Acknowledgements

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This investigation deals with the measurement of the shape and magnitude of pressure waveforms generated by the growth and collapse of bubbles in nucleate boiling of water. The measurements were made at liquid pressures of 1.0, 3.0, 7.0 and 14.7 psia and liquid subcoolings of up to 20 deg. C. The heating surface, a 0.072 in. diameter stainless steel tube mounted between electrodes 2 in. apart, was immersed in a pool of distilled de-ionised water contained in a Pyrex glass vessel of cruciform shape. The pressure pulses were detected by a Kistler quartz crystal transducer, type 410B, connected to an oscilloscope through a Kistler charge amplifier. Using a Hycam rotating prism camera, simultaneous recording on film of bubble motion and associated pressure pulse was made at speeds of up to 5000 frames per sec. An expression has been developed which relates the bubble radius to the liquid pressure. This expression, when compared with experimental data obtained from the films, was found to give reasonable agreement.

A study has been made of the sound produced by single bubbles in nucleate boiling of water at pressures of 1.0, 3.0, 7.0 and 14.7 psia with liquid subcoolings of up to 15 deg. C. The sound, sensed by the Kistler quartz transducer, was recorded on a persistence oscilloscope and photographed using a Polaroid camera. A frequency analysis of the sound showed that the experimental spectra rose at 12 to 16 dB per octave at frequencies much lower than that equivalent to the reciprocal of the bubble lifetime. At high frequencies, the spectra diminished at rates varying from 1.0 to 4 dB per octave. At intermediate frequencies, the spectra were irregular.

It is shown that the pressure amplitude of the sound increases as the $3/2$ power of the bubble maximum radius and as the $3/4$ power of the
bubble lifetime.

It is further shown that only a small fraction, of the order of $10^{-5}$, of the maximum energy of the bubble is radiated as sound and that this fraction decreases with increase of ambient liquid pressure.

Another phenomenon that has been observed is that sound is radiated when a bubble divides into two or more smaller bubbles.

The effect of dissolved gas content has been investigated on the sound emitted in nucleate boiling of water at atmospheric pressure with liquid subcoolings of 5, 25 and 35 deg.C. By bubbling oxygen gas through the water for specified intervals of time, the dissolved oxygen content of the water was varied from 6.2 to 29.0 parts per million. The boiling sounds were sensed by the same quartz transducer and recorded on a Ferrograph tape recorder. The recordings were made for two heat fluxes, 76000 and 116000 Btu per hr. per sq. ft. A frequency analysis of the sounds recorded on tape was carried out in the Noise Analysis Laboratory of the UKAEA, Dounreay. The equipment consisted of a tape recorder, a Bruel and Kjaer frequency analyser, type 2107, and a logarithmic recorder. The analysis showed that the spectra cut off at a frequency which was usually in the range 10–14 kHz. The rate of decay after cut-off was very sharp, varying from 33 to 90 dB per octave. Before cut-off, however, the spectra diminished 6 dB per octave. Both the cut-off frequency and the rate of decay of the spectra were found to be weakly dependent on the oxygen content.

The experimentally determined bubble growth rates in saturated boiling at 1.0, 3.0, 7.0 and 14.7 psia have been compared with previously reported work. At atmospheric pressure of 14.7 psia, the theories available in the literature give good agreement with experimental results. At sub-atmospheric pressures, the theoretical predictions depart considerably from the experimental values, the
extent of departure increasing with the increase in Jakob number. An analysis of bubble growth from a cylindrical surface has been presented and the equations programmed for solution on a computer. Only a few results have been presented because of the limited computer time available.
TRANSIENT PHENOMENA ASSOCIATED WITH HEAT TRANSFER IN NUCLEATE BOILING

by

R. A. KUMAR

Thesis submitted for the degree of

Doctor of Philosophy

University of Edinburgh

1970.
LIST OF FIGURES

Figure Nos. 1 - 24 of Chapter 2  
Pages  
112 - 133

Figure Nos. 25 - 35 of Chapter 3  
134 - 144

Figure Nos. 36 - 50A of Chapter 4  
145 - 163

Figure Nos. 51 - 59 of Chapter 5  
164 - 170
<table>
<thead>
<tr>
<th>CONTENTS</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acknowledgements</td>
<td>i</td>
</tr>
<tr>
<td>Summary</td>
<td>ii</td>
</tr>
<tr>
<td>Notation</td>
<td>x</td>
</tr>
<tr>
<td><strong>Chapter 1.</strong> Introduction</td>
<td>1</td>
</tr>
<tr>
<td><strong>Chapter 2.</strong> PRESSURE TRANSIENTS IN A LIQUID DUE TO GROWTH AND COLLAPSE OF VAPOUR BUBBLES.</td>
<td>6</td>
</tr>
<tr>
<td>2.1. Introduction and Survey of relevant literature.</td>
<td>6</td>
</tr>
<tr>
<td>2.2. Derivation of the relevant equations and their solution.</td>
<td>10</td>
</tr>
<tr>
<td>2.3. Description of Experimental Apparatus.</td>
<td>15</td>
</tr>
<tr>
<td>a. Boiler Assembly.</td>
<td>15</td>
</tr>
<tr>
<td>b. The Heating Surface</td>
<td>15</td>
</tr>
<tr>
<td>c. Power Circuitry and Instrumentation</td>
<td>17</td>
</tr>
<tr>
<td>d. High Speed Photography</td>
<td>19</td>
</tr>
<tr>
<td>2.4. Experimental Procedure</td>
<td>21</td>
</tr>
<tr>
<td>2.5. Method of film analysis and Interpretation of records.</td>
<td>22</td>
</tr>
<tr>
<td>2.6. Results and Discussion</td>
<td>24</td>
</tr>
<tr>
<td>2.7. Conclusions</td>
<td>29</td>
</tr>
<tr>
<td><strong>Chapter 3.</strong> SOUND GENERATED BY SINGLE BUBBLES IN SATURATED AND SUBCOOLED NUCLEATE BOILING.</td>
<td>30</td>
</tr>
<tr>
<td>3.1. Introduction</td>
<td>30</td>
</tr>
<tr>
<td>3.2. Survey of Literature</td>
<td>30</td>
</tr>
<tr>
<td>a. Sound generated by gas bubbles entrained in a liquid.</td>
<td>30</td>
</tr>
<tr>
<td>b. Boiling and cavitation noise.</td>
<td>31</td>
</tr>
<tr>
<td>3.3. Experimental Apparatus</td>
<td>33</td>
</tr>
</tbody>
</table>
### Chapter 3

3.4. Results and Discussion  
- a. Frequency spectrum of noise due to single bubbles.  
- b. Noise spectral behaviour at low frequencies  
- c. Noise spectral behaviour at high frequencies  
3.5. Pressure pulse amplitude in relation to bubble lifetime and maximum radius.  
3.6. Energy content of the noise generated by single bubbles.  
3.7. Noise generated due to bubble division.  

### Chapter 4

SOUND EMISSION DURING NUCLEATE BOILING AND ITS VARIATION WITH DISSOLVED OXYGEN CONTENT OF BULK LIQUID.  
4.1. Introduction and survey of relevant literature.  
4.2. Measurement of dissolved oxygen content of bulk liquid.  
4.3. Experimental Apparatus.  
- a. Experimental setup.  
- b. Recording system.  
4.4. Experimental Procedure.  
4.5. Analysis of Tape Recordings.  
4.6. Results and Discussion.  
4.7. Conclusions

### Chapter 5

THE GROWTH RATE OF VAPOUR BUBBLES FROM A CYLINDRICAL SURFACE IN NUCLEATE BOILING.  
5.1. Introduction  
5.2. Survey of Literature.
5.3. Theoretical Analysis
   a. The bubble growth model and the derivation of the relevant equations.
   b. Temperature distribution in the liquid around a heated cylinder.
   c. Growth of vapour bubbles from a cylindrical surface.

5.4. Results and Discussion

5.5. Conclusions

Chapter 6. CONCLUSIONS AND REMARKS

Appendix 1 Deduction of outside surface temperature of heating tube.

Appendix 2 Coding for solution of equation (16) Chapter 2.

Appendix 3 Determination of bubble radius from simple acoustic theory.

Appendix 4 Coding for determination of frequency spectra of sound due to single bubbles.

Appendix 5 I Temperature distribution in the liquid surrounding a heated cylinder.

II Bubble growth from a heated cylindrical surface.

Appendix 6 Determination of bubble growth rates from a cylindrical surface.

Appendix 7 Estimation of errors.

