extraction of other features from the return signal.

It has been shown that the return signal from the rotating aircraft blades can be considered as consisting of a series of blade flashes, which in turn can be considered as multiple echoes of a single blade flash. This interpretation has led to an extension of cepstrum analysis, which in turn has led to a theoretical basis for using the power cepstrum to extract \( \Delta f \).
CHAPTER FIVE

PRACTICAL CONSIDERATIONS

5.1 Introduction

This chapter discusses some of the practical considerations which are associated with the analysis of the return signal from rotating aircraft blades. Because of this, the chapter is of interest with respect to the implementation of feature extraction and aircraft classification algorithms.

Only those practical considerations which are associated with the analysis of the return signal from rotating aircraft blades are discussed. Those which are associated with pulse Doppler radars in general, e.g. PRF selection, range ambiguity resolution, Doppler ambiguity resolution, etc., are not discussed.

The chapter only considers the case of aircraft for which there is no significant amount of AM of the return signal from the rotating blades, i.e. aircraft which have double-sided spectra. However, all of the theory and results in the chapter can also be extended to apply to the case of aircraft for which there is a significant amount of AM of the return signal from the rotating blades, i.e. aircraft which have single-sided spectra. Four main subjects are discussed: the radar parameters, window functions, relative acceleration and aircraft classification.

The structure of the chapter is as follows.

Section 5.2 discusses the effects of the radar parameters on the return signal. The section discusses three of the main parameters which affect the nature of the return signal from rotating aircraft blades: the radar wavelength, the PRF and the observation interval. The section shows that there is a minimum value that the radar wavelength can have, in order for the frequency spectrum to contain
sidebands, and that there is also a minimum bandwidth that the frequency spectrum can have in order for the frequency spectrum to contain sidebands. The section also shows that there is a minimum value that the PRF can have in order to avoid aliasing of the frequency spectrum, and that this value will be different for different classes of aircraft. The section further shows that there is a minimum value that the observation interval can have in order for the modulation period to be observed in the return signal, and that this value will also be different for different classes of aircraft. It is believed that this is the first time that a detailed analysis of the effects of the radar parameters on the return signal from rotating aircraft blades has appeared in the literature.

The effects of window functions on the frequency spectrum are discussed in Section 5.3. The section discusses the reasons for applying window functions, and the main effects that they will have on the frequency spectrum. The section discusses three of the main parameters by which the performance of window functions can be measured: the 3-dB bandwidth, $B_{3\text{ dB}}$, the processing gain, $G_p$, and the scalloping loss, $L_s$. For each of these parameters, the parameter is defined, its significance is explained, and the approximate range of values of the parameter is given for different types of window function. The section does not present any new results. However, it does indicate the effects of window functions on the frequency spectrum.

Section 5.4 discusses the effects of relative acceleration on the frequency spectrum. The section presents a detailed mathematical analysis of the effects of acceleration on the frequency spectrum, and includes simulation results which shows these effects. The section also presents an acceleration compensation system, which can reduce the effects of acceleration, if good estimates of the acceleration can be obtained, or can remove the effects of acceleration, if the acceleration is known exactly, and includes simulation results which show the performance of the system. It is believed that this is the first time that a detailed mathematical analysis of the effects of acceleration on the frequency spectrum has appeared in the literature.
Moreover, it is believed that this is the first time that an acceleration compensation system has appeared in the literature.

Aircraft classification is discussed in Section 5.5. The section presents an algorithm in which the number of sets of aircraft parameters which provide a solution set of the aircraft features can be reduced dramatically, and includes an example which shows the performance of the algorithm. The algorithm is also used to obtain some general results, regarding the maximum number of solution sets of the aircraft features for main rotors, tail rotors and propellers, both when the blade symmetry is known, and when the blade symmetry is unknown. It is believed that this is the first time that such an algorithm has appeared in the literature.

Finally, Section 5.6 draws some conclusions.

5.2 Radar Parameters

Almost every radar parameter will affect the return signal in some way. However, this section only discusses those parameters which will affect the nature of the return signal from rotating aircraft blades. There are three main radar parameters which will affect the nature of the return signal: the radar wavelength, the PRF and the observation interval.

5.2.1 Radar Wavelength

There is a maximum value that the radar wavelength can have, in order for the frequency spectrum to contain sidebands. This is shown as follows.

The number of sidebands in the frequency spectrum is given by

\[ N_2 = \frac{8\pi (L_2 - L_1) \cos(\theta)}{N\lambda} \]  

(2.36)

The minimum number of sidebands that the frequency spectrum can have is two. Therefore, Equation (2.36) can be rearranged to give

\[ \frac{8\pi (L_2 - L_1) \cos(\theta)}{N\lambda} \geq 2 \]  

(5.1)
Equation (5.1) can be rearranged in terms of $\lambda$ to give

$$\lambda \leq \frac{4\pi(L_2 - L_1)\cos(\theta)}{N} \tag{5.2}$$

Equation (5.2) gives the maximum value that the radar wavelength can have, in order for the frequency spectrum to contain sidebands. If the radar wavelength is smaller than this value, then the frequency spectrum will not contain any sidebands.

There is also a minimum bandwidth that the frequency spectrum can have in order for the frequency spectrum to contain sidebands. This is shown as follows.

The bandwidth of the frequency spectrum is given by

$$B_2 = \frac{8\pi f_r (L_2 - L_1)\cos(\theta)}{\lambda} \tag{2.38}$$

Equation (5.2) can be substituted into Equation (2.38) to give

$$B_2 \geq 2Nf_r \tag{5.3}$$

Equation (2.32) can be substituted into Equation (5.3) to give

$$B_2 \geq 2\Delta f \tag{5.4}$$

Equation (5.4) gives the minimum bandwidth that the frequency spectrum can have, in order for the frequency spectrum to contain sidebands.

Equation (5.4) merely confirms what we would intuitively expect: when there are only two sidebands in the frequency spectrum, the bandwidth of the frequency spectrum is equal to twice the blade-passing frequency of the aircraft.

In the case where there are only two sidebands, this is analogous to narrowband PM, i.e. [86]

$$B_2 = 2f_m \tag{5.5}$$

5.2.2 PRF

Shannon’s sampling theorem states that any signal can be completely deter-
mined by a set of samples of the signal taken at a frequency which is greater than or equal to twice the highest frequency in the signal, i.e. \[ f_s \geq 2B \] (5.6)

where \( B \) = bandwidth,
\( f_s \) = sampling frequency.

This is shown as follows.

Consider an input signal which is sampled at a frequency of \( f_s \). The sampled input signal is given by \[ f(nT_s) = \sum_{n=-\infty}^{\infty} f(t) \delta(t - nT_s) \] (5.7)

where \( f(nT_s) \) = sampled input signal,
\( T_s = \frac{1}{f_s} \) = sampling interval.

The Fourier transform of Equation (5.7) is given by \[ F(f) = f_s \sum_{n=-\infty}^{\infty} F(f - nf_s) \] (5.8)

It can be seen from Equation (5.8) that the frequency spectrum of a sampled input signal consists of an infinite number of copies of the input signal spectrum, weighted by the sampling frequency, and separated by the sampling frequency \( f_s \).

If the sampling frequency is less than twice the highest frequency in the input signal, then aliasing will occur \([91]\). Aliasing occurs when adjacent copies of the input signal spectrum overlap such that it is not possible to determine the input signal from the sampled input signal spectrum \([92]\). In this case, the aliased frequencies are given by
\[ f_a = f_u \mod \frac{f_s}{2} \] (5.9)

where \( f_a \) = aliased frequency,
\( f_u \) = unaliased frequency.
If aliasing occurs, then this may affect the performance of any feature extraction or aircraft classification algorithms.

There is a minimum value that the PRF can have, in order to avoid aliasing of the frequency spectrum. In order to avoid aliasing, the PRF must be greater than or equal to twice the highest frequency in the frequency spectrum, i.e.

$$\text{PRF} \geq \frac{8\pi frL_c \cos(\theta)}{\lambda} \quad (5.10)$$

Equation (2.42) can be substituted into Equation (5.10) to give

$$\text{PRF} \geq \frac{4v_b \cos(\theta)}{\lambda} = \frac{4v_b}{\lambda} \quad \text{if } \cos(\theta) = 1 \quad (5.11)$$

If we assume that the maximum blade tip speeds of main rotors, tail rotors and propellers are 250 m/s, 250 m/s, and 300 m/s, respectively, then the maximum Doppler frequency shifts caused by the blade tips will be $500/\lambda$ Hz, $500/\lambda$ Hz, and $600/\lambda$ Hz, respectively. Therefore, in order to avoid aliasing of the frequency spectrum, the PRFs for main rotors, tail rotors and propellers must be greater than or equal to $1/\lambda$ kHz, $1/\lambda$ kHz, and $1.2/\lambda$ kHz, respectively.

If the PRF is greater than or equal to $1.2/\lambda$ kHz, then there will be no aliasing of the frequency spectra of main rotors, tail rotor or propellers. If the PRF is greater than or equal to $1/\lambda$ kHz, but less than $1.2/\lambda$ kHz, then there will be no aliasing of the frequency spectra of main rotors or tail rotors, but there will be aliasing of the frequency spectra of propellers. If the PRF is less than $1/\lambda$ kHz, then there will be aliasing of the frequency spectra of main rotors, tail rotor and propellers.

If we assume that there is no a priori knowledge of the class of aircraft, then in order to avoid aliasing of the frequency spectrum of any rotor or propeller aircraft, the PRF must be greater than or equal to the minimum PRF for the class of aircraft which has the highest blade tip speed, i.e. the PRF must be greater than or equal to $1.2/\lambda$ kHz, corresponding to the minimum PRF for propellers.
Note that the analysis above assumes that there is no overlap of the blade tip speeds of main and tail rotors, and propellers. In practice, there will be some overlap, which will cause the minimum PRF to be increased in some cases, and reduced in other cases.

5.2.3 Observation Interval

There is also a minimum value that the observation interval can have in order for the frequency spectrum to contain sidebands. In order for the frequency spectrum to contain sidebands, the observation interval must be greater than or equal to twice the blade-passing interval of the aircraft, i.e.

\[ T_o \geq \frac{2}{N_f}, \quad (5.12) \]

If the observation interval is less than twice the blade-passing interval of the aircraft, then the nature of the frequency spectrum will depend on whether the return signal contains any blade flashes.

If the observation interval is less than twice the blade-passing interval, and the return signal contains any blade flashes, then the frequency spectrum may or may not contain sidebands, depending on the actual return signal, and the processing of the return signal, i.e.

\[ N_2 \leq \frac{8\pi (L_2 - L_1) \cos(\theta)}{N \lambda}, \quad (5.13) \]

However, the frequency spectrum will have a bandwidth corresponding to the maximum blade tip speed, i.e.

\[ B_2 = \frac{8\pi f_r (L_2 - L_1) \cos(\theta)}{\lambda}, \quad (2.38) \]

In the case where the frequency spectrum does not contain any blade flashes, the frequency spectrum will be continuous.