References

Table 1

Figures
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Area, constant</td>
</tr>
<tr>
<td>A', A''</td>
<td>Constants</td>
</tr>
<tr>
<td>b</td>
<td>Thickness of superheated layer in Griffith's analysis.</td>
</tr>
<tr>
<td>C</td>
<td>Velocity of sound in liquid, curvature factor.</td>
</tr>
<tr>
<td>d</td>
<td>Function of non-dimensional variable s, heater outside diameter</td>
</tr>
<tr>
<td>D</td>
<td>Bubble diameter.</td>
</tr>
<tr>
<td>f</td>
<td>Frequency, function of non-dimensional variable s.</td>
</tr>
<tr>
<td>g(f)</td>
<td>Fourier Transform of pressure p(r,t).</td>
</tr>
<tr>
<td>G</td>
<td>Gas constant</td>
</tr>
<tr>
<td>h_{fg}</td>
<td>Latent heat of vapourisation.</td>
</tr>
<tr>
<td>h</td>
<td>Enthalpy, defined by equation (7), Chapter 2.</td>
</tr>
<tr>
<td>I</td>
<td>Average sound intensity.</td>
</tr>
<tr>
<td>k</td>
<td>Function of non-dimensional variable s, liquid thermal conductivity.</td>
</tr>
<tr>
<td>m</td>
<td>Constant</td>
</tr>
<tr>
<td>M</td>
<td>Constant, Molecular weight</td>
</tr>
<tr>
<td>M'</td>
<td>Constant</td>
</tr>
<tr>
<td>N</td>
<td>Number of active sites per unit area.</td>
</tr>
<tr>
<td>N_{Ja}</td>
<td>Jakob Number</td>
</tr>
<tr>
<td>p</td>
<td>Pressure in liquid at distance r at time t.</td>
</tr>
<tr>
<td>P_{w}</td>
<td>Ambient liquid pressure.</td>
</tr>
<tr>
<td>P_{m}</td>
<td>Maximum pressure in liquid during cavity collapse.</td>
</tr>
<tr>
<td>P_{L}</td>
<td>Liquid pressure at bubble wall, introduced in Chapter 5.</td>
</tr>
<tr>
<td>P</td>
<td>Liquid pressure at boundary of cavity or bubble in Chapters 2, 3</td>
</tr>
<tr>
<td>P_{v}</td>
<td>Pressure of vapour within bubble</td>
</tr>
<tr>
<td>q_{w}</td>
<td>Heat flux applied to heating surface.</td>
</tr>
<tr>
<td>R</td>
<td>Bubble or cavity radius at time t.</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>( R_i )</td>
<td>radius of embryonic nucleus, initial bubble radius in saturated boiling.</td>
</tr>
<tr>
<td>( R_m )</td>
<td>mean radius of bubble in volume pulsation.</td>
</tr>
<tr>
<td>( R_o )</td>
<td>maximum radius of bubble in saturated and subcooled boiling.</td>
</tr>
<tr>
<td>( \bar{R} )</td>
<td>bubble or cavity boundary velocity</td>
</tr>
<tr>
<td>( r )</td>
<td>radial coordinate</td>
</tr>
<tr>
<td>( S(f) )</td>
<td>Spectral density of sound at frequency ( f ).</td>
</tr>
<tr>
<td>( s )</td>
<td>non-dimensional bubble radius.</td>
</tr>
<tr>
<td>( T )</td>
<td>Temperature, Duration of pressure pulse.</td>
</tr>
<tr>
<td>( T_{sat} )</td>
<td>Saturation temperature corresponding to pressure ( P_m ).</td>
</tr>
<tr>
<td>( T_L )</td>
<td>Saturation temperature corresponding to liquid pressure ( P_L ).</td>
</tr>
<tr>
<td>( T_w )</td>
<td>Wall temperature</td>
</tr>
<tr>
<td>( T_{\infty} )</td>
<td>liquid temperature far from bubble.</td>
</tr>
<tr>
<td>( t )</td>
<td>time</td>
</tr>
<tr>
<td>( t_o )</td>
<td>time for complete collapse</td>
</tr>
<tr>
<td>( U )</td>
<td>Cavity boundary velocity, non-dimensional temperature</td>
</tr>
<tr>
<td>( u )</td>
<td>liquid particle velocity</td>
</tr>
<tr>
<td>( V )</td>
<td>non-dimensional bubble radius, introduced in Chapter 5.</td>
</tr>
<tr>
<td>( \int p^2 dt )</td>
<td></td>
</tr>
<tr>
<td>( y )</td>
<td>( (r - R)/R_i )</td>
</tr>
<tr>
<td>( z )</td>
<td>( R/R_o )</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>thermal diffusivity of liquid.</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Growth constant</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Ratio of specific heats</td>
</tr>
<tr>
<td>( \delta )</td>
<td>Superheated layer thickness</td>
</tr>
<tr>
<td>( \theta )</td>
<td>polar angle</td>
</tr>
<tr>
<td>( \mu )</td>
<td>(-\cos \theta)</td>
</tr>
</tbody>
</table>
\[ \tau = \frac{ct}{R_i^2} \]

\( \rho_v \) vapour density

\( \rho \) density of liquid

\( \varnothing \) velocity potential, angle.

\( \sigma \) surface tension of liquid

Note: i) Throughout this report, British system of units
    \((\text{lb}_f, \text{lb}_m, \text{ft., sec.})\) has been used.

ii) Equations are numbered afresh in each chapter.

iii) Symbols not included above are defined as and when they occur in the text.
CHAPTER 1

Introduction

It is now generally recognised that the growth and collapse of vapour bubbles in a liquid give rise to several physical effects, two of which are

1) Pressure fluctuations in the liquid and

ii) The emission of sound.

High speed photographs of bubble motion indicate that normally bubbles grow from a microscopic nucleus to a maximum size in a short interval of time, of the order of a few milliseconds. Subsequently the bubbles may collapse provided that either the liquid is subcooled or the pressure in the liquid is greater than the pressure in the bubble. The first condition is commonly encountered in boiling systems while the second occurs in cavitation. Starting from a microscopic nucleus, if the bubble grows slowly, any disturbance at the bubble wall will be propagated instantaneously through the liquid at a speed which is effectively infinite. The liquid may then be considered incompressible and the bubble motion determined from the results of ideal fluid theory.

If the bubble radius changes with time more rapidly, the disturbance emanating at the bubble wall will be propagated as a pressure wave through the liquid at a speed equal to the sonic speed in the liquid. It is then more realistic to treat the liquid as possessing a finite speed of sound for purposes of analysis. Published data on bubble growth and collapse in nucleate boiling indicate that such rapid changes of radius with time are likely to occur immediately following bubble nucleation and during collapse. There appears to be little experimental data on pressure waveforms under these conditions and to obtain such data, in detail, is one of the aims of the present investigation.

Finally, if the bubble radius changes with time extremely rapidly,
at a speed comparable to the sonic speed in the liquid, the pressure in the liquid may become so large that the speed of propagation of disturbances starting at the bubble wall will tend to increase. This is equivalent to saying that the sonic speed in the liquid can no longer be considered a constant but becomes a function of the pressure. The result is that pressure waves generated at time $t$ will tend to overtake those generated at earlier time $t_1$. Consequently a wave with a steep front, a shock wave, may be generated. Examples of this behaviour are the movement of the gas globe in an underwater explosion (1) and the final stages of the collapse of a vapour bubble in a cavitating liquid (4). In the latter instance, the violent character of the bubbles is well known, inasmuch as they can give rise to extremely high pressures upon collapse and one of the physical effects produced thereby is the erosion of solid surfaces in a short interval of time.

The sounds emitted in boiling and cavitation are a consequence of the bubble induced pressure waves. The intensity and frequency of the sounds emitted depend on the rate of bubble growth or collapse. In cavitation, the sound emitted has a metallic quality and the numerous investigations reported to date appear to indicate that the noise spectra extend to very high frequencies, in the megacycle range (21,23). In boiling, however, the literature is not so extensive. The few studies reported show that, in the nucleate boiling regime, the intensity of the sound emitted bears a linear relation to the temperature difference between the heated surface and the liquid (16,20); the spectrum of sound in nucleate boiling is found to be confined to the low frequency region, roughly 25-1000 Hz (16).

Numbers in brackets denote references included at the end of the thesis.
The influence on the sounds emitted of the dissolved gas in a liquid has long been recognised. In boiling systems, the gas may be trapped in pits or crevices in the solid surfaces and the bubbles formed may contain some non-condensible gas. The reduction in the intensity of radiated sound with increase in gas content has been attributed to the damping of the pressure waves by the gas-filled bubbles. In cavitation, the compression of the gas in the bubble during the collapse phase will result in a rebound after the bubble has reached a minimum size. The presence of the gas in the bubble is a moderating influence on the pressures generated by the cavity collapse. With increase of gas content, the reduction in the critical pressure, for inception of cavitation has been noted (22); similarly in boiling, the reduction in heat flux required to produce the same temperature difference has been reported (48).

The study reported here deals with the pressure transients due to vapour bubble growth and collapse in the nucleate boiling of water at atmospheric and sub-atmospheric pressures. Detailed information on the shape and magnitude of the pressure pulses generated by individual bubbles have been obtained. High speed photographs of the bubble motion, at speeds of up to 5000 frames per second, yielded data on the bubble radius-time history. An expression has been developed which relates the bubble radius to the liquid pressure; this expression has been compared with experimental data and was found to give reasonable agreement.

The noise emitted by individual bubbles in nucleate boiling of saturated and subcooled water has been recorded. A frequency analysis of the noise indicated that the spectrum rose as $f^4$ i.e., at 12 dB per octave at low frequencies. At high frequencies, the spectra decayed, the rate of decay varying from 1 to 4 dB per octave. At intermediate
frequencies the spectrum was irregular. Analysis of high speed photographs of bubble motion showed that the pressure pulse amplitude varied as the $\frac{3}{2}$ power of the bubble maximum radius and the $\frac{3}{4}$ power of the bubble lifetime.

Further the influence of dissolved gas content on the sound generated in nucleate boiling of water at atmospheric pressure has been studied. The dissolved oxygen content of the water varied from 6 to 28 parts per million with subcoolings of 5 to 35 deg. C. The cut-off frequency of the noise spectra varied from 10 to 14 kHz but was only slightly affected by the dissolved gas content. Before cut off, the spectra fell off at approximately 6 dB per octave.

The experimentally determined bubble growth rates in saturated boiling at 1.0, 3.0, 7.0 and 14.7 psia have been compared with previously reported work. An analysis of bubble growth from a cylindrical test surface has been presented and the equations programmed for solution on a digital computer. Only a few results at 7.0 psia have been obtained on account of the limited time available on the computer used.

Some of the results of this investigation may be relevant to practical engineering situations, as for example in the assessment of safety levels in nuclear reactors. It is of considerable importance to estimate the pressure rise in the reactor coolant due to the rapid growth of a large number of voids and some related experiments are in progress in at least one laboratory (49). In this connection, the measurements reported here of the magnitude of bubble induced pressure transients may be relevant.

To detect local overheating in a reactor channel due to boiling of coolant following a power surge, the noise generated by bubbles may be used as a guide. Ledwidge (50) has reported experiments dealing with such detection in a fast reactor, in which it was noted that
boiling noise could be detected over and above background noise provided measurements are made in the high frequency range. However, as noted in Chapter 4 of this investigation, the boiling noise levels at high frequencies may be much reduced due to the presence of dissolved gas and this may have to be considered in the interpretation of measured noise spectra.
CHAPTER 2
PRESSURE VARIATIONS IN A LIQUID DUE TO THE GROWTH AND COLLAPSE OF VAPOUR BUBBLES

2.1. Introduction and review of relevant literature

The growth and collapse cycle of a vapour bubble may be divided into three stages. The first stage in which the bubble grows from a microscopic nucleus to observable size is very rapid resulting in accelerations large compared to gravity. Following this is the growth of the bubble to a maximum size and subsequent collapse. During this growth and most of the collapse phase, the velocities attain only moderate values. The third and final stage, the terminal collapse, is accompanied by large velocities which may reach values close to or exceeding the sonic speed in the liquid. (In saturated boiling, however, no collapse occurs and the bubbles after growing to a maximum size rise through the body of the liquid). This sub-division is possible notwithstanding the several methods by which a bubble may form in a liquid. The bubble motion will have an effect on the surrounding liquid. In this liquid there will be a time-dependent pressure field generated by the motion of the bubble.

In this chapter is reported the measurements made to determine the form of the pressure field in nucleate boiling at pressures 1.0, 3.0, 7.0 and 14.7 psia with liquid subcoolings of up to 20 deg.C. A review of the literature on the subject will be presented first.

Several investigators, the first of whom was Lord Rayleigh, have derived expressions for the pressure field in a liquid surrounding a cavity (2, 3, 4). The terms cavity and bubble are considered to be identical in meaning unless otherwise mentioned. Rayleigh (2) considered the case of an empty cavity collapsing under the influence of the external liquid pressure. The liquid was assumed to be
incompressible. By equating the change in kinetic energy of the liquid to the rate of work done on the liquid by the cavity, the following expression was obtained for the cavity boundary velocity $U$:

$$U^2 = \left[ \frac{2(p_\infty - P)}{3\rho} \right] \frac{R_0^3}{R^3} - 1 \quad \ldots \ldots (1)$$

Here $P$ is the pressure at the boundary of the cavity, $R_0$ its maximum radius and $R$ its radius at time $t$. $p_\infty$ and $\rho$ are the pressure and density of the liquid far from the cavity. For an empty cavity, $P$ is zero. This expression for $U$ was inserted into the Bernoulli equation to yield the following expression for the pressure in the liquid at a point distant $r$ from the cavity.

$$p(r,t) - p_\infty = \frac{R}{r}(P - p_\infty) + \frac{R}{r}(1 - \frac{R_0^3}{R^3}) \frac{\rho U^2}{2} \quad \ldots \ldots (2)$$

For points not too far from the cavity, the second term on the right hand side of equation (2) is predominant. This second term has a maximum value of $\frac{3}{8} U^2/4^{1/3}$ at $r = 4^{1/3} R$. Substituting this value in equation (2), the maximum pressure, $p_m$, in the liquid is given by the relation

$$p_m - p_\infty = 4^{-4/3}(p_\infty - P)(R_0/R)^3 \quad \ldots \ldots (3)$$

for $R \ll R_0$. The behaviour of the liquid pressure $p(r,t)$ is shown in figure 1 for four values of the cavity radius, 2.5, 1.75, 1.25 and 1.0 cm. The pressure distribution which is initially dispersed becomes progressively steeper as the cavity shrinks, indicating that in the final stages of collapse extremely high pressures are developed in the liquid near the cavity. In theory, the velocity of the cavity boundary also increases without limit as the cavity collapses; for example, for a cavity in water at a pressure of 1 atm., at a radius one-fortieth of the initial radius the velocity exceeds the sonic speed. This velocity is shooting up as the cavity shrinks further. Therefore, during the final stages of collapse the compressibility of the liquid becomes an
important factor and should be considered if an accurate description of the cavity motion is desired. Such an analysis has been carried out by Gilmore (3), among others (4, 5). Gilmore derived a simple expression for the pressure field in a compressible liquid by assuming that, for the ratio $U/C$, powers higher than the first could be neglected. This expression is as follows:

$$p(r,t) - p_{\infty} = (P - p_{\infty}) \frac{R}{r} + \frac{R}{r} \left[ 1 - \frac{R^3}{r^3} \right] \frac{U^2}{2} + \left( 1 - \frac{R}{r} \right) \left[ \rho U^2 - 2(P - p_{\infty}) - \frac{dP}{dr} \frac{U}{C} \right]$$

Here $C$ is the sonic speed in the liquid and the other terms have the meanings indicated previously.

The first two terms on the right hand side of equation (4) represent the pressure field in an incompressible liquid. The third term, therefore, represents a first order correction to the compressibility of the liquid. Corrections of higher order have been derived by Gilmore but they are complicated expressions and will not be discussed here.

The determination of $p(r,t)$ from equation (4) for a cavity other than an empty one requires a knowledge of the pressure $P$ at the boundary. This pressure is dependent on the cavity motion. For growth or collapse in a compressible liquid, an equation relating the radius of the cavity and the velocity and acceleration of the liquid has been derived by Gilmore (3). Solution of this equation in explicit form is difficult and numerical methods must be adopted. Once the cavity motion is determined, equation (4) may be used for the determination of the pressure field.

Numerous experimental studies of the collapse of bubbles in a liquid are available in the literature (4, 6, 7). Knapp and Hollander (6) studied photographically the collapse of bubbles subjected to a fluctuating environmental pressure. Their experimental results were analysed by Plesset (7) who extended the form of the Rayleigh equation (2)
to include the fluctuating external pressure. Despite certain limiting idealisations in the theory, the theoretical curve correctly described the incompressive stages of the collapse of the bubbles.

Mellen (4) made a careful experimental study of the collapse of spherical cavities in water. The bubbles were formed by spark ignition and the pressure waves measured by a probe hydrophone. The hydrophone was coupled to an oscilloscope and the traces recorded photographically. On account of the frequency response limitations of the hydrophone, fairly large cavities, ranging in diameter from 2 to 4 cm. were studied. The cavities gave two pressure impulses, one at growth and the other at the end of collapse. The occurrence of the two pulses, one at the beginning and the other at the end of the bubble radius cycle, was confirmed by simultaneous photographs of bubble and oscilloscope traces. The pressure pulse at collapse had an extremely short rise time, of the order of a microsecond and thereafter decayed exponentially. The magnitudes of the pressure pulses compared favourably with those obtained from Gilmore's theoretical results. By plotting a pressure pulse magnitude versus time curve from experimental results, the time of arrival of a shock wave was determined. At this instant, the pressure magnitude jumped discontinuously to a much higher value. The cavity boundary velocity at this instant was estimated to be very nearly equal to the sonic speed in the liquid.