If the observation interval is less than twice the blade-passing interval, and the return signal does not contain any blade flashes, then, again, the frequency spec-
trum may or may not contain sidebands, depending on the actual return signal, and the processing of the return signal. However, the frequency spectrum will have a bandwidth corresponding to less than the maximum blade tip speed, i.e.

$$B_2 \leq \frac{8\pi f_r (L_2 - L_1) \cos(\theta)}{\lambda}$$

(5.14)

If we assume that the minimum blade-passing frequencies of main rotors, tail rotors and propellers are 10 Hz, 50 Hz, and 50 Hz, respectively, then the maximum blade-passing intervals will be 0.1 s, 0.02 s, and 0.02 s, respectively. Therefore, in order for the frequency spectrum to contain sidebands, the observation intervals for main rotors, tail rotors and propellers must be greater than or equal to 0.2 s, 0.04 s, and 0.04 s, respectively.

If the observation interval is greater than or equal to 0.2 s, then the frequency spectra of main rotors, tail rotors and propellers will contain sidebands. If the observation interval is greater than or equal to 0.04 s, but less than 0.2 s, then the frequency spectra of tail rotors and propellers will contain sidebands, but the frequency spectra of main rotors will be continuous. If the observation interval is less than 0.04 s then the frequency spectra of main rotors, tail rotors and propellers will be continuous.

If we assume that there is no a priori knowledge of the class of aircraft, then in order for the frequency spectrum of any rotor or propeller aircraft to contain sidebands, the observation interval must be greater than or equal to the minimum observation interval for the class of aircraft which has the longest blade-passing interval, i.e. the observation interval must be greater than or equal to 0.2 s, corresponding to the minimum observation interval for main rotors.

Note that the analysis above assumes that there is no overlap of the blade-passing intervals of main rotors, and tail rotors and propellers. In practice, there will be some overlap, which will cause the minimum observation interval to be increased in some cases, and reduced in other cases.
If no window function is applied to the frequency spectrum, then the frequency resolution of the frequency spectrum will be equal to the reciprocal of the observation interval, i.e. [93]

$$\Delta F = \frac{1}{T_o}$$  \hspace{1cm} (5.15)

where $\Delta F =$ frequency resolution.

If a window function is applied to the frequency spectrum then the frequency resolution of the frequency spectrum will be less than this value, i.e.

$$\Delta F > \frac{1}{T_o}$$ \hspace{1cm} (5.16)

In this case, the frequency resolution will be determined by the actual window function applied.

5.3 Window Functions

Window functions are weighting functions which are applied to signals in order to reduce the leakage caused by finite observation intervals [94].

If the observation interval is finite and does not correspond to a whole number of periods of the input signal, then this will cause a discontinuity in the input signal, at the ends of the observation interval [95]. This is equivalent to multiplying the input signal by a rectangular function, which is equivalent to convolving the frequency spectrum with a sinc function [96]. This will cause leakage - the greater the discontinuity, the greater the leakage will be.

Window functions are tapered weighting functions which are applied to the input signal, such that the weight of the input signal is reduced near to the discontinuity [97]. This has the effect of reducing the magnitude of the discontinuity, and therefore has the effect of reducing the amount of leakage [98].

All tapered window functions will have lower sidelobes than those of a rectangular window function, therefore all tapered window functions will reduce leak-
age, compared with that of a rectangular window function [99]. However, all tapered window functions will also have a wider main lobe than that of a rectangular window function, therefore all tapered window functions will also reduce the frequency resolution, compared with that of a rectangular window function [100].

There are many different types of window function. Each one has both advantages and disadvantages, and no single window function is optimum for all applications [101]. The optimum window function for a given application will depend on the actual application [102].

There are a number of parameters by which the performance of window functions can be measured [103]. However, in this section we will only discuss three of the main parameters: the 3-dB bandwidth, the processing gain, and the scalloping loss.

5.3.1 3-dB Bandwidth

The 3-dB bandwidth of a window function is defined as the frequency at which the gain of each Doppler bin is reduced from its maximum value by 3-dB, measured in bin widths, i.e.

\[ B_{3\text{-dB}} = \frac{f_0 - f_{3\text{-dB}}}{\Delta F} \]  

(5.17)

where \( B_{3\text{-dB}} \) = 3-dB bandwidth, measured in bin widths,

\( f_0 \) = centre frequency of the bin,

\( f_{3\text{-dB}} \) = frequency at which the gain of the bin is reduced from its maximum value by 3-dB.

The 3-dB bandwidth is a measure of the frequency resolution of each bin. The minimum 3-dB bandwidth occurs for a rectangular window function, in which case

\[ B_{3\text{-dB}}_{\text{min}} = 0.9 \]  

(5.18)

where \( B_{3\text{-dB}}_{\text{min}} \) = minimum 3-dB bandwidth, measured in bin widths.

The minimum 3-dB bandwidth occurs for this window function, because in this case
there will be no tapering of the window function, therefore Equation (5.17) will be a minimum.

Values of $B_{\text{3-dB}}$ range from about 0.9 for a rectangular window function to about 1.9 for a Hann-Poisson window function. Any window function other than a rectangular window function will cause some increase in the 3-dB bandwidth.

### 5.3.2 Processing Gain

The processing gain of a window function is defined as the mean gain of the window function, i.e.

$$G_p = \frac{1}{N_s} \sum_{n=0}^{N_s-1} w(nT_s) = W(0)$$  \hspace{1cm} (5.19)

where $G_p =$ processing gain,

$N_s =$ number of FFT sample points,

$w(nT_s) =$ window function,

$W(m\Delta f) =$ Fourier transform of $w(nT_s)$.

The processing gain is a measure of the reduction in gain caused by the window function. The maximum processing gain occurs for a rectangular window function, in which case

$$G_{p_{\text{max}}} = 1$$  \hspace{1cm} (5.20)

where $G_{p_{\text{max}}} =$ maximum processing gain.

The maximum processing gain occurs for this window function, because in this case the weight of each window function sample will be a maximum, therefore Equation (5.19) will be a maximum.

Values of $G_p$ range from about 0.2 for a Poisson window function to 1 for a rectangular window function. Any window function other that a rectangular window function will cause some reduction in processing gain.
5.3.3 Scalloping Loss

The scalloping loss of a window function is defined as the ratio of the gain for a frequency located at a fraction of a bin from an FFT sample point to the gain for the frequency located at the FFT sample point, i.e.

\[ L_s = \frac{\sum_{n=0}^{N/2-1} w(nT_s) e^{-j2\pi f_i n/N_s} }{\sum_{n=0}^{N/2-1} w(nT_s)} = \frac{|W(f_i)|}{W(0)} \]

(5.21)

where \( f_i \) = input frequency,

\( L_s \) = scalloping loss.

The scalloping loss is a measure of the loss for a frequency located at a fraction of a bin from an FFT sample point. The maximum scalloping loss occurs for a frequency located at half a bin from an FFT sample point, in which case

\[ L_{s,\text{max}} = \frac{\sum_{n=0}^{N/2-1} w(nT_s) e^{-j2\pi f_i n/N_s} }{\sum_{n=0}^{N/2-1} w(nT_s)} = \frac{|W(\frac{\Delta F}{2})|}{W(0)} \]

(5.22)

where \( L_{s,\text{max}} \) = maximum scalloping loss.

The maximum scalloping loss occurs for this frequency, because in this case the frequency will be located at the greatest distance from the nearest bin, therefore Equation (5.21) will be a minimum.

Values of \( L_{s,\text{max}} \) range from about 0.8 for a rectangular window function to about 0.9 for a Hann window function. Any frequencies which are not at FFT sample points will have some scalloping loss.

5.4 Relative Acceleration

If the aircraft has any relative acceleration, then this may affect the frequency spectrum and power cepstrum, and therefore may affect the performance of any
feature extraction or aircraft classification algorithms. Acceleration will have three main effects on the frequency spectrum:

1. It will cause a Doppler frequency spread of the aircraft spectrum.
2. It will cause a Doppler frequency shift of the aircraft spectrum.
3. It will cause a reduction in the magnitude of the aircraft spectrum, due to the frequency spread of the spectrum.

Aircraft can have two types of acceleration: translational and rotational. With translational acceleration, the acceleration is caused by a change in the translational velocity of the aircraft. With rotational acceleration, the acceleration is caused by a change in the direction of motion of the aircraft.

In practice, aircraft will have a maximum possible translational acceleration of about $\pm 10 \text{ m/s}^2$. Therefore, if the radar is ground-based, then there will be maximum possible relative acceleration of about $\pm 10 \text{ m/s}^2$, since the radar platform will be stationary. However, if the radar is airborne, then there will be maximum possible relative acceleration of about $\pm 20 \text{ m/s}^2$, since the radar platform may also be accelerating. Similarly, aircraft will have a maximum possible rotational acceleration of about $\pm 100 \text{ m/s}^2$. Therefore, if the radar is ground-based, then there will be a maximum possible relative acceleration of about $\pm 100 \text{ m/s}^2$. However, if the radar is airborne, then there will be a maximum possible relative acceleration of about $\pm 200 \text{ m/s}^2$.

Note that when in level flight, most aircraft will cruise at a constant velocity, and will have no translational or rotational acceleration. Translational acceleration is most likely to occur when the aircraft is climbing or descending, or when the aircraft is near to take-off or landing, and rotational acceleration is most likely to occur when the aircraft changes its direction of motion.

This section only considers the case of translational acceleration. However, all of the theory and results in the section can also be extended to apply to the case of rotational acceleration.
Consider the return signal from the airframe of an aircraft which consists of a single scatterer and which has a constant acceleration. The return signal is given by

\[ v_r(t) = A_r e^{j(\omega_0 - 4\pi(a + \frac{\pi^2}{2})t)} \]  

where \( a \) = relative acceleration of the aircraft.

The Fourier transform of Equation (5.23) is

\[ V_r(f) = A_r e^{j(f - \frac{\omega_0}{4a} + \text{sgn}(a)\frac{\pi}{4})} \]  

where \( A_r = A_r \left( \frac{\lambda}{2|a|} \right)^{\frac{1}{2}} e^{-j\left(\frac{4\pi(a + \text{sgn}(a)\frac{\pi}{4})}{4a}\right)} \),

since

\[ F(e^{j\omega^2}) = \left( \frac{\pi}{|a|} \right)^{\frac{1}{2}} e^{-j\left(\frac{\omega^2}{4a} + \text{sgn}(a)\frac{\pi}{4}\right)} \]  

Equation (5.25) shows that the Fourier transform of a sinusoidal function which has a squared variable is also a sinusoidal function which has a squared variable. This is analogous to the way in which the Fourier transform of a Gaussian function is also a Gaussian function, i.e. [104]

\[ F(e^{-a^2}) = \left( \frac{\pi}{|a|} \right)^{\frac{1}{2}} e^{-\frac{a^2}{4a}} \]  

Figure 5.1 shows the real component of the return signal from the airframe of an aircraft which consists of a single scatterer in which \( a = -10 \text{ m/s}^2 \), and \( \lambda = 1 \text{ m} \). It can be seen that the return signal consists of a sinusoidal function in which the frequency increases linearly with time. The increase in frequency with time is because the aircraft has a positive acceleration - if the aircraft had a negative acceleration, then this would cause a decrease in frequency with time.