From the brief review presented above, it is clear that much attention has been directed towards the determination of the features which characterise the motion of bubbles in cavitating systems. In the field of boiling however, there appear to be few studies dealing with the pressure fluctuations caused by the growth of vapour bubbles. That pressure transients are generated by the formation and growth of bubbles in saturated nucleate boiling has been established by Carg and
Patten(3). A somewhat related study has been reported by Thomas and Williams (3) who measured the pressure waveform generated in air by spark ignited "bubbles" of combustible gas. Fig. 2 shows the shape of a typical waveform obtained in their study. Also shown on the same figure is the waveform calculated using simple acoustic theory. This theory does not appear to predict the details of the rarefaction spike correctly. The linear rise of the pressure with time in the early stages of the motion was also noted by Garg and Patten.

The present chapter describes in detail measurements made of the pressure field generated by the growth and collapse of vapour bubbles in saturated and subcooled nucleate boiling of water. The bubble motion was then computed from the measured pressure waveforms. The results were compared with data obtained from high speed films of the actual growth and collapse cycle of the bubbles.

2.2. Derivation of the relevant equations and their solution

Consider a bubble growing in a uniform liquid extending to infinity and at rest there. It is required to relate the motion of the bubble to the acceleration of the liquid. The assumptions used in this analysis are

1) The liquid has a finite but constant sonic speed.
2) The motion is spherically symmetric.
3) Translational velocity effects are negligible.
4) The viscosity of the liquid is negligible.

Assumption 3) implies that this analysis is not applicable after the bubble acquires a translatory motion due to buoyancy forces.

With the above assumptions, the equations of continuity and momentum are

\[ \frac{Dp}{Dt} + \rho \left( \frac{3u}{3r} + \frac{2u}{r} \right) = 0 \quad \ldots \ldots (5) \]
Here $\phi$ is the velocity potential, $u$, $p$, $\rho$ the liquid particle velocity, liquid pressure and density respectively at the point distant $r$ from the bubble centre. $p_{\infty}$ is the liquid pressure far from the bubble. In deriving equation (6), two assumptions are made. Firstly, the liquid density is assumed to be a function of pressure only and secondly, the liquid is assumed to be at rest at infinity so that no constant need appear on the right hand side of equation (6). In addition, the viscosity of the liquid has been neglected.

Following Gilmore (3), the quantity $h$ is defined by the relation

$$h = \int_{p_{\infty}}^{p} \frac{dp}{\rho} \quad \cdots \quad (7)$$

If the liquid velocities are small so that the squares of these velocities can be neglected, equation (6) has the well known solution (1):

$$\phi = \frac{1}{r} f(t - \frac{x}{C}) \quad \cdots \quad (8)$$

where $C$ is the sonic speed in the liquid.

Substituting for $\phi$ from equation (8) in equation (6) gives

$$f'(t - \frac{x}{C}) = r(h + u^2/2) \quad \cdots \quad (9)$$

Equation (9) indicates that the quantity $r(h + u^2/2) = r \frac{\partial \phi}{\partial t}$ is propagated through the liquid at a speed equal to the sonic speed $C$.

This assumption was first used by Herring (5) for the investigation of pulsations of the gas globe formed in an underwater explosion. A more accurate assumption, that the quantity $r \frac{\partial \phi}{\partial t}$ is propagated with a variable velocity ($C + u$) is used by Gilmore (3). The former assumption will be used here because all of the observed growth and collapse of the bubbles occurred in the low velocity region. With this assumption, the following relation is obtained:

$$\frac{\partial}{\partial t}[r(h + u^2/2)] + C \frac{\partial}{\partial r} [r(h + u^2/2)] = 0 \quad \cdots \quad (10)$$
In the spherically symmetrical situation, equation (6) may be written

\[ \frac{Du}{Dt} = -\frac{\partial h}{\partial r} \] \hspace{1cm} \text{(11)}

Equation (11) is obtained by substituting equation (7) in equation (6) and differentiating the latter equation with respect to \( r \).

For small amplitude pressure waves, the sonic speed \( C \) in the liquid may be defined by the relation (1),

\[ C^2 = \frac{dp}{dp} \] \hspace{1cm} \text{(12)}

From equation (7),

\[ \frac{\partial h}{\partial r} = \frac{1}{\rho} \frac{\partial p}{\partial r} \text{ and } \frac{\partial h}{\partial t} = \frac{1}{\rho} \frac{\partial p}{\partial t}. \]

Using the above relations, \( \frac{dp}{Dt} \) may be written

\[ \frac{dp}{Dt} = \frac{1}{c^2} \left( \frac{\partial h}{\partial t} + \frac{rp}{c^2} \frac{\partial h}{\partial r} \right) \text{ and hence } \frac{dp}{Dt} = \frac{p}{c^2} \left( \frac{\partial h}{\partial t} \right) \]

Substituting the above value of \( \frac{dp}{Dt} \) in equation (5) gives

\[ \frac{1}{c^2} \cdot \frac{\partial h}{\partial t} = -\left( \frac{3u}{r} + \frac{2u}{r} \right) \] \hspace{1cm} \text{(13)}

Equation (10) may be written in the form

\[ \frac{D}{Dt} \left[ r(h + u^2/2) \right] + \frac{C}{2} \frac{\partial}{\partial r} \left[ r(h + u^2/2) \right] - u \frac{\partial}{\partial r} \left[ r(h + u^2/2) \right] = 0 \]

The above equation yields, after simplification,

\[ (r - ru/C + ru^2/C^2)\frac{\partial h}{\partial t} + Ch + Cr(2u/C - 1)\frac{Du}{Dt} - \frac{3}{2}Cu^2(1 - 4u/3C) = 0 \]

This may be written in the form

\[ rC(1 - 2u/C)\frac{Du}{Dt} + \frac{3}{2} u^2C(1 - 4u/3C) - Ch - r \frac{\partial h}{\partial t}(1 - u/C + u^2/C^2) = 0 \] \hspace{1cm} \text{(14)}

Equation (14) is identical with that derived by Herring (5) by a different method. For the present purpose this equation may be simplified on account of the low velocities encountered in this study. The terms \( u/C \) and \( u^2/c^2 \) are small compared with 1 and may be neglected. Equation (14) may then be written in the form

\[ \frac{r\partial h}{\partial t} + Ch - rC \frac{Du}{Dt} - \frac{3}{2} Cu^2 = 0 \]
Alternatively,

\[ r \frac{Du}{Dt} + \frac{3}{2} u^2 = \frac{p}{C} \frac{Dh}{Dt} + h \]  

Equation (15) is the equation relating the liquid acceleration to the pressure at a point distant \( r \) from the bubble. It can be related to the bubble motion as follows:

Let \( R \) be the radius of the bubble at time \( t \) and \( \overline{R} \) its wall velocity.

Using the equation of continuity, the liquid velocity \( u \) is given by

\[ u = \frac{R^2 \overline{R}}{r^2} \]

Now,

\[ r \frac{Du}{Dt} = r(\frac{3u}{3t} + u \frac{u}{3r}) = \frac{R^2 \overline{R}^2 + 2R^2}{r} - 2R^4 \frac{\overline{R}^2}{r^4} \]

and \( \frac{3}{2} u^2 = 3R^4 \frac{\overline{R}^2}{2r^4} \), \( \frac{1}{C} \frac{Dh}{Dt} = \frac{1}{\rho C} \frac{dp}{dt} \)

where \( \overline{R} = \frac{d^2 P}{dt^2} \) and \( u/C \ll 1 \).

Substitution of these relations into equation (15) gives

\[ \frac{R^2 \overline{R}^2 + 2R^2}{r} - R^4 \frac{\overline{R}^2}{r^4} = \frac{(p - p_\infty)}{\rho} + \frac{(r/\rho C) dp}{dt} \]

Equation (16) with the first term only on the right hand side is the well known Rayleigh equation for an incompressible liquid. The second term on the right hand side represents a correction for a finite but constant sonic speed of the liquid.

Relations obtained from simple acoustic theory.

In simple acoustic theory, the velocities everywhere are small such that their squares may be neglected. The momentum equation takes the form

\[ (p - p_\infty) = \rho \frac{\partial \phi}{\partial t} \]

For the radially symmetric situation, \( \phi = R^2 \overline{R}/r \).

Substitution in equation (17) gives

\[ \frac{R^2 \overline{R}^2 + 2R^2}{r} = \frac{(p - p_\infty)}{\rho} \]

This may be written as
\[
\frac{1}{3\pi} \frac{d}{dt} \left( d \frac{R^3}{dt} \right) = \left( p - p_a \right) / \rho \quad \ldots \ldots (19)
\]

Now \((p - p_a)\) is the excess pressure above the static pressure \(p_a\). This is the acoustic or sound pressure, \(p_a\). In terms of the bubble volume \(V\), equation (19) becomes

\[
\frac{1}{3\pi} \frac{d}{dt} \left( dV / dt \right) = \frac{p_a}{\rho} \quad \ldots \ldots (20)
\]

The sound pressure is therefore proportional to the rate of change of the rate of increase of bubble volume.

**Solution of the Equations.**

Equations (16) and (19) permit the determination of the bubble radius \(R\) once the temporal variation of the acoustic pressure is known. Equation (16) cannot be solved analytically and therefore its integration was performed numerically. In this study the temporal variation of the acoustic pressure was obtained by experiment. Figure 3 shows a waveform of the acoustic pressure, typical of many obtained in this study. The period of the pulse was divided into a large number of intervals, for example 40 or as many as 72 in some cases. The pressures measured at each of the intervals constituted the data for the numerical solution of equation (16). Using a fourth order Runge-Kutta method, a suitable computer program was developed for the integration. The accuracy of the results was checked by halving the time interval and recomputing. In the early stages of the solution, time intervals of the order of one-tenth of a millisecond were used on account of the rapid pressure variation in this region. Larger time steps were used for the subsequent integration. The computer program is included in appendix 2.

The solution of equation (16) requires the specification of two constants. For saturated boiling these were taken to be the initial bubble radius \(R_i\) and growth rate \(\dot{R}_i\). The value chosen for \(R_i\) was...
sufficiently small so that the subsequent growth of the bubble was unaffected by the choice. The initial growth rate \( \dot{R} \) was set equal to zero. For subcooled boiling, the initial conditions were taken at the instant of maximum bubble radius. At this instant \( \dot{R} \) is zero. Integration was performed forward to represent collapse and backward to represent growth. The theoretical curve has been fitted to the experimental values at this one point.

The integration of equation (19) is straightforward. The constants to be specified for its solution are the same as those for equation (16). The acoustic pressure pulse of Fig. 3 may be divided into several regions, in each of which the pressure may be represented either by a linear or by a simple quadratic function of time. For example, in the region OA, the relation between \( p_a \) and \( t \) is of the form \( p_a = A t \) where \( A \) is a constant. Starting with region OA, the relation between \( p_s \) and \( t \) is inserted into the right hand side of equation (19) and two integrations performed. This yields an equation between \( R^3 \) and \( t \) from which the bubble radius \( R \) may be determined. The solution may be continued until the entire pulse period has been covered. A sample calculation by this method is indicated in Appendix 3.

2.3. Experimental Apparatus

a. Boiler Assembly

The boiling liquid was contained in a Pyrex glass vessel of cruciform shape. Two views of the boiling vessel are shown in figure 4. The horizontal limb was 4 in. in diameter and 16 in. long while the vertical limb was 6 in. in diameter and 12 in. long; the vessel thickness was roughly \( \frac{3}{4} \) in. Glass windows measuring 5\( \frac{1}{2} \) in. diameter by \( \frac{3}{4} \) in. thick located at each end of the horizontal arm by suitable flanges permitted photography and visual observation. The top and bottom flanges were made of mild steel and measured 10\( \frac{1}{2} \) in. diameter.
by 8 in. thick. To prevent rusting with consequent contamination of the test liquid, these flanges were chrome plated and polished. Figures 5A and 5B show details of the top and bottom flanges. A 1.5 kW glass enclosed heater maintained the test liquid (water in the present experiments) at the desired temperature. In addition an Electrothermal heating tape wound around the outside of the vessel provided further heat as required and enabled the water to be maintained at specified temperatures to within ±0.5 deg.F. To reduce heat loss, the entire apparatus was wrapped with mineral wool insulation.

Connections for temperature and pressure measurement and a 1 in. diameter vapour exhaust pipe were provided through the top flange. Sub-atmospheric pressures could be maintained by means of an Edward vacuum pump. An air admittance valve near the suction end of the pump and a needle valve in the manometer line allowed the vacuum to be maintained at prescribed values accurate to within ± 0.05 inch of mercury. A schematic diagram of the experimental arrangement appears in figure 6. The water-cooled condenser in the vacuum line significantly reduced the flow of vapour to the pump.

b. The Heating Surface.

The heating surface was a 0.072 in. diameter stainless steel tube (type 410) with an average wall thickness of 0.012 in. Because one of the objectives of this study was to determine the shape and magnitude of the pressure waveform generated by single bubbles (free of interference from neighbouring bubbles), it was necessary to remove most naturally occurring cavities on the tube surface. These cavities may be in the form of pits or depressions of geometry suitable for trapping vapour or gas (6). The removal of these cavities was achieved by polishing the tube. A 6 in. length of tube, which was
reasonably smooth when observed under a low power (x 20) microscope, was selected and polished with fine silicon carbide paper, until most pits and scratches were removed. Further polishing was carried out with DIALAP diamond compound, which was in the form of a paste. Differing grades of the compound were used, with particles varying in size from \( \frac{6}{3} \) down to \( \frac{1}{4} \). The tube was gripped in a high speed lathe and polished lengthwise by applying small quantities of the compound with a special felt cloth supplied by the makers. The tube was examined under a microscope at a magnification of 20 at various stages of polishing and when the surface appeared mirror smooth with no observable pits or scratches, it was assumed that the naturally occurring cavities had been removed and the polishing was therefore discontinued.

The polished tube was cut to the required length of the test section (2.75 in.) held in a suitable jig and a cavity made centrally using a sharpened gramophone needle. The diameter of the cavity was measured using a metallurgical microscope with a calibrated eyepiece at a magnification of 100. In the tests reported here, the cavity diameter was found to be 0.016 in. with an estimated accuracy of \( \pm 0.0002 \) in. No attempt was made to measure the depth of the cavity.

The tube was mounted between two silver coated brass electrodes, \( \frac{3}{8} \) in. diameter, 10 in. long with the cavity positioned centrally and facing up. The length of the tube between the electrodes was 2 in. Details of the arrangement are shown in figure 5D. Good electrical contact was ensured by wrapping the ends of the tube with gold foil before mounting. The electrodes were located in insulating tufnol bushes in the top flange (fig.5A).

c. Power Circuitry and Instrumentation.

DC power was supplied to the heating surface by means of a full-wave bridge rectifier. DC was chosen in preference to AC to avoid any
in-phase generation of bubbles at mains frequency. The maximum current available was roughly 50 A at 2.2 V with less than 1 per cent ripple at this voltage. This current was equivalent to a heat flux of approximately 120,000 Btu per hour per sq. ft. A sketch of the power circuit is shown in fig. 7. The voltage across the test section was measured by a voltmeter and the current by an ammeter. The maximum error in the measurement of heat flux was estimated to be ± 20 per cent. The error decreased to ± 6.5 per cent at the highest heat flux reported in this study. The estimation of these errors is shown in Appendix 7. The power input to the heating tube was varied by controlling the setting of the Variac transformer feeding the rectifier.

For the measurement of the required temperatures, 0.0076 in. diameter Chromel-Alumel (T₁ - T₂) thermocouples, supplied by British Driver-Harris, were used. The thermocouple junctions were formed by butt welding the wires on a Spembly capacitor discharge welder. Calibration was carried out at ice and steam points and thermocouples with an error greater than 0.5 per cent from the tabulated values were rejected. To measure the tube temperature, two thermocouples were placed one on either side of the centre of the tube internally along the horizontal centre line. The ends of the tube were then sealed with epoxy resin and covered with SILASTOMER rubber. Care was taken to ensure that the thermocouple junctions were isolated from the tube wall. The effect of the current through the tube was investigated by reversing the direction of the current and noting the change in the thermocouple emf. No readable change was observed, even at the maximum current of 50 A. The inside of the tube was under essentially adiabatic conditions and therefore the thermocouples measured the inside tube surface temperature. This temperature was then used in the estimation of the outside surface temperature from a knowledge of the thermal properties.
of the tube material as provided by the manufacturer. The bulk liquid temperature was measured by a thermocouple located in the horizontal plane of the heating tube but \( \frac{1}{2} \) in. behind it. All thermocouple wires were led out through the top plate to a cold junction maintained at zero deg.C. Thermocouple emfs were measured on a Croydon precision potentiometer reading to 1 microvolt.

For the measurement of sub-atmospheric pressures in the boiling vessel, a mercury manometer was used. The pressure could be maintained constant to within \( \pm 0.05 \) in. of mercury and down to 0.5 psia.