Figure 5.2 shows the real component of the frequency spectrum of the aircraft of Figure 5.1. It can be seen that the frequency spectrum also consists of a sinusoidal function. However, in this case, the periodicity decreases linearly with
Figure 5.1.
Return signal from the airframe of an aircraft which consists of a single scatterer in which $a = -10 \text{ m/s}^2$, and $\lambda = 1 \text{ m}$.
Figure 5.2.
Frequency spectrum of the airframe of an aircraft which consists of a single scatterer in which $a = -10$ m/s$^2$, and $\lambda = 1$ m.
frequency. The decrease in periodicity with frequency is because of the sign reversal of the exponential terms in Equation (5.25). Note that care must be taken with the terminology here, since we are referring to a sinusoidal function in the frequency domain, i.e. we are referring to a function which has frequency in the frequency domain, hence the use of the term "periodicity", instead of "frequency".

In the example of Figure 5.1, it was assumed that the observation interval was infinite. In practice, the observation interval will be finite.

Now consider the return signal from the airframe of an aircraft which consists of a single scatterer which has a constant acceleration, and where the observation interval is finite. The return signal is given by

\[
v_r(t) = A_r e^{j \left( \omega_c - \frac{4\pi}{\lambda} (\pi + \omega^2/2) \right)} \int \frac{t - T_o}{T_o} \text{rect} \left( \frac{t - T_o}{T_o} \right)
\]

The Fourier transform of Equation (5.27) is given by

\[
V_r(f) = A_r e^{j \left( \omega_c - \frac{4\pi}{\lambda} (\pi + \omega^2/2) \right)} \int \frac{t - T_o}{T_o} \text{rect} \left( \frac{t - T_o}{T_o} \right)
\]

where

\[
A_r = A_r \left( \frac{\lambda}{4|a|} \right)^{1/2} e^{-j \frac{4\pi a}{\lambda}}
\]

\[
\times \left[ (C \left( \frac{T_o}{\lambda} \right) + \text{sign}(a)2f \left( \frac{\lambda}{4|a|} \right)^{1/2} - C \left( \text{sign}(a)2f \left( \frac{\lambda}{4|a|} \right)^{1/2} \right)) \right. \\
\left. - j \text{sign}(a) \left( S \left( \frac{T_o}{\lambda} \right) + \text{sign}(a)2f \left( \frac{\lambda}{4|a|} \right)^{1/2} - S \left( \text{sign}(a)2f \left( \frac{\lambda}{4|a|} \right)^{1/2} \right) \right) \right]
\]

\[
C(T_o) = \int_{0}^{T_o} \cos \left( \frac{\pi}{2} t^2 \right) dt = \text{Fresnel cosine integral},
\]

\[
S(T_o) = \int_{0}^{T_o} \sin \left( \frac{\pi}{2} t^2 \right) dt = \text{Fresnel sine integral},
\]

since

\[
F \left( e^{j \omega t} \text{rect} \left( \frac{t - T_o}{T_o} \right) \right) = \left( \frac{\pi}{2|a|} \right)^{1/2} e^{-j \frac{\omega^2}{4a}}
\]
\[(C \left( \frac{T_o}{2|a|} \right)^{\frac{1}{2}} \text{sign}(a)2f \left( \frac{\pi}{2|a|} \right)^{\frac{1}{2}})

- \left( \frac{T_o}{2|a|} \right)^{\frac{1}{2}} \text{sign}(a)2f \left( \frac{\pi}{2|a|} \right)^{\frac{1}{2}})

+j \text{sign}(a) \left( \frac{T_o}{2|a|} \right)^{\frac{1}{2}} \text{sign}(a)2f \left( \frac{\pi}{2|a|} \right)^{\frac{1}{2}})

- \left( \frac{T_o}{2|a|} \right)^{\frac{1}{2}} \text{sign}(a)2f \left( \frac{\pi}{2|a|} \right)^{\frac{1}{2}})

\]

Equation (5.29) shows that the Fourier transform of a sinusoidal function which has a squared variable, and which has a finite duration, is also a sinusoidal function which has a squared variable, but is multiplied by a Fresnel integral function. Again, this is analogous to the way in which the Fourier transform of a Gaussian function which has a finite duration is also a Gaussian function, but is multiplied by a Fresnel integral function, i.e.

\[F(e^{-a^2 \text{rect}(t - \frac{T_o}{2})}) = \left( \frac{\pi}{2|a|} \right)^{\frac{1}{2}} e^{-\frac{a^2}{4|a|^2} - \frac{\pi}{4}}\]

Figure 5.3 shows the real component of the return signal from the airframe of an aircraft which consists of a single scatterer in which \(a = -10 \text{ m/s}^2\), \(T_o = 4 \text{ s}\), and \(\lambda = 1 \text{ m}\). It can be seen that Figure 5.3 is identical to Figure 5.1, except that the observation interval is finite. This signal is similar to those which occur in linear FM pulse compression [105]. (Note that in the case of rotational acceleration, the signal will be similar to those which occur in non-linear FM pulse compression.) In
Figure 5.3.
Return signal from the airframe of an aircraft which consists of a single scatterer in which $a = -10 \text{ m/s}^2$, $T_o = 4 \text{ s}$, and $\lambda = 1 \text{ m}$. 
linear FM pulse compression, the frequency of the transmitted pulse is increased or decreased linearly, with respect to time [106]. The received pulse is then passed through a pulse compression filter which has a time delay v. frequency characteristic, such that it linearly delays one end of the pulse, with respect to the other [107]. This produces a shorter pulse with a greater magnitude at the output of the filter, than the pulse at the input of the filter [108].

Figure 5.4 shows the frequency spectrum of the aircraft of Figure 5.3, with no window function applied. It can be seen that there is a Doppler frequency spread and a Doppler frequency shift of the frequency spectrum, due to the acceleration. There will also be a reduction in the magnitude of the frequency spectrum, due to the acceleration. It can also be seen that the frequency spectrum is symmetrical and consists of an approximately rectangular pulse which has a Fresnel ripple superimposed on it. The Doppler frequency spread of the frequency spectrum given by

$$B_a = \frac{2|a|T_0}{\lambda}$$

(5.31)

where $B_a =$ cyclic Doppler frequency spread of the frequency spectrum, due to relative acceleration.

The Doppler frequency shift of the frequency spectrum is given by

$$f_{d_a} = -\frac{aT_0}{\lambda}$$

(5.32)

where $f_{d_a} =$ cyclic Doppler frequency shift of the frequency spectrum, due to relative acceleration.

It can be seen that $B_a = 80$ Hz, and $f_{d_a} = 40$ Hz.

It can be seen from Equation (5.31) that $B_a$ is a function of $a$, $T_0$, and $\lambda$. It can also be seen from Equations (5.31) and (5.32) that $f_{d_a}$ is half of $B_a$, since the frequency spectrum is symmetrical, therefore $f_{d_a}$ is also a function of $a$, $T_0$, and $\lambda$.

Figure 5.5 shows the frequency spectrum of the aircraft of Figure 5.3, with a Hann window function applied. It can be seen that Figure 5.5 is identical to Figure 153.
Figure 5.4.
Frequency spectrum of the airframe of an aircraft which consists of a single scatterer in which $a = -10 \text{ m/s}^2$, $T_o = 4 \text{ s}$, and $\lambda = 1 \text{ m}$, with no window function applied.
Figure 5.5.
Frequency spectrum of the airframe of an aircraft which consists of a single scatterer in which $a = -10 \text{ m/s}^2$, $T_o = 4 \text{ s}$, and $\lambda = 1 \text{ m}$, with a Hanning window function applied.
except that the frequency spectrum has been smoothed. The window function
has the effect of removing the Fresnel ripples from the frequency spectrum, and
tapering the frequency spectrum at the upper and lower cut-off frequencies. The
window function also has the effect of reducing further the magnitude of the fre-
quency spectrum.

Now consider the return signal from the rotating blades of an aircraft which
has a constant acceleration, and where the observation interval is finite. The return
signal is given by

$$v_r(t) = \sum_{n=0}^{N-1} A_n \sin\left(\frac{4\pi}{\lambda} \frac{(L_2-L_1)}{2}\cos(\theta)\sin(\omega_r t + \frac{2\pi n}{N})\right)$$

where $A_n = A_r (L_2-L_1)$.

The Fourier transform of Equation (5.33) is given by

$$V_r(f) = \sum_{k=-N_2/2}^{N_2/2} c_{nk} e^{-j \frac{(\omega - \omega_0 - Nk \omega_r)^2}{4 \Delta n \lambda}}$$

Figure 5.6 shows the frequency spectrum of a rotor aircraft in which $N=4$,
$L_2=5 \text{ m}$, $f_r=6 \text{ Hz}$, $\theta=0 \text{ rad.}$, $T_o=4 \text{ s}$, and $\lambda=1 \text{ m}$, with a Hann window function
applied. It can be seen that $N=30$, $\Delta f=24 \text{ Hz}$, and $B=720 \text{ Hz}$. Figure 5.7
Figure 5.6.
Frequency spectrum of a rotor aircraft in which $N=4, L_2=5\text{ m}, f_r=6\text{ Hz}, \theta=0$ rad., $T_o=4\text{ s}$, and $\lambda=1\text{ m}$.
Figure 5.7.
Power cepstrum of a rotor aircraft in which $N=4$, $L_2=5$ m, $f_r=6$ Hz, $\theta=0$ rad., $T_o=4$ s, and $\lambda=1$ m.
shows the power cepstrum of the aircraft of Figure 5.6. Again, it can be seen that \( \Delta f = 24 \text{ Hz} \).

Figure 5.8 shows the frequency spectrum of an aircraft which is identical to that of Figure 5.6, except that in this case \( a = -10 \text{ m/s}^2 \). It can be seen that it is no longer possible to determine the correct values of \( \Delta f \) and \( B_2 \), since the frequency spectrum has been spread, due to the acceleration. Note that the frequency spectrum has also been Doppler frequency shifted, due to the acceleration. It can also be seen that \( B_1 = 80 \text{ Hz} \), and \( f_{d_2} = 40 \text{ Hz} \).

Figure 5.9 shows the power cepstrum of the aircraft of Figure 5.8. It can be seen that it is no longer possible to determine the correct value of \( \Delta f \), since the spreading of the frequency spectrum has caused the power cepstrum to indicate a value of \( \Delta f = 0.7 \text{ Hz} \), i.e. \( \Delta f = 24/33.6 \text{ Hz} \), instead of \( \Delta f = 24 \text{ Hz} \).

It can be seen from the example above that if there is any acceleration, then this may affect the frequency spectrum and power cepstrum of the aircraft, and therefore may affect the performance of any feature extraction or aircraft classification algorithms.

Figure 5.10 shows an acceleration compensation system, which can reduce the effects of any acceleration. The system can reduce the effects of the acceleration, if good estimates of the acceleration can be obtained, or can remove the effects of the acceleration, if the acceleration is known exactly. The system operates by using the estimates of the acceleration to remove the phase component of the return signal, due to the acceleration. It can be seen that the system would be easy to implement in software or in hardware. Unfortunately, the acceleration has to be estimated, since acceleration cannot be measured directly by the radar, therefore there will be some estimation error. However, no additional hardware would be required in order to obtain estimates of the acceleration, in the case of tracking radars, since acceleration is routinely estimated for aircraft tracking.

If the return signal is applied to the system, then the output signal from the
Figure 5.8.
Frequency spectrum of a rotor aircraft in which $N=4$, $L_2=5$ m, $f_r=6$ Hz, $\theta=0$ rad., $a=-10$ m/s$^2$, $T_0=4$ s, and $\lambda=1$ m.
Figure 5.9.
Power cepstrum of a rotor aircraft in which $N=4$, $L_2=5\ m$, $f_r=6\ Hz$, $\theta=0\ rad.$, $a=-10\ m/s^2$, $T_o=4\ s$, and $\lambda=1\ m$. 
Figure 5.10.
An acceleration compensation circuit.
system is given by

\[
\nu_n(t) = \sum_{n=0}^{N-1} A_n \text{sinc} \left( \frac{4\pi}{\lambda} \left( \frac{L_2 - L_1}{2} \cos(\theta) \sin(\omega, t + \frac{2\pi n}{N}) \right) \right)
\]

where \( A_n = A, (L_2 - L_1) \),

\( a_{err} = a - a_{est} \) = error relative acceleration of the aircraft,

\( a_{est} \) = estimated relative acceleration of the aircraft.

The Fourier transform of Equation (5.35) is given by

\[
V_r(f) = \sum_{k=-N/2}^{N/2-1} c_{nk} e^{j \frac{\omega_0 - \omega n}{2 |a_{err}|}}
\]

where \( c_{nk} = \sum_{n=0}^{N-1} \frac{1}{4\pi} \cos(\theta) \)

\[
\cdot \left( J_{N/2+1} \left( \frac{4\pi}{\lambda} (L_2 \cos(\theta)) \right) - J_{N/2+1} \left( \frac{4\pi}{\lambda} - L_1 \cos(\theta) \right) \right) e^{-\frac{4\pi n}{\lambda}}
\]

\[
\cdot \left( (C \left( \frac{T_n}{\lambda} \right) + \text{sign}(a) 2f \left( \frac{\lambda}{4 |a_{err}|} \right)^{\frac{1}{2}} \right)
\]

\[
- C \left( \text{sign}(a) 2f \left( \frac{\lambda}{4 |a_{err}|} \right)^{\frac{1}{2}} \right)
\]

\[
-j \text{sign}(a) \left( S \left( \frac{T_n}{\lambda} \right) + \text{sign}(a) 2f \left( \frac{\lambda}{4 |a_{err}|} \right)^{\frac{1}{2}} \right)
\]

\[
- \left( S \left( \text{sign}(a) 2f \left( \frac{\lambda}{4 |a_{err}|} \right)^{\frac{1}{2}} \right) \right).
\]

As \( a_{err} \to 0 \) m/s², Equation (5.35) will tend to a down-converted form of Equation (2.11), i.e.

\[
\nu_n(t) = \sum_{n=0}^{N-1} A_n \text{sinc} \left( \frac{4\pi}{\lambda} \left( \frac{L_2 - L_1}{2} \cos(\theta) \sin(\omega, t + \frac{2\pi n}{N}) \right) \right)
\]

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Similarly, Equation (5.36) will tend to a down-converted form of Equation (2.25), i.e.

\[ V_r(f) = \sum_{k=-N_2}^{N_2} c_{Nk} \delta(f - f_d - Nkf_r) \]

(5.38)

where

\[ c_{Nk} = \frac{2(-1)^{\alpha(k)N}}{4\pi \lambda \cos(\theta)} \]

\[ \cdot (J_{N|k|+2l+1}^N(\frac{4\pi \lambda}{L} - L_1 \cos(\theta)) - J_{N|k|+2l+1}^N(\frac{4\pi \lambda}{L} + L_1 \cos(\theta)))e^{-\frac{j2\pi f}{\lambda}} \]

since

\[ F(e^{j\omega t^2})|_{\omega=\omega_0} = \delta(f) \]

(5.39)

Figures 5.11-5.15 show the frequency spectrum of the aircraft of Figure 5.8 after the return signal has been applied to the system, as \(a_{err}\) is reduced from 0 m/s\(^2\) to -10 m/s\(^2\) in steps of 2.5 m/s\(^2\), i.e. as \(|a_{err}|\) is reduced from 10 m/s\(^2\) to 0 m/s\(^2\) in steps of 2.5 m/s\(^2\). It can be seen that as \(|a_{err}|\) - 0 m/s\(^2\), the Doppler frequency spread and the Doppler frequency shift, due to the acceleration, are reduced. There will also be an increase in the magnitude of the frequency spectrum, as \(|a_{err}|\) - 0 m/s\(^2\). It can also be seen that when \(|a_{err}|\) = 10 m/s\(^2\), the frequency spectrum is continuous, and no sidebands can be seen; as \(|a_{err}|\) - 0 m/s\(^2\), the effects of the acceleration are reduced and sidebands begin to appear in the frequency spectrum; and when \(|a_{err}|\) = 0 m/s\(^2\), the original frequency spectrum, i.e. the frequency spectrum with no acceleration, is obtained.

Figures 5.16-5.20 show the power cepstrum of the aircraft of Figure 5.8 after the return signal has been applied to the system, as \(a_{err}\) is reduced from 0 m/s\(^2\) to -10 m/s\(^2\) in steps of 2.5 m/s\(^2\). As \(|a_{err}|\) - 0 m/s\(^2\), there will be an increase in the magnitude of the power cepstrum. It can be seen that when \(|a_{err}|\) = 10 m/s\(^2\), the
Figure 5.11.
Frequency spectrum of a rotor aircraft in which $N = 4$, $L_2 = 5$ m, $f_r = 6$ Hz, $\theta = 0$ rad., $a = -10$ m/s$^2$, $a_{rms} = 0$ m/s$^2$, $T_o = 4$ s, and $\lambda = 1$ m.
Figure 5.12.
Frequency spectrum of a rotor aircraft in which $N = 4$, $L_z = 5$ m, $f_r = 6$ Hz, $\theta = 0$ rad., $a = -10$ m/s$^2$, $a_{eff} = -2.5$ m/s$^2$, $T_o = 4$ s, and $\lambda = 1$ m.
Figure 5.13.
Frequency spectrum of a rotor aircraft in which $N = 4$, $L_z = 5$ m, $f_r = 6$ Hz, $\theta = 0$ rad., $a = -10$ m/s$^2$, $a_{err} = -5$ m/s$^2$, $T_0 = 4$ s, and $\lambda = 1$ m.
Figure 5.14.
Frequency spectrum of a rotor aircraft in which $N=4$, $L_2=5$ m, $f_r=6$ Hz, $\theta=0$ rad., $a=-10$ m/s$^2$, $a_{\text{rr}}=-7.5$ m/s$^2$, $T_o=4$ s, and $\lambda=1$ m.
Frequency spectrum of a rotor aircraft in which $N=4$, $L_2=5$ m, $f_r=6$ Hz, $\theta=0$ rad., $a=-10$ m/s$^2$, $a_{\alpha}=-10$ m/s$^2$, $T_o=4$ s, and $\lambda=1$ m.

Figure 5.15.
Figure 5.16.
Power cepstrum of a rotor aircraft in which $N=4$, $L_2=5$ m, $f_r=6$ Hz, $\theta=0$ rad.,
$a=-10$ m/s$^2$, $a_{ax}=0$ m/s$^2$, $T_o=4$ s, and $\lambda=1$ m.
Figure 5.17.
Power cepstrum of a rotor aircraft in which $N=4$, $L_2=5$ m, $f_r=6$ Hz, $\theta=0$ rad.,
$a=-10$ m/s$^2$, $a_{\theta}=2.5$ m/s$^2$, $T_o=4$ s, and $\lambda=1$ m.
Figure 5.18.
Power cepstrum of a rotor aircraft in which $N=4$, $L_2=5$ m, $f_r=6$ Hz, $\theta=0$ rad.,
$a=-10$ m/s$^2$, $a_{ext}=-5$ m/s$^2$, $T_0=4$ s, and $\lambda=1$ m.
Figure 5.19.
Power cepstrum of a rotor aircraft in which \( N = 4, L_2 = 5 \text{ m}, f_r = 6 \text{ Hz}, \theta = 0 \text{ rad.}, a = -10 \text{ m/s}^2, a_{\text{eff}} = -7.5 \text{ m/s}^2, T_o = 4 \text{ s}, \) and \( \lambda = 1 \text{ m}. \)
Figure 5.20.
Power cepstrum of a rotor aircraft in which $N = 4$, $L_2 = 5$ m, $f_r = 6$ Hz, $\theta = 0$ rad., $a = -10$ m/s$^2$, $a_{in} = -10$ m/s$^2$, $T_o = 4$ s, and $\lambda = 1$ m.
power cepstrum indicates an incorrect value for $\Delta f$; as $|a_{err}| = 0 \text{ m/s}^2$, the effects of the acceleration are reduced; and when $|a_{err}| = 0 \text{ m/s}^2$, the original power cepstrum is obtained.

The frequency spectra of Figures 5.11-5.15 and the power cepstra of Figures 5.16-5.20 were somewhat unrealistic, in that, in each case, it was assumed that the acceleration and estimated acceleration, therefore the error acceleration, were constant. In practice, the acceleration and estimated acceleration, therefore the error acceleration, may vary from pulse to pulse. This will reduce the performance of the system - the greater the error acceleration, the greater the reduction will be.

The system has the advantage that if the acceleration does vary from pulse to pulse, then the system will still reduce the effects of any acceleration, provided that good estimates of the acceleration can still be obtained, i.e. provided that the average value of $|a_{err}|$ is less than the average value of $|a|$. However, it is likely that if the acceleration does vary from pulse to pulse, then this will reduce the accuracy of the estimates - the greater the variation, the greater the reduction will be. Again, this will reduce the performance of the system.

The system could also be extended to deal with higher order rates of change of motion, i.e. $\dot{a}$, $\ddot{a}$, etc. Unfortunately, higher order rates are not usually estimated for aircraft tracking, therefore additional software or hardware would be required. Also, higher order rates would be more difficult to estimate accurately than lower order rates. However, the values of higher order rates would be smaller than those of lower order rates, therefore they would have less effect on the frequency spectrum and power cepstrum.

5.5 Aircraft Classification

Although this thesis is concerned with the analysis of the return signal from rotating aircraft blades, and not with the classification of aircraft using these return signals, it is still possible to make some progress toward aircraft classification.
In Chapter 2 it was shown that there are a countable number of sets of aircraft parameters which provide a solution set of the aircraft features. However, the number of solution sets can be reduced dramatically, by using the results of Chapters 2, 3 and 4. This can be done by using the following algorithm:

1. Extract the aircraft features from the return signal.
2. Compile a table of sets of aircraft parameters which provide a solution set of the aircraft features.
3. Remove all solution sets in which the blade symmetry is not equal to that extracted from the return signal.
4. Remove all solution sets in which the aircraft parameters are not within the limits for actual aircraft.
5. Remove all solution sets in which one or more of the aircraft parameters do not correspond to those of any actual aircraft.
6. Remove all solution sets in which the combination of the aircraft parameters does not correspond to that of any actual aircraft.

The operation of this algorithm can be shown by the following example.

Figure 5.21 shows the frequency spectrum of a rotor aircraft in which $N = 4$, $L_2 = 5$ m, $f_r = 6$ Hz, $\theta = 0$ rad., and $\lambda = 1$ m. It can be seen that $N_2 = 30$, $\Delta f = 24$ Hz, and $B_2 = 720$ Hz. Table 5.1 shows a number of sets of aircraft parameters which provide a solution set of the aircraft features.

In Chapter 4 it was shown that the blade symmetry can be extracted from the power cepstrum of the return signal. Therefore, if we remove all solution sets in which the blade symmetry is not even, then Table 5.1 can be reduced to give Table 5.2.