For the detection and recording of pressure waveforms generated by single bubbles, the equipment consisted of a pressure transducer, amplifier, oscilloscope and a high speed camera. A sketch showing their relative positions is given in figure 8. Reference may also be made to plate 1 which is a photograph of the overall experimental set-up. The Kistler quartz crystal pressure transducer, type 410B has a frequency response, flat to within 1 per cent, in the range DC to 40 KHz. The calibration constant of the transducer is 154 picocoulomb per atmosphere (pC\(\text{b}/\text{atm.}\)). The output of the transducer was fed to the matched Kistler charge amplifier type 566 and then to a Solartron oscilloscope. The gain on the amplifier could be varied in steps up to a maximum of 100 mV/pC\(\text{b}\). The overall noise level of the system at maximum gain was less than 1 per cent of the signal amplitude. However, in the tests reported here, it was sufficient to use a gain of 50, with consequent reduction of the noise content to negligible levels.

d. High speed photography

High speed photographs of the vapour bubbles were taken with a Hycam rotating prism camera, model K2054E, at speeds of up to 5000 frames per second. Ilford Mark V 16 mm negative films were employed, in the form of 100 ft. and 200 ft. daylight loading spools. Because
of the short exposure times involved, suitable lighting was ascertained by trial and error. Two 100 ft. spools were used before the correct lighting system was obtained. This lighting system consisted of three 240 V, 500 W Photoflood reflector lamps, operated through dimmer switches. Two lamps were arranged, as shown in plate 1, to the right hand side of the boiling vessel. These lamps were operated at full voltage. A third lamp was placed behind the heating tube. Sheets of tracing paper interposed between the rear lamp and the boiling vessel provided diffused lighting. The voltage of this lamp was adjusted so that the total light intensity was sufficient for the required frame speed. Care was taken to locate the lamp at such distances that no significant heating of the water occurred.

The camera axis was in the same plane but perpendicular to the axis of the heating tube. A Cosmicar 50 mm. f 1.4 lens with a ½ in. extension tube was used with the aperture set at f4. The lens to object distance was 19 in. The field of view was such that the entire length of the heating tube (2 in.) and the transducer face could be seen. The diameter of the transducer face of 0.547 in. was used as a reference length in the subsequent analysis of the films.

Simultaneous recording on the film of the pressure transient was obtained through an oscilloscope lens attachment available on the camera. A 50 mm. f 1.8 Pentax lens was used, with the aperture set at f 1.8. The lens to oscilloscope distance was 16 in. Looking through the objective lens the pressure trace on the oscilloscope screen was positioned so that its image on the film was within the dimensions of the frame. The objective lens was also used for focussing the oscilloscope lens. A second trace on the oscilloscope screen was located a specified distance from the first, and this distance was used as a reference length during pulse amplitude measurements from
the film. The time base of the oscilloscope was then switched off.

Timing marks were placed on the edge of the film at one millisecond intervals, by a Wollensak timing light generator so that the average speed over a short length of film could be accurately determined from these timing marks.

2.4. Experimental Procedure

The pressure waveforms generated by individual bubbles were measured and recorded at pressures of 1.0, 3.0, 7.0 and 14.7 psia, for saturated and subcooled boiling conditions. These pressures were chosen after taking into account the variation of the properties of water over the range 1.0 to 14.7 psia. Over the whole range while the surface tension and viscosity decrease by 20 per cent and 54 per cent respectively, the specific volume of saturated vapour decreases by a factor of 12. The thermal conductivity of water increases by about 8 per cent over the same range.

Before each test the top plate was removed and the boiling vessel thoroughly cleaned with hot water and detergent and finally rinsed with de-ionised water. The heating surface, brass electrodes, the transducer and the top plate were cleaned first with hot water and detergent. After drying, they were then cleaned with acetone to remove any deposits or foreign matter from the metallic surfaces and finally rinsed in distilled de-ionised water. This procedure was repeated after each test which normally lasted for two hours.

The apparatus was then assembled and the vessel filled with freshly prepared distilled deionised water of resistivity greater than 1 megohm per cm. to a level ½ in. below the top plate. By means of the bulk heaters the temperature of the water was increased to saturation temperature corresponding to half the nominated test pressure. Subsequently the pressure in the vessel was reduced gradually to half the test pressure. Some bulk boiling occurred, after which the water was
boiled on the heating surface at test pressure for about half an hour in order to remove any air trapped on the tube. When steady state conditions were reached, as indicated by the temperatures, data recording was started. The bulk liquid was maintained at saturation temperature or at the temperature equivalent to a prescribed degree of subcooling by monitoring the voltage to the wall heaters. With the above procedure, the reproducibility of the test data, as can be seen in fig. 9, was found to be satisfactory.

For each pressure and subcooling of test liquid the following data were taken:

i) voltage on the wall heater,
ii) voltage across heating tube and current,
iii) thermocouple emfs and cold junction temperature,
iv) amplifier and oscilloscope settings,
v) barometer reading,
vi) pressure in the test vessel.

To avoid convection currents in the bulk liquid during a test run, the bulk heater was switched off while recordings were being made.

2.5. Method of film analysis and Interpretation of records

In order to obtain quantitative information on the dynamics of bubble motion and the associated pressure waveforms, the high speed films were analysed frame by frame on a a Lytax film analyser which gave a 3 x magnification of film size to viewed image. In plate 2 is shown a sequence of frames of subcooled boiling at 3.0 psia. The position of zero time was fixed at the frame preceding that in which the bubble first became visible. In plate 2, the bubble was first observed in frame no.500. The origin of time is therefore fixed at frame no.499. There is, however, a synchronisation error between the bubble signal and pressure amplitude measurement.
Referring again to plate 2, the pressure pulse associated with the formation and growth of the bubble can be seen. Its magnitude is given by the deflection of the oscilloscope trace from its initial position; this initial position is located approximately at the centre of the frame in frame no. 500. Subsequent frames show the growth of the bubble and the behaviour of the pressure pulse. The pulse reaches a maximum positive value in frame no. 500, then decreases and reaches zero in frame no. 503. Thereafter it attains negative values with a maximum in frame no. 505, increases again and, in frames not included here, goes through one more cycle before finally reaching zero.

For each of the films examined, the following data were recorded:

a) Frame speed, position of bubble at instant of formation in relation to pressure transducer

b) Bubble diameter vs. time

c) Magnitude of the pressure pulse vs. time.

During growth, the bubbles remained essentially spherical in shape, reached a maximum diameter and then departed from the tube surface thus acquiring a translational velocity.

Subsequently, two types of bubble behaviour were observed. In the first, after a short interval of time, the front surface of the bubble slowed down; the rear surface folded inwards from the pole and a tongue of liquid penetrated the bubble from the back; the bubble thus acquired a toroidal shape. In the second, the bubble was not penetrated by a tongue of liquid and no folding of the rear surface was observed. The bubble, however, assumed an ellipsoidal shape soon after leaving the tube.

The bubbles were measured in two perpendicular directions and the arithmetic average taken to represent the diameter. When the bubbles became distorted, they were considered as ellipsoids and the horizontal and vertical axes were measured. The third axis, which was unknown, was
taken equal to the vertical axis. The bubble diameter for comparative purposes was taken to be that of a sphere with the same volume as the ellipsoid. If the axes of the ellipsoid are $2a$, $2b$, $2c$, the bubble diameter in the present case, with $b$ equal to $c$, would be $(ab^2)^{1/3}$.

At pressures of 1.0, 3.0 and 7.0 psia, the bubble enclosed the heating tube. A correction for this enclosed heater volume was applied when calculating the bubble diameter. The heater volume enclosed by a bubble of diameter $D$ is $\pi d^2D$ where $d$ is the heater outside diameter. The corrected volume of the bubble is, therefore, $\pi D^3/6 - \pi d^2D$. The diameter of a sphere with a volume equal to this corrected volume was taken to be the bubble diameter. This correction was not necessary at atmospheric pressure, because the bubbles did not enclose the heater at this pressure.

In all bubble size measurements, the known diameter of the transducer face was used as a reference length. The frame rate was determined by counting the timing marks. For the pressure pulse data, the distance between the two oscilloscope traces, when no bubble was present on the frame, was taken as a reference length. Only those bubbles which were free of interference from neighbouring bubbles were considered for measurement.

By this method, data were obtained for some eighty-two bubbles; the results are presented and discussed in the next section.

Accuracy of bubble size measurements.

The bubble sizes could be measured to an accuracy of $\pm 0.0068$ in. The average error, over the bubble growth period, for growth at atmospheric pressure was estimated to be $\pm 8$ per cent. The calculations are indicated in Appendix 7.

2.6. Results and Discussion

In figures 10 to 23 are shown the bubble diameter as a function
of time for the pressures and subcoolings studied in this investigation. The full lines show the computed growth and collapse history, obtained by a computer solution of equation (16) as discussed earlier. Bubble growth rates observed in this study will be discussed in detail in Chapter 5.

The pressure waveforms generated by individual bubbles for saturated boiling at atmospheric pressure and sub-atmospheric pressures of 7.0 and 1.0 psia are shown in figs. 10, 11 and 13 respectively. For subcooled conditions, some selected waveforms are given in figs. 15, 17 and 22. These were typical of many obtained in this investigation.

Pressure waveforms in relation to bubble history

It will be seen from figs. 10 and 22 that the pressure waveforms at saturated and subcooled boiling are similar during the period of growth of the bubble. In subcooled boiling, however, the pressure goes through one more cycle before finally attaining a zero value. At the instant of collapse of the bubble, (defined here as the instant of disappearance), in the subcooled case shown in figure 22, the pressure pulse did not rise to a sharp peak which is contrary to the observations on cavitation bubbles (4).