In Chapter 3 it was shown that the value of $N$ for main rotors ranges from 2 to 8. Therefore, if we remove all solution sets in which the value of $N$ is not in this range, then Table 5.2 can be reduced to give Table 5.3.
Figure 5.21.
Frequency spectrum of a rotor aircraft in which $N=4$, $L_2=5$ m, $f_r=6$ Hz, $\theta=0$ rad., and $\lambda=1$ m.
Table 5.1.
Table showing a number of sets of aircraft parameters which provide a solution set of the aircraft features of a rotor aircraft in which \( N = 4 \), \( L_2 = 5 \text{ m} \), \( f_r = 6 \text{ Hz} \), \( \theta = 0 \) rad., and \( \lambda = 1 \text{ m} \).

<table>
<thead>
<tr>
<th>( N )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>( \cdots )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_2 ) (m)</td>
<td>1.25</td>
<td>2.5</td>
<td>3.75</td>
<td>5</td>
<td>6.25</td>
<td>( \cdots )</td>
</tr>
<tr>
<td>( f_r ) (Hz)</td>
<td>24</td>
<td>12</td>
<td>8</td>
<td>6</td>
<td>4.8</td>
<td>( \cdots )</td>
</tr>
</tbody>
</table>
Table 5.2.

Table showing a number of sets of aircraft parameters which provide a solution set of the aircraft features of a rotor aircraft in which $N=4$, $L_2=5$ m, $f_r=6$ Hz, $\theta=0$ rad., and $\lambda=1$ m, after removing all solution sets in which the blade symmetry is not even.
Table 5.3.  
Table showing the sets of aircraft parameters which provide a solution set of the aircraft features of a rotor aircraft in which \( N = 4 \), \( L_2 = 5 \) m, \( f_r = 6 \) Hz, \( \theta = 0 \) rad., and \( \lambda = 1 \) m, after removing all solution sets in which the value of \( N \) is not in the range of 2 to 8.

<table>
<thead>
<tr>
<th>( N )</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_2 ) (m)</td>
<td>2.5</td>
<td>5</td>
<td>7.5</td>
<td>10</td>
</tr>
<tr>
<td>( f_r ) (Hz)</td>
<td>12</td>
<td>6</td>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>
It was also shown in Chapter 3 that the value of $L_2$ for main rotors ranges from about 3.5 m to 15 m. Therefore, if we remove all solution sets in which the value of $L_2$ is not in this range, then Table 5.3 can be reduced to give Table 5.4.

It was further shown in Chapter 3 that the value of $f_r$ for main rotors ranges from about 2 Hz to 10 Hz. Therefore, if we remove all solution sets in which the value of $f_r$ is not in this range, then in some cases it may be possible to reduce Table 5.4 even further.

Each column of Table 5.4 corresponds to a solution set in which the aircraft parameters are within the limits for rotor aircraft. However, although it is possible to have a rotor aircraft in which $L_2=10$ m, and $f_r=3$ Hz, in the sense that these parameters are within the limits for rotor aircraft, it may be that there is no actual rotor aircraft which has one or more of these parameters. In this example, we will assume that there is no actual rotor aircraft in which $L_2=10$ m. Therefore, if we remove all solution sets in which one or more of the aircraft parameters do not correspond to those of any actual aircraft, then Table 5.4 can be reduced to give Table 5.5.

Again, each column of Table 5.5 corresponds to a solution set in which the aircraft parameters are within the limits for rotor aircraft. However, although it is possible to have a rotor aircraft in which $N=6$, $L_2=7.5$ m, and $f_r=4$ Hz, in the sense that these parameters are within the limits for rotor aircraft, and in the sense that they correspond to the parameters of actual aircraft, it may be that there is no actual rotor aircraft which has this combination of parameters. In this example, we will assume that there is no actual rotor aircraft in which $N=6$, $L_2=7.5$ m, and $f_r=4$ Hz. Therefore, if we remove all solution sets in which the combination of the aircraft parameters does not correspond to that of any actual aircraft, then Table 5.5 can be reduced to give Table 5.6.

Note that, in order to remove all solution sets in which the parameters do not correspond to those of any actual aircraft, and all solution sets in which the combi-
Table 5.4.
Table showing the sets of aircraft parameters which provide a solution set of the aircraft features of a rotor aircraft in which $N=4$, $L_2=5$ m, $f_r=6$ Hz, $\theta=0$ rad., and $\lambda=1$ m, after removing all solution sets in which the value of $L_2$ is not in the range of 3.5 to 15 m.

<table>
<thead>
<tr>
<th>$N$</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_2$ (m)</td>
<td>5</td>
<td>7.5</td>
<td>10</td>
</tr>
<tr>
<td>$f_r$ (Hz)</td>
<td>6</td>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>
Table 5.5.
Table showing the sets of aircraft parameters which provide a solution set of the aircraft features of a rotor aircraft in which $N = 4$, $L_2 = 5$ m, $f_r = 6$ Hz, $\theta = 0$ rad., and $\lambda = 1$ m, after removing all solution sets in which one or more of the aircraft parameters do not correspond to those of any actual aircraft.

<table>
<thead>
<tr>
<th>$N$</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_2$ (m)</td>
<td>5</td>
<td>7.5</td>
</tr>
<tr>
<td>$f_r$ (Hz)</td>
<td>6</td>
<td>4</td>
</tr>
</tbody>
</table>
Table 5.6.
Table showing the sets of aircraft parameters which provide a solution set of the aircraft features of a rotor aircraft in which $N = 4$, $L_2 = 5$ m, $f_r = 6$ Hz, $\theta = 0$ rad., and $\lambda = 1$ m, after removing all solution sets in which the combination of the aircraft parameters does not correspond to that of any actual aircraft.

<table>
<thead>
<tr>
<th>$N$</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_2$ (m)</td>
<td>5</td>
</tr>
<tr>
<td>$f_r$ (Hz)</td>
<td>6</td>
</tr>
</tbody>
</table>
nations of parameters do not correspond to those of any actual aircraft, it would be necessary to have a library of aircraft parameters. This library would include not only the parameters of aircraft which are of interest, but also the parameters of aircraft which are not of interest, but which provide a solution set of the aircraft features.

By using the algorithm in the example above, we have reduced the number of solution sets from a countable number of sets to a single set, i.e. we have uniquely classified the aircraft, assuming that there is only one type of aircraft which has this solution set. Note that it may not always be possible to reduce the number of solution sets to a single set by using this algorithm. In some cases, after the algorithm has been used, there may still be more than a single solution set. In such cases it will not be possible to uniquely classify the aircraft by using this algorithm. The algorithm can also be applied to the case of propeller aircraft. In this case, it may also be possible to include $L_1$ in the algorithm.

We can use the algorithm above to obtain some general results. In Chapter 3 it was shown that values of $N$ for main rotors, tail rotors and propellers range from 2 to 8, 2 to 6, and 2 to 6, respectively. Therefore, if the blade symmetry of the aircraft is unknown, then the maximum number of solution sets for main rotors, tail rotors and propellers, will be 7, 5 and 5, respectively. If the blade symmetry of the aircraft is even, then the maximum number of solution sets will be 4, 3 and 3, respectively. If the blade symmetry of the aircraft is odd, then the maximum number of solutions sets will be 3, 2 and 2, respectively. These results are shown in Table 5.7.

5.6 Conclusions

This chapter has discussed some of the practical considerations which are associated with the analysis of the return signal from rotating aircraft blades.

The effects of the radar parameters on the return signal have been discussed.
Table showing the maximum number of sets of aircraft parameters which provide a solution set of the aircraft features for main rotors, tail rotors and propellers when the blade symmetry is unknown, even and odd.

<table>
<thead>
<tr>
<th></th>
<th>unknown</th>
<th>even</th>
<th>odd</th>
</tr>
</thead>
<tbody>
<tr>
<td>main rotor</td>
<td>7</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>tail rotor</td>
<td>5</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>propeller</td>
<td>5</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>
It has been shown that if there is no a priori knowledge of the class of aircraft, then in order to avoid aliasing of the frequency spectrum of any rotor or propeller aircraft, the PRF must be greater than or equal to the minimum PRF for the class of aircraft which has the highest blade tip speed, i.e. the PRF must be greater than or equal to \( 1.2/\lambda \) kHz, corresponding to the minimum PRF for propellers. It has also been shown that if there is no a priori knowledge of the class of aircraft, then in order for the frequency spectrum of any rotor or propeller aircraft to contain sidebands, the observation interval must be greater than or equal to the minimum observation interval for the class of aircraft which has the longest blade-passing interval, i.e. the observation interval must be greater than or equal to 0.2 s, corresponding to the minimum observation interval for main rotors.

The chapter has also discussed the effects of window functions on the frequency spectrum. It has been shown that applying window functions will reduce the leakage caused by finite observation intervals, but will also reduce the resolution of the frequency spectrum.

The effects of acceleration on the frequency spectrum have also been discussed. It has been shown that if there is any acceleration, then this will have three main effects on the frequency spectrum:

1. It will cause a Doppler frequency spread of the aircraft spectrum.
2. It will cause a Doppler frequency shift of the aircraft spectrum.
3. It will cause a reduction in the magnitude of the aircraft spectrum, caused by the frequency spread of the spectrum.

It has been shown that \( B_a \) is a function of \( a \), \( T_0 \) and \( \lambda \). It has also been shown that \( f_{da} \) is half of \( B_a \), since the frequency spectrum is symmetrical, therefore \( f_{da} \) is also a function of \( a \), \( T_0 \) and \( \lambda \). An acceleration compensation system, which can reduce the effects of any acceleration, has been presented. It has been shown that the system can reduce the effects of the acceleration, if good estimates of the acceleration can be obtained, or can remove the effects of the acceleration, if the acceleration is
known exactly.

The chapter has also discussed aircraft classification. An algorithm has been presented in which the number of sets of aircraft parameters which provide a solution set of the aircraft features can be reduced dramatically, by using the results of Chapters 2, 3 and 4. It has been shown that, by using this algorithm, it is possible to reduce the number of solution sets from a countable number of sets to a limited number of sets, or, in some cases, to a single set. It has also been shown that if the blade symmetry of the aircraft is unknown, then the maximum number of solution sets for main rotors, tail rotors and propellers, will be 7, 5 and 5, respectively; if the blade symmetry of the aircraft is even, then the maximum number of solution sets for main rotors, tail rotors and propellers, will be 4, 3 and 3, respectively; and if the blade symmetry of the aircraft is odd, then the maximum number of solution sets for main rotors, tail rotors and propellers, will be 3, 2 and 2, respectively.
CHAPTER SIX

CONCLUSIONS

6.1 Achievements

This thesis has presented a detailed mathematical analysis of the return signal from rotating aircraft blades. It has been shown that the modulation caused by the rotation of the blades is a form of PM, that the frequency spectrum consists of an infinite number of sidebands, and that each sideband can be approximated by a finite series of Bessel functions. It has also been shown that the frequency spectrum can be approximated by a finite number of sidebands, and that each sideband can be approximated by a finite series of Bessel functions. It has also been shown that when $L_1 \neq 0$, the sidebands which are nearest to the centre frequency will be approximately zero, causing a frequency notch about the centre frequency. The larger the value of $L_1$, the larger this notch will be. It has further been shown that there are an infinite number of sets of blade parameters which provide a solution set of the blade features, although in practice the number of sets will be limited because the extent to which the aircraft parameters for different types of aircraft can vary will be limited. A detailed mathematical analysis of the effects of blade pitch on the return signal has also been presented. It has been shown that if there is any blade pitch, then this will tend to cause a periodic variation of the cross section of each blade, which will cause AM of the return signal from each blade. It has also been shown that the exact nature of the AM will depend on whether $\theta$ is positive or negative, and that this may affect the performance of any feature extraction or aircraft classification algorithms. It has further been shown that some rotating objects will not tend to modulate the return signal, and that, in general, a rotating object will not tend to modulate the return signal if it is a body of revolution about its axis.
A detailed survey of the physical configurations, physical parameters, and radar features of rotor and propeller aircraft ranging in size from light to heavy aircraft has also been presented. (This includes some features which have not previously been used, or even considered, for aircraft classification.) It is this type of information which would be used to classify aircraft, therefore it is this type of information which will determine how feature extraction and aircraft classification algorithms should be implemented, and also how they will perform.