In all cases, the initial sharp rise of the pressure pulse indicates that the velocities and accelerations are large during initiation and a short time afterwards. The rise time of the pulse was typically around ½ millisecond, which is equivalent to a frequency of 2 kHz. This is well below the resonant frequency of the transducer and therefore it is unlikely that any distortion of the pressure wave occurred. Another factor which may affect the shape of the wave is the reflection from free and solid surfaces. Preliminary experiments showed that the effect of reflections from free and solid surfaces was negligible. In these preliminary experiments, the heating tube assembly was mounted in a
rectangular Perspex tank, 11 in. x 7 in. x 12 in., with the free surface of the bulk liquid about 7½ in. above the level of the tube. The transducer was placed about 1 in. below the tube. The pressure waveforms obtained under these conditions were similar to those obtained and reported here. Altering the distance between the transducer and the tube altered the amplitude of the pressure waveform, but the shape remained the same. These considerations would appear to indicate that reflections from free and solid surfaces had little effect on the waveforms measured in the present study.

The extent of the agreement between the experimental and computed growth curves may be seen in figs. 11 to 23 and is considered good. The simple acoustic theory appears to give results close to the experimental values during the growth phase of the bubbles. This is shown in fig. 24 where a comparison is made between the experimental growth and collapse curves for three bubbles and those curves determined from theory. During collapse, however, the simple acoustic theory yields values which depart considerably from the experimental results. This is probably due to the large velocities occurring in this phase of the bubble history.

In the Rayleigh theory, the bubble radius as a function of time is determined from equation (1). The time of complete collapse, \( t_o \), is given by [2],

\[
t_o = 0.915 \left[ \frac{\rho}{(\rho_m - \rho)} \right]^{\frac{1}{2}}
\]

To determine the bubble radii as a function of time, let \( \frac{R}{R_0} = z \). Then equation (1) may be written

\[
R_0 \frac{dz}{dt} = \left[ \frac{2(\rho_m - \rho)}{3\rho} \right]^{\frac{1}{2}} \left[ \frac{1 - z^2}{z^2} \right]^{\frac{3}{2}}
\]

The time, \( t \), for the bubble to collapse to a radius \( R \) is given by

\[
t = R_0 \left[ \frac{3\rho}{2(\rho_m - \rho)} \right]^{\frac{1}{2}} \int \frac{1}{z^{3/2}} \frac{dz}{(1 - z^2)^{3/2}}
\]
Dividing both sides of the above equation by \( t_0 \) gives

\[
\frac{t}{t_0} = 1.34 \int_{z}^{1} \frac{z^{3/2}}{(1 - z^2)^{3/2}} \, dz
\]

\[
\ldots \ldots (24)
\]

This relation involves no fluid properties; and once the time of total collapse \( t_0 \) and the maximum radius \( R_0 \) are known, the time \( t \) for the bubble to collapse to any radius \( R \) may be determined. The value of the integral for various values of \( z \) has been tabulated in reference (6). In this manner, the collapse rates for the two bubbles in fig. 24 were determined. It is seen that equation (24) predicts bubble diameters considerably larger than the experimentally measured values and a much slower collapse rate than actually occurs.

It is apparent from figure 24 that equation (16), which is a computer solution, describes the collapse phase, at least the initial incompressive stages, correctly. It is suggested that the deviation from the measured values is due to the following:

a) the bubble acquires a significant translational velocity just before the beginning of the collapse phase,

b) the shape of the bubble is not absolutely spherical,

c) the distance \( r \), which is the distance of the bubble centre from the transducer varies with time. Strictly speaking, equation (16) would not be applicable from the instant when the bubble, after reaching a maximum diameter, begins to rise through liquid and acquires a translational velocity. The translational velocity effects b), c) have not been allowed for in the derivation of equation (16). It is seen that equation (16) predicts a much faster collapse than actually occurs.

The above agreement suggests that provided the relative position of the bubble centre and the transducer is known, the bubble size may be computed directly from the recorded pressure trace without observation of the bubble. Subject to these limits then it is possible to use the
acoustic detection equipment to give information about the bubble characteristics.

Pressure Transients as a factor in bubble nucleation.

The rapid changes in pressure which follow the growth of a bubble will have the effect of changing the total pressure at adjacent sites. The consequences of the change in pressure will be severe, at the instant of and immediately after nucleation, in terms of the temperature difference required for nucleation. Some numerical results may be obtained from fig. 22. Immediately following the initiation of bubble no.24-1, the pressure amplitude reaches a value of roughly 0.225 lb. per sq. in. Near the bubble, this value will be much larger, around 0.3 lb. per sq. in. This is equivalent to an increase in the liquid pressure from 3.0 to 3.3 lb. per sq. in., which may correspond to an increase in saturation temperature of 5 deg.F. Consequently, initiation of a bubble from an adjacent site may be retarded. Approximately 4 milliseconds after initiation the liquid pressure has fallen by about 0.1 sq. in. to 2.9 lb. per sq. in. The consequent decrease in the superheat required for initiation, which is roughly 1.5 deg.F, will be of some assistance for nucleation. Therefore the effect of the pressure wave may be to retard nucleation at the instant of bubble formation but to encourage it immediately afterward. No definite evidence of this effect could be obtained from the films because, in this study, the main object was to nucleate single bubbles and record the associated pressure waveforms.

The analysis of Henley and Hummel (47) lends support to the hypothesis that bubble induced pressure waves may influence nucleation from adjacent sites. Five rolls of film of boiling of water on stainless steel strips were analysed. By a statistical examination of these films, Henley and Hummel concluded that there was a strong tendency for
a bubble to nucleate one frame after the nucleation of a previous bubble. For example, in roll no.1, the nucleation of a bubble followed one frame later by nucleation of second bubble at a different site occurred 22 times. In roll no.2, the same phenomenon was counted 15 times. The occurrence of this phenomenon was attributed to some form of "sonic interaction of a bubble with its surroundings".

2.7. Conclusions.

The following conclusions may be drawn from the results presented.

a) Bubbles generate a pressure wave at the instant of nucleation. The shape and magnitude of these waves have been measured.

b) The pressure waveforms may be related to the growth and collapse history of the bubbles and one may be computed from the other. The experimental and computed radius-time curves agree satisfactorily.

c) It is possible that the pressure transients will have the effect of retarding nucleation at instant of bubble formation but favour it shortly afterwards. This effect has not been proved in the present study.

d) In this investigation, the bubbles produced no sharp pressure impulses upon collapse.
CHAPTER 3

SOUND GENERATED BY SINGLE BUBBLES IN SATURATED AND
SUBCOOLED NUCLEATE BOILING

3.1. Introduction

Whenever boiling or cavitation occurs, a characteristic feature is the growth and collapse of bubbles at preferred locations either on a solid surface or in the body of the liquid. The pressure pulses, of varying magnitude and frequency, generated by the bubbles will arrive at a point in the liquid in a more or less random manner. The only meaningful measurement of radiated noise in such systems is the noise spectrum associated with boiling or cavitation. It will be difficult, if not impossible, to identify the contribution of an individual bubble to the overall spectrum. However, it may be noted that the form of the total spectrum of a series of pulses arriving in a random manner at any point is similar in shape to that of a single pulse. The spectrum of noise from single bubbles may therefore be relevant to boiling noise. In view of these considerations, a study of single bubble noise has been investigated and reported here. The noise generated by single bubbles has been measured at atmospheric and sub-atmospheric pressures with subcooling of the bulk liquid up to 20 deg. C. The spectra of the noise generated were computed using Fourier Transform methods.

3.2. Survey of literature

a. Sound generated by gas bubbles entrained in a liquid.

The sound produced by gas bubbles entrained in water was first studied by Minnaert (10). The sound was associated with the volume pulsations of the bubble. An equation for the frequency of the volume pulsations was derived by Minnaert.

\[
f = \frac{(3\gamma P_0/\rho)^{1/2}}{2\pi R_m}
\]

\[\ldots (1)\]

\(P_0\) is the ambient liquid pressure, \(\gamma\) the ratio of specific heats for
the gas within the bubble, \( R_m \) the mean radius of the bubble and \( \rho \), the density of the liquid. Observations on bubbles forming at a nozzle showed that equation (1) was in substantial agreement with experimental results.