The thesis has also discussed the extraction of aircraft features from the return signal. Algorithms have been presented which extract the blade features from the return signal: \( f, L_2 \) was extracted from the frequency spectrum using tracking data and a knowledge of flight geometry, and \( \Delta f \) and the blade symmetry were extracted from the power cepstrum of the aircraft. It has been shown that the return signal from the rotating aircraft blades can be considered as consisting of a series of blade flashes, which in turn can be considered as multiple echoes of a single blade flash. This interpretation has led to an extension of cepstrum analysis, which in turn has led to a theoretical basis for using the power cepstrum to extract \( \Delta f \).

Some of the practical considerations which are associated with the analysis of the return signal from rotating aircraft blades have also been presented. The effects of the radar parameters on the return signal have also been discussed, as have the effects of window functions on the frequency spectrum. A detailed mathematical analysis of the effects of acceleration on the frequency spectrum has been presented, together with an acceleration compensation system, which can reduce the effects of any acceleration, if good estimates of the acceleration can be obtained. Aircraft classification has been discussed, and an algorithm has been presented, in which the number of sets of aircraft parameters which provide a solution set of the aircraft features can be reduced dramatically, by using the results of Chapters 2, 3 and 4.
6.2 Limitations

There are three main limitations to the work which has been presented in the thesis.

First, there has been no analysis of real data. As was noted in Chapter 1, this was mainly because of the difficulty which was encountered in obtaining real data, due to military classification. Despite this, a limited amount of real data has appeared in the literature, and this has tended to verify the work which has been presented in the thesis. Moreover, a limited amount of simulated data has also appeared in the literature, and this has further tended to verify the work which has been presented in the thesis.

Second, there has been no analysis of the return signals from rotating aircraft blades in the presence of noise or clutter. This is because the thesis has been concerned mainly with the presentation of new theory and results, and not with the performance analysis of existing theory and results, in the presence of noise and clutter. The presence of noise and clutter may affect the detection of blade modulation in the return signal, either as blade flashes, or as sidebands, and therefore may affect the performance of any feature extraction and aircraft classification algorithms. In Chapter 5 it was shown that in order to avoid aliasing of the frequency spectrum of any rotor or propeller aircraft, the PRF must be greater than or equal to twice the highest frequency in the target spectrum. However, in the case of airborne radar, moving targets may appear in the clutter region of the frequency spectrum. In order to avoid this, the PRF can be increased, which, depending on the relative velocity of the target, will cause the target spectrum to appear in the clutter-free region of the frequency spectrum. The increase in the PRF will be determined by the speed of the radar platform. Therefore, in the case of airborne radar, the PRF will be determined not only by the highest frequency in the target spectrum, but also by the speed of the radar platform.

Third, there has been no analysis of the return signals from aircraft which
have more than one rotor or propeller. Some provisional work has actually been carried out by the author, but has not been presented here, because of limitations of scope and space.

6.3 Future Work

There are many areas in which future work could be conducted. The most important of these are the three areas which were discussed in Section 6.2, and which form the main limitations to the work.

Future work could include extending the results of Chapter 2 to include analysing real data for both rotor and propeller aircraft, in order to compare theoretical with practical results. For a general analysis, this could involve analysing the return signals from aircraft in normal flight, either as cooperative trials aircraft, or as aircraft of opportunity, or could involve analysing the return signals from aircraft at close range. A more detailed analysis could involve analysing the return signals from rotating aircraft blades in isolation, ideally in an anechoic chamber. Other work could include analysing the return signals from aircraft which have more than one rotor or propeller.

The results of Chapter 3 could be extended to include collecting and collating further information on the features which have already been discussed. For example, values of $L_1$ are not usually published, although they could be used for aircraft classification, therefore it would be useful to obtain values of $L_1$ for different types of aircraft. This would probably involve examining scale drawings or models, or obtaining the information from the manufacturers. Other work could include collecting and collating information on other types of feature, e.g. the maximum rate of climb of the aircraft, the maximum rate of descent of the aircraft, the maximum rate of turn of the aircraft, etc.

Further work could also include extending the results of Chapter 4 to include testing the feature extraction algorithms on real data. This would enable the algo-
rithms to be improved, since it would then be possible to take steps towards making the algorithms more optimal or robust, with respect to the return signal. Other work could include developing algorithms which extract the same features from the return signal, frequency spectrum and power cepstrum of the aircraft, and then comparing the performance of each of these algorithms on simulated and real data. Other work could also include extending the algorithms which extracts $f, L_2$ from the return signal, to extract $f, L_1$ from the return signal, and developing an algorithm to extract the number of rotors or propellers from the return signal.

The results of Chapter 5 could be extended to include an analysis of the effects of rotational acceleration on the frequency spectrum. Other work could include the classification of aircraft which have more than one rotor or propeller.
REFERENCES


[98] Churchill, F. E., "Sidelobe levels of weighting functions with a platform when the number of samples (N) is small", *Transactions of the IEEE*, vol. 72, no. 12, pp. 1818-1820, December 1984.


APPENDIX A

PUBLICATIONS


ABSTRACT

It has long been known that the radar return signals of some airborne targets contain modulation components as well as the component due to the airframe of the target. However very little information has appeared in the literature on this subject, with the result that the theory which describes this modulation is not widely known. This paper will attempt to analyse the theoretical radar return signal from aircraft propeller blades. It will describe the basic theory involved, will examine some simulation results, and will also look at some practical considerations. Finally, it will draw some conclusions.

INTRODUCTION

It is well known that moving objects tend to cause a Doppler frequency shift of the return signal. However rotating objects tend to cause a modulation of the return signal. Almost all airborne targets have rotating parts (rotor blades, propeller blades, or jet engine compressor and turbine blades), therefore almost all airborne targets can cause this modulation. The modulation is a form of target noise and depends on a number of target parameters. More specifically, the modulation is a form of frequency modulation and results in a number of sidebands about the centre frequency of the target. By analysing the modulation contained in the return signal it may be possible in some cases to identify a particular type of target.

BASIC THEORY

A model which describes the theoretical return signal from an aircraft propeller is given by

\[ v(t) = \sum_{n=-\infty}^{\infty} A_n (L_2-L_1)e^{i2\pi n t} \sin(\frac{4\pi}{\lambda} (L_2-L_1) \cos(\theta)) \sin(\omega_c (t + \frac{2\pi n}{N})) \]

where \( A_n \) is a scale factor, \( L_2 \) is the distance of the blade roots from the centre of rotation, \( L_1 \) is the distance of the blade tips from the centre of rotation, \( N \) is the number of blades, \( R \) is the range of the centre of rotation, \( t \) is time, \( \nu \) is the radial velocity of the centre of rotation with respect to the radar, \( \lambda \) is the wavelength of the transmitted signal, \( \theta \) is the angle between the plane of rotation and the line of sight from the radar to the centre of rotation, \( \omega_c \) is the radial frequency of rotation.

A number of assumptions are made in the derivation of Equation (1):

1. Each blade acts as a homogeneous, linear, rigid antenna [1].
2. Each blade is always visible to the radar, i.e. there is no shielding of the blades.
3. The propeller is in the far field of the radar.

The above assumptions result in a good theoretical model of practical targets, however the following points should be noted:

1. Each blade is actually a rotating aerofoil, having camber, twist, etc. Therefore the centre of rotation will change with its distance from the centre of rotation, with its angular position with respect to time, and with the aspect angle of the propeller.
2. At some aspect angles shielding of the blades will occur.
3. The propeller will almost always be in the far field of the radar. (If the propeller is not in the far field then modulation of the return signal can still occur, but it will not be described by Equation (1).)

The Fourier transform of Equation (1) is given by

\[ V(f) = \sum_{k=0}^{N'-1} c_{NM} B (f - f_s - f_{kN}) \]

where \( c_{NM} = \sum_{n=-N}^{N} 2(-1)^{n} \delta_{n0} N \) and

\[ B(f) = \text{sinc}(\frac{\lambda f}{2}) \text{sinc}(\frac{\lambda f}{2}) \]

(3)

When \( L_1 = 0 \), Equation (1) describes the return signal from a helicopter rotor and Equation (2) describes a frequency modulated signal with \( N_1 \) pairs of sidebands about the centre frequency of the target, each separated by \( N f_{1} \). When \( L_1 \neq 0 \), Equation (1) describes the return signal from an aircraft propeller and Equation (2) still describes a frequency modulated signal, however, depending on \( L_1 \), the sidebands nearest to the centre frequency will be approximately zero, resulting in a frequency notch about the centre frequency. The greater the value of \( L_1 \), the greater this notch will be. In general:

When \( L_1 = 0 \)

\[ N' = \frac{8\pi L_1 \cos(\theta)}{N \lambda} \]

(3)

\[ \Delta f = \frac{N f_{1}}{B} \]

(4)

When \( L_1 \neq 0 \)

\[ N' = \frac{8\pi (L_2-L_1) \cos(\theta)}{N \lambda} \]

(3)

\[ \Delta f = \frac{8 f_s (L_2-L_1) \cos(\theta)}{B} \]

(4)

Equations (6) and (7) correspond to Equations (3) and (5), respectively, and describe a frequency modulated signal in which there is a frequency notch about the centre frequency.

A better understanding of Equation (3) can be obtained by rearranging \( B \) as follows.

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It can be seen that Equation (8) is similar to the equation which describes the Doppler frequency shift of a target which is moving with a constant radial velocity with respect to the radar [3]. This is what one would intuitively expect: the upper sidebands are due to reflections from the blades as they move towards the radar, and the lower sidebands are due to reflections from the blades as they move away from the radar. Also, the sidebands nearest to the centre frequency are due to reflections from the blade roots, and the sidebands furthest from the centre frequency are due to reflections from the blades as they move away from the radar tips.

It can also be seen from Equations (3)-(7) that the modulation is due to six main variables: \( N, L_1, L_2, f_j, \lambda, \) and \( \theta \), four of which are parameters of the propeller blades: \( N, L_1, L_2 \) and \( f_j \), one of which depends on the radar, \( \lambda \), and one of which depends on the aspect angle of the propeller, \( \theta \).

In the case where each blade is not a homogeneous, linear, rigid antenna, Equation (1) can be modified to give

\[
V_\lambda (f) = \frac{8 \pi f_r L_1 \cos (\theta)}{\lambda} \int \left( \sum_{k=-1}^{N-1} A_k f(r, \vartheta) e^{i 2 \pi (\lambda f + 2 \pi k f_\lambda - 4 \pi \vartheta \cos (\theta))} \right) \, dr
\]

where \( f(r, \vartheta) \) is a function of the distance of each incremental chord-wise section of blade from the centre of rotation, \( \lambda \) is the angular position of each blade with respect to time, and \( \theta \) is the aspect angle of the propeller.

**SIMULATION RESULTS**

Figure (1) shows the normalised simulated return signal from the propeller of an aircraft in which \( N = 4, L_1 = 0.25 \) m, \( L_2 = 1 \) m, \( f_j = 40 \) Hz, \( v = 0 \) m/s, \( \lambda = 0.20 \) m, and \( \theta = 0 \) rad. Since we are only interested in the basic theory at this stage, there is no component due to the airframe of the target, nor is there any noise or clutter.

It can be seen that \( v_\lambda (r) \) is real, even and periodic with period \( \Delta f = 1/Nf_\lambda \). The maxima correspond to points where \( v_{\lambda} \) is real and odd, with \( \lambda = 0, \frac{1}{2}, \frac{1}{2} \), i.e. to points where two blades are orthogonal to the line of sight from the radar to the centre of rotation, and therefore reflect the greatest amount of energy.

Figure (2) shows the normalised Fourier transform of \( v_\lambda (r) \). It can be seen that \( V_\lambda (f) \) is real and even, with \( N' = 24, \Delta f' = 160 \) Hz, and \( B = 3.52 \) kHz. Note that there is a frequency notch of 1.28 kHz about the centre frequency of the target.

Figures (3) and (4) show the real and imaginary components, respectively, of the simulated return signal from the propeller of an aircraft which is identical to that of Figure (1), except that in this case \( N = 5 \).

It can be seen that \( v_\lambda (r) \) is complex, the real component being even and periodic with \( \Delta f = 1/Nf_\lambda \), and the imaginary component being odd and periodic with \( \Delta f = 1/Nf_\lambda \). The maxima correspond to points where one blade is orthogonal to the line of sight from the radar to the centre of rotation.

Figure (5) shows the Fourier transform of \( v_\lambda (r) \). It can be seen that \( V_\lambda (f) \) is real and unsymmetrical, with \( N' = 20, \Delta f' = 200 \) Hz, and \( B = 3.60 \) kHz. Note that there is a frequency notch of 1.20 kHz about the centre frequency of the target. Although \( V_\lambda (f) \) is unsymmetrical, Figures (6), (7), and (8), respectively, show that the component of \( V_\lambda (f) \) due to \( \text{Re}(V_\lambda (f)) \) is real and odd, and that the component due to \( \text{Im}(V_\lambda (f)) \) is real and even, so that \( V_\lambda (f) \) is real and even. In general:

When \( N \) is even:

- \( v_\lambda (r) \) is real, even and periodic with \( \Delta f = 1/Nf_\lambda \).
- \( V_\lambda (f) \) is real and even.

When \( N \) is odd:

- \( v_\lambda (r) \) is complex, the real component being even and periodic with \( \Delta f = 1/Nf_\lambda \), and the imaginary component being even and periodic with \( \Delta f = 1/Nf_\lambda \).
- \( V_\lambda (f) \) is real and unsymmetrical, although \( V_\lambda (f) \) is real and even.

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The above theory applies in the case where \( V_\lambda (f) \) and \( V_\lambda (f) \) are continuous signals. In the discrete case the theory is modified slightly, in that, in most cases, \( V_\lambda (f) \) will be complex, not real. This is due to the discrete nature of the discrete Fourier transform (DFT) algorithm and does not affect any other previous results.

**PRACTICAL CONSIDERATIONS**

It should be noted that some rotating objects will not tend to cause a modulation of the return signal. In general, a rotating object will not tend to cause a modulation of the return signal if it is a body of revolution about its axis of rotation, e.g. a flat disc, a sphere, etc. This is why there is usually a frequency notch about the centre frequency of the target in the case of propeller aircraft - because the propeller spinner (a streamlined fairing which fits over the propeller hub) is a body of revolution.

It can be seen from Equations (6) and (7) that \( N', \Delta f' \) and \( B \) are inherently ambiguous, since there are four unknowns (assuming that \( \lambda \) and \( \theta \) are known), but only two linearly independent equations.

Figure (9) shows the simulated Fourier transform of the return signal from the propeller of an aircraft in which \( N = 8, L_1 = 0.50 \) m, \( L_2 = 2 \) m, \( f_j = 20 \) Hz, \( v = 0 \) m/s, \( \lambda = 0.2 \) m, and \( \theta = 0 \) rad. It can be seen that this target has exactly three sidebands - \( N', \Delta f' \) and \( B \) as that of Figure (1) - only the amplitudes of the sidebands are different.

In theory, there are a countably infinite number of ambiguous sets of \( N, L_1, L_2 \) and \( f_j \). In practice, however, the number of sets will be finite, since \( N, L_1, L_2 \) and \( f_j \) will only have values corresponding to practical parameter values.

So far, little mention has been made of \( \lambda \). It can be seen from Equations (3), (5), (6) and (7) that \( N' \) and \( B \) depend on \( \lambda \): the greater the value of \( \lambda \), the smaller the values of \( N' \) and \( B \), and vice versa. It is interesting to note that, by controlling \( \lambda \), \( N' \) and \( B \) can be controlled by the radar.

Little mention has also been made of \( \theta \). It can be seen from Equations (3), (5), (6) and (7) that \( N' \) and \( B \) depend on \( \theta \): the greater the value of \( \theta \), where \( 0 \leq \theta \leq \pi/2 \) rad., the smaller the values of \( N' \) and \( B \), and vice versa.

In the previous simulation results we have only been interested in the basic theory, therefore there has been no component due to the airframe of the target, nor has there been any noise or clutter. In practice, however, the return signal will almost always include a component due to the airframe, as well as noise and, in some cases, clutter.

Figure (10) shows the Fourier transform of the simulated return signal from an aircraft which is identical to that of Figure (1) except that in this case \( v = 256 \) m/s, and \( \theta = 4 \) rad. The return signal includes the component due to the airframe of the target, and zero mean white Gaussian noise.

It can be seen from Figure (10) that there is a spread of the Doppler frequencies of the airframe and sidebands. This spread is due to random variations of the velocity of the aircraft, caused by air turbulence, mechanical vibrations, etc., and is a function of the variance of the velocity of the aircraft, and of the wavelength of the transmitted signal [5]. In practice, these random variations will cause the previous theory to be modified, in that \( v_\lambda (r) \) and \( V_\lambda (f) \) will be complex and unsymmetrical.

It can also be seen from Figure (10) that the amplitude of the upper sidebands is approximately zero. This is due to periodic variations of the radar cross sections of the propeller blades. In practice, the amplitude of the upper sidebands will be approximately zero at some aspect angles, and the amplitude of the lower sidebands will be approximately zero at other aspect angles.

**CONCLUSIONS**

This paper has attempted to analyse the theoretical return signal from aircraft propeller blades. It has described the basic theory involved, has examined some simulation results, and has also looked at some practical considerations.

It has been shown that the modulation contained in the return signal is a form of frequency modulation and results in an number of sidebands about the centre frequency of the target.
It has also been shown that the modulation is due to six main variables: $N_1$, $L_1$, $L_2$, $f_1$, $\lambda$, and $\theta$, four of which are parameters of the propeller blades: $N$, $L_1$, $L_2$ and $f_1$, one of which depends on the radar: $\lambda$, and one of which depends on the aspect angle of the propeller, $\theta$.

Also, it has been shown that the values of $N$, $\Delta f$ and $\theta$ are inherently ambiguous.

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REFERENCES


Figure 1.
Simulated return signal.
($N=4$, $L_1=0.25$ m, $L_2=1$ m, $f_1=40$ Hz, $\lambda=0.20$ m, $\theta=0$ rad.)

Figure 3.
Real component of simulated return signal.
($N=5$, $L_1=0.25$ m, $L_2=1$ m, $f_1=40$ Hz, $\lambda=0.20$ m, $\theta=0$ rad.)

Figure 2.
Fourier transform of simulated return signal.
($N=4$, $L_1=0.25$ m, $L_2=1$ m, $f_1=40$ Hz, $\lambda=0.20$ m, $\theta=0$ rad.)

Figure 4.
Imaginary component of simulated return signal.
($N=5$, $L_1=0.25$ m, $L_2=1$ m, $f_1=40$ Hz, $\lambda=0.20$ m, $\theta=0$ rad.)
Figure 5.
Fourier transform of simulated return signal.
\((N=5, L_1=0.25 \text{ m}, L_2=1 \text{ m}, f_s=40 \text{ Hz}, \lambda=0.20 \text{ m}, \theta=0 \text{ rad.})\)

Figure 8.
Modulus of Fourier transform of simulated return signal.
\((N=5, L_1=0.25 \text{ m}, L_2=1 \text{ m}, f_s=40 \text{ Hz}, \lambda=0.20 \text{ m}, \theta=0 \text{ rad.})\)

Figure 6.
Fourier transform of real component of simulated return signal.
\((N=5, L_1=0.25 \text{ m}, L_2=1 \text{ m}, f_s=40 \text{ Hz}, \lambda=0.20 \text{ m}, \theta=0 \text{ rad.})\)

Figure 9.
Fourier transform of simulated return signal.
\((N=5, L_1=0.25 \text{ m}, L_2=1 \text{ m}, f_s=40 \text{ Hz}, \lambda=0.20 \text{ m}, \theta=0 \text{ rad.})\)

Figure 7.
Fourier transform of imaginary component of simulated return signal.
\((N=5, L_1=0.25 \text{ m}, L_2=1 \text{ m}, f_s=40 \text{ Hz}, \lambda=0.20 \text{ m}, \theta=0 \text{ rad.})\)

Figure 10.
Modulus of Fourier transform of simulated return signal.
\((N=5, L_1=0.25 \text{ m}, L_2=1 \text{ m}, f_s=40 \text{ Hz}, \lambda=0.20 \text{ m}, \theta=0 \text{ rad.})\)
AN AIRBORNE PULSE DOPPLER RADAR MODEL

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This paper will describe an airborne pulse Doppler radar model which simulates the radar return signal from airborne targets, with particular attention paid to the frequency spectrum of the return signal. The model simulates the return signal from the airframe and rotor or propeller blades of the target. The return signal also includes noise and ground clutter.

1. INTRODUCTION

Over the years various airborne pulse Doppler radar models have appeared in the literature, e.g. [1]-[4]. Most of these have attempted to model the return signal from ground clutter, not that from airborne targets.

Pulse Doppler radars are used mainly in airborne applications which require the detection of moving targets in a ground clutter environment. In pulse Doppler radar, the radar signal is transmitted as a series of coherent pulses. From the return signal the radar is able to calculate the range and Doppler frequency shift of the target.

Although the model is described as an airborne pulse Doppler radar model, in practice it can also be used as a ground-based pulse Doppler radar model, in which case there will be no ground clutter.

The velocity of the airframe will tend to cause a Doppler frequency shift of the return signal. The rotation of the rotor or propeller blades will tend to cause a modulation of the return signal. The modulation is a form of frequency modulation and results in a number of sidebands about the airframe line of the target.

The model is useful for two main reasons. First, target acquisition and tracking is sometimes achieved using the Doppler frequency shift of the airframe [5]. At the development stage of acquisition and tracking algorithms it would obviously be much less expensive to test the algorithms on simulated data, than to pay for flight trials in order to test them on real data. Second, target classification and identification is sometimes achieved using the sidebands of the target [6]. As with acquisition and tracking algorithms, it would obviously be much less expensive to test any classification and identification algorithms on simulated data, than on real data.

2. MODEL STRUCTURE

Figure 1 shows the basic structure of the model. The radar signal is transmitted by the radar and travels towards the target. The signal is then reflected by the airframe and rotor or propeller blades of the target and travels back towards the radar. At some antenna angles the signal is also reflected by the ground and this also travels back towards the radar. Zero-mean white Gaussian noise is then added to the signal. The return signal is then received by the radar, and after pre-processing is down-converted into real and imaginary components. The fast Fourier transform (FFT) of the signal is then taken. Finally, the time series and frequency spectrum of the signal are

3. AIRFRAME

A model which describes the theoretical return signal from an airframe is given by

\[ v_e(t) = A_e e^{j(\omega_0 t + \frac{2\pi}{\lambda} + \frac{v_r}{c} t + \frac{2\pi}{\lambda} \frac{v_r}{c} t)} \]

where

- \( A_e \) = a scale factor,
- \( R_e \) = range of the airframe,
- \( t \) = time,
- \( v_r \) = radial velocity of the airframe with respect to the radar,
- \( \lambda \) = wavelength of the transmitted signal, and
- \( \omega_0 \) = radian frequency of the transmitted signal.

The Fourier transform of Equation (1) is given by

\[ V_e(f) = A'_e \delta(f - f_s - f_d) \]

where

- \( A'_e \) = \( A_e e^{-\frac{2\pi f}{\lambda}} \),
- \( f \) = cyclic frequency,
- \( f_s \) = cyclic frequency of the transmitted signal, and
- \( f_d = -2v_r/\lambda \) = cyclic Doppler frequency shift of the airframe.

\( \delta(f) \) = unit impulse function.

In the theoretical case, the airframe will consist of a single scatterer, having a single cross section, range and velocity. In the practical case, however, the airframe will consist of many scatterers, each having random variations of its cross section, range, and velocity.

The cross section will have a Rayleigh or chi-square probability density function (pdf), corresponding to Swerling's four fluctuation models, and the range and velocity will have a Gaussian pdf. In practice these pdfs will not always correspond to the pdfs of practical targets, however in many cases they will be a reasonable approximation.

The random variations of the cross section, range, and velocity are mainly caused by random variations of the position and orientation of the airframe. The position and orientation of the airframe can be resolved into six degrees of freedom: roll, pitch and yaw, i.e. rotation about three orthogonal axes; and surge, sway and heave, i.e. translation along the three axes, respectively.
In the theoretical case, the airframe line will consist of a single spectral line. In the practical case, the random variations of the velocity will cause the airframe line to have a Doppler frequency spread. The variance of the spread depends mainly on the variance of the velocity and the wavelength of the transmitted signal.

4. ENGINES

A model which describes the theoretical return signal from the blades of an aircraft propeller is given by

\[ v_b(t) = \sum_{n=0}^{N-1} A_n (L_2-L_1) \]

\[ \times \exp \left( j \left( \frac{2 \pi}{\lambda} (L_2 - L_1) \cos(\theta) \sin(\omega_u t + \frac{2 \pi n}{N}) \right) \right) \]

where \( A_n \) is a scale factor,
\( L_1 \) = distance of the blade roots from the centre of rotation,
\( L_2 \) = distance of the blade tips from the centre of rotation,
\( N \) = number of blades,
\( R_s \) = range of the centre of rotation,
\( v_b \) = radial velocity of the centre of rotation with respect to the radar,
\( \theta \) = angle between the plane of rotation and the line of sight from the radar to the centre of rotation,
\( \omega_u \) = radian frequency of rotation.

A number of assumptions are made in the derivation of Equation (1):
1. Each blade acts as a homogeneous, linear, rigid antenna.
2. Each blade is always visible to the radar, i.e. there is no shielding of the blades.
3. The rotor or propeller is in the far field of the antenna.

The Fourier transform of Equation (3) is given by

\[ V_s(f) = \sum_{n=-\infty}^{\infty} c_{Nn} \delta(f-f_n-Nf_s) \]

where \( c_{Nn} = \sum_{k=-\infty}^{\infty} (-1)^{k+N} N \int_{-\infty}^{\infty} \bigg( J_k(4\pi L_1 \cos(\theta)) - J_k(4\pi L_2 \cos(\theta)) \bigg) \]

\[ f_n = -2\omega_u \lambda \cos(\theta) \]

\[ f_s = \text{cyclic Doppler frequency shift of the centre of rotation}, \]

\( J_k(\cdot) \) = Bessel function of the 1st kind and \( k \) th order,
\( N \) = highest significant sideband,
\( \alpha \) = unit step function.

When \( L_1 = 0 \), Equation (3) describes the return signal from a helicopter rotor and Equation (4) describes a frequency modulated signal with \( N \) pairs of sidebands about the airframe line, each separated by \( f_s \). When \( L_1 \neq 0 \), Equation (3) describes the return signal from an aircraft propeller and Equation (4) still describes a frequency modulated signal, however, depending on \( L_1 \), the sidebands nearest to the airframe line will be approximately zero, resulting in a frequency notch about the airframe line. The greater the value of \( L_1 \), the greater the notch will be. In general:

When \( L_1 = 0 \)

\[ N' = \frac{8\pi L_1 \cos(\theta)}{N \lambda} \]

\[ \Delta f = Nf_s \]

\[ B = \frac{8\pi f_s L_1 \cos(\theta)}{\lambda} \]

When \( L_1 \neq 0 \)

\[ N' = \frac{8\pi (L_2-L_1) \cos(\theta)}{N \lambda} \]

\[ \Delta f = \frac{8\pi f_s (L_2-L_1) \cos(\theta)}{\lambda} \]

where \( N' = \) number of significant sidebands,
\( \Delta f = \) frequency separation of each sideband,
\( B = \) bandwidth of the sidebands.

As with the airframe, in the theoretical case, each incremental chord-wise section of blade will consist of a single scatterer, having a single cross section, range and velocity. In the practical case, each section of blade will consist of many scatterers, each having random variations of its cross section, range and velocity.

In the theoretical case, each sideband will consist of a single spectral line. In the practical case, the random variations of the velocity will cause each sideband to have a Doppler frequency spread.

It should be noted that some rotating objects will not tend to cause a modulation of the return signal. In general, a rotating object will not tend to cause a modulation of the signal if it is a body of revolution about its axis of rotation, e.g. a flat disc, a sphere, etc. This is why there is usually a frequency notch about the airframe line in the case of propeller aircraft - because the propeller spinner (a streamlined fairing which fits over the propeller hub) is a body of revolution about its axis of rotation.

In the case of propeller aircraft, the propeller blades can have significant blade pitch. This will tend to cause a periodic variation of the cross section of each incremental chord-wise section of blade, which will tend to cause a second type of modulation of the return signal. The modulation is a form of amplitude modulation and results in the attenuation of the upper or lower sidebands, depending on the aspect angle.

5. CLUTTER

At some antenna angles the return signal will include ground clutter. Various clutter types are included in the model, using various pdfs (e.g. exponential, Gaussian, log-normal, Rayleigh and Weibull). In each case the clutter is uncorrelated from pulse to pulse, although in practice the clutter is sometimes correlated.

6. SIMULATION RESULTS

Figure 2 shows the frequency spectrum of a helicopter aircraft in which \( N = 4 \), \( L_1 = 5 \text{ m} \), \( f_s = 6 \text{ Hz} \), \( \lambda = 2 \text{ m} \), and \( \theta = 0 \text{ rad} \), and in which the airframe and rotor blades each consist of a single scatterer (i.e. \( N_s = 1 \) and \( L_s = 1 \)), each having a single cross section, range and velocity.
It can be seen that $N' = 16$, $\Delta f = 24$ Hz, and $B = 384$ Hz. Note that the airframe line and sidebands each consist of a single spectral line, and that the frequency spectrum is approximately symmetrical about the airframe line.

Figures 3-5 show the frequency spectrum of the same target as the number of scatterers in the airframe and rotor blades are each increased to 8, 64, 512 and 4096, respectively. In each case, the cross section has a Rayleigh pdf, corresponding to a Swerling Case 2 model, and the range and velocity have a Gaussian pdf.

It can be seen that, for a small number of scatterers, the frequency spectrum is unrealistically noisy, and that as the number of scatterers is increased, the frequency spectrum becomes similar to that for practical targets.

Figure 6 shows the frequency spectrum of a propeller aircraft in which $N' = 9$, $\Delta f = 160$ Hz, and $B = 128$ kHz. Note that there is a frequency notch of 480 Hz about the airframe line, caused by the propeller spinner, and that the upper sidebands are approximately zero, caused by amplitude modulation of the return signal from the propeller blades.

7. FURTHER WORK

There are a number of ways in which the model could be improved. First, the model could be adapted to simulate cross section, range, and velocity pdfs other than those described above. Second, the model could be developed to simulate the return signal from the compressor and turbine blades of jet engines. (The model would then have to simulate multiple reflections from the airframe and engine cowling.) Third, the model could be extended to simulate volume clutter.

8. CONCLUSIONS

This paper has described an airborne pulse Doppler radar model which simulates the return signal from airborne targets. The return signal also includes noise and ground clutter.

The model is useful at the development stages of target acquisition and tracking algorithms, and target classification and identification algorithms.

It has been shown that, for a small number of scatterers, each having random variations of its cross section, range and velocity, the frequency spectrum is unrealistically noisy, and that as the number of scatterers is increased, the frequency spectrum becomes similar to that for practical targets.

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REFERENCES


![Figure 1](image-url)  
*Figure 1.* Basic structure of the airborne pulse doppler radar model.
Figure 2. Frequency spectrum of a helicopter aircraft in which $N_s = 1$ and $N_t = 1$.

Figure 3. Frequency spectrum of target with $N_s = 8$ and $N_t = 8$.

Figure 4. Frequency spectrum of target with $N_s = 64$ and $N_t = 64$.

Figure 5. Frequency spectrum of target with $N_s = 512$ and $N_t = 512$.

Figure 6. Frequency spectrum of target with $N_s = 4096$ and $N_t = 4096$.

Figure 7. Frequency spectrum of a propeller aircraft in which $N_s = 4096$ and $N_t = 4096$. 
The disk which is included with this thesis contains files which contain some of the main programs, functions and subroutines which were used in the simulations. The programs were written in the BASIC Programming Language to be run on the Hewlett-Packard HP9000 computer. The files are as follows.

**MODEL**  This file contains the main program which was used in the simulations. The program contains three main subroutines, viz. FREQUENCY, QUEFRENCEY and TIME, which were used for frequency, quefreny and time domain simulations, respectively.

**MATHS**  This file contains a library of mathematical functions and subroutines, some of which were used in the simulations.

**NOISE**  This file contains a library of random noise generation functions, some of which were used in the simulations.

**GRAPHS**  This file contains a two-dimensional graphics plotting subroutine, which was used to plot the simulations.