Strasberg (11) in a thorough investigation of the sound generated by entrained gas bubbles in a liquid compared the sound pressure radiated by bubbles oscillating in the zeroth, first and higher modes. The zeroth mode corresponds to simple volume pulsation and the frequency is given by equation (1). The first mode is simple translation. Higher modes correspond to oscillations in the shape of the bubble. When the amplitude of the oscillations was small, the contribution of all but the zeroth mode to the sound pressure was negligible. The sound pressure, at a distance \( r \), was calculated from the following relation derived from simple acoustic theory.

\[ p_s = \left( \frac{\rho}{4\pi r} \right) \frac{d^2V}{dt^2} \]  

.....(2)

In equation (2), \( p_s \) is the sound pressure and \( V \), the instantaneous bubble volume. Experimental results were reported for air bubble formation and break-off at a nozzle. A pressure pulse was emitted at the instant of bubble break-off and thereafter the pulse decayed sinusoidally. The values of the frequency \( f \) and the pressure \( p_s \) calculated from equations (1) and (2) agreed closely with the experimental results.

For bubble coalescence and division, Strasberg gives some theoretical results only. For two bubbles uniting to form a single bubble of radius \( \frac{1}{3} \) cm., the resultant peak sound pressure at 1 metre distance was roughly 0.3 microbar but no experimental information was provided.

**b. Boiling and Cavitation Noise**

The behaviour of bubbles formed in boiling or cavitation is, of course, different from that of gas bubbles. In boiling and cavitation, the growth and collapse of bubbles is a transient phenomenon, occurring
in a very short time with accompanying pulsations of large amplitude. Under these conditions, it may be expected that the sound pressure waveform will be different from the decaying sinusoidal characteristic of volume pulsations.

The noise generated by cavitation bubbles has been extensively studied and reported in the literature (4,12,13). These studies indicate that cavitation noise differs markedly from that generated by entrained gas bubbles considered before. A sharp peak occurs in the pressure waveform near the end of collapse and the frequency of this peak can be as high as 100 kHz. Mellen (12) studied the spectrum of sound radiated from cavitation generated by moving a rod in a liquid. The spectrum rose to a peak between 1 and 5 kHz and then fell off roughly as $-6\text{dB per octave}$.

A careful study of single bubble cavitation noise in non-uniform pressure fields was reported by Harrison (13). Two experimental methods were employed. In the first method, cavitation bubbles formed in water flowing through a venturi were studied. The pressure pulse was measured by a hydrophone placed outside the lucite venturi tube and immersed in a tank of water. The bubbles and the associated pulse were photographed simultaneously using a high speed camera at speeds of up to 8000 frames per second. In the second method, bubbles were generated in a tank of water by means of a spark. The pressure pulses emitted by these bubbles were recorded on an oscilloscope and then photographed. The air content of the water was carefully controlled.

In the venturi nozzle tests, the cavitation bubbles formed at air nuclei present in the flowing water. The amplitude of the radiated pressure pulse was sensitive to the time life of the nuclei. The pulse magnitude increased in a linear fashion with increase in the time allowed for the nuclei to dissolve, but eventually reached a limiting value. The
reason for this behaviour probably lies in the fact that as the air content decreases, the speed of collapse increases. Consequently compressibility effects become important in determining the cavity motion and because the energy available is finite, the pressure amplitude may be expected to tend towards a limiting value.

Noise spectra of the cavitation bubbles are also reported by Harrison (13). No details of the form of the pressure pulse emitted by these bubbles were presented, although it was mentioned that the spectrum decayed as $s^{-2}$, i.e., as $-6$ dB per octave. This result agrees with the observations of Mellen (12). However, the pressure pulses measured by Harrison are open to some doubt. This is because the pressure waves travelled through the lucite wall of the venturi tube and then through water before reaching the hydrophone. Because the acoustic impedances of lucite and water are greatly different, considerable distortion of the pressure wave would have occurred at the interface between the two media. The behaviour of the spark bubbles was identical with that of the venturi tube bubbles formed at a large air nucleus as regards magnitude of pressure pulse, duration of pulse and spectrum of the generated noise. Attempts to form a spark bubble that behaved like a venturi bubble of low air content were reported to be unsuccessful.

In boiling systems, there appears to be no comparative study of the noise generated by single bubbles. All the studies reported to date (14, 15, 16) deal with the overall noise spectrum for a variety of experimental conditions. The present study deals with the simplest case, that of a single bubble growing and collapsing in a large body of liquid.

3.3. *Experimental Apparatus*

The experimental apparatus used for the study of a single bubble noise was essentially the same as described in Chapter 2, section 3. Of primary interest in the present study were two aspects of single bubble
noise, namely,

a) The spectral distribution of the noise,

b) Variation of pressure pulse amplitude with bubble lifetime and maximum radius.

For the second of the two aspects, the simultaneous measurement of pressure pulse and bubble motion on high speed film provided the requisite information. For the measurement of noise spectra, however, it was necessary to know only the details of the pressure pulse, which were obtained on an oscilloscope and recorded on Polaroid film.

A complete description of the high speed photography has been given in Chapter 2. With this technique, data were obtained at atmospheric pressure for two values of subcooling, 15 deg. C and 5 deg. C and at 3.0 psia for a subcooling of 15 deg. C.

For the measurement of noise spectra, the arrangement of the equipment was identical to that shown in figure 9, except that, in this case, the high speed camera was removed. The oscilloscope was of the persistence type which permitted the pulses to be photographed conveniently. A Polaroid camera, with type 107 film was used for purposes of photography. An exposure time of 1/10 sec. with the aperture set at f8 gave satisfactory results. With this equipment, pressure pulse traces were obtained on film at sub-atmospheric pressures ranging from 1.0 to 7.0 psia and liquid subcooling of up to 15 deg. C. Plot 3 shows a photograph of the pressure pulse recorded at 3.0 psia with a subcooling of 15 deg. C.

3.4. Results and Discussion

a. Frequency spectrum of Noise due to Single Bubbles

The measurement of the pressure pulses permitted the determination of the noise spectra. If \( p(r,t) \) represents the pressure in a liquid at a distance \( r \) at time \( t \), the spectral distribution of the sound is given by the relation

\[
S(f) = |g(f)|^2
\]

\[\textbf{(3)}\]
where

\[ g(f) = \int_{-\infty}^{\infty} p(r,t) \exp(-2\pi if t) \, dt \quad \ldots \ldots (4) \]

\( g(f) \) is the Fourier transform of the pressure \( p(r,t) \). Some normalising factor may be introduced into the right hand side of equation (3) but this will not affect the form of the spectrum.

Because the pressure waveform could not be represented by a simple analytical expression, the integral in equation (4) was evaluated numerically, at each frequency, using Simpson's rule. The limits of the integral in equation (4) extend from \(-\infty\) to \(+\infty\). However, it may be seen from Plate 3 that the pressure is zero everywhere except in the interval \((0,T)\), where \( T \) is the period of the pulse. Therefore the equation for \( g(f) \) may be written

\[ g(f) = \int_{0}^{T} p(r,t) \exp(-2\pi if t) \, dt \quad \ldots \ldots (5) \]

which is the finite Fourier transform of \( p(r,t) \). The period \( T \) was divided into a large number of small intervals and the pressure ordinate at each interval was measured. These values were then fed into a suitable computer program developed to evaluate equations (5) and (3). This program is shown in Appendix 4. The frequencies at which \( g(f) \) was evaluated ranged from 20 Hz to 100 kHz. Substitution of the computed value of \( g(f) \) at each frequency into equation (3) gave \( S(f) \). The noise generated by single bubbles were computed at atmospheric and several sub-atmospheric pressures. These spectra are shown in figs. 25 to 32.

It may be seen from figs. 26 to 32 that initially, in the low frequency range 20-100 Hz, the spectra rise at approximately 12 to 15 dB per octave. Thereafter, the spectra remain irregular up to a frequency of about 10 kHz. Beyond 10 kHz, the spectra decay, the rate of decay varying from 1 to 4 dB per octave.
The experimentally observed noise spectral behaviour at low frequencies can be predicted, with good accuracy, from equations (2), (3) and (4). Using the same equations, the behaviour at high frequencies, however, may be predicted only approximately.

b. Noise Spectral behaviour at low frequencies.

The frequencies considered here are low compared to the reciprocal of the bubble lifetime, $1/T$. For the bubbles with spectra as shown in figs. 25 to 32, the average bubble lifetime, $T$, was approximately 30 msec. The frequency corresponding to this lifetime is therefore 33 Hz. The arguments given below are therefore expected to be valid at frequencies below 33 Hz.

The spectral density of the sound is given by equation (3)

$$S(f) = |g(f)|^2 \quad \ldots \ldots (3)$$

where $g(f)$, the Fourier transform of the pressure $p(r,t)$, is given by

$$g(f) = \int_{-\infty}^{\infty} p(r,t) \exp(-2\piift) dt \quad \ldots \ldots (4)$$

Consider for simplicity, the case of incompressible flow with low velocities so that the expression for $p(r,t)$ is given by equation (2). Substitution of this expression for $p(r,t)$ in equation (4) gives

$$g(f) = A \int_{-\infty}^{\infty} \frac{d^2V}{dt^2} \exp(-2\piift) dt \quad \ldots \ldots (6)$$

where $A = \rho/\mu_w r$

Upon integration by parts, equation (6) becomes

$$g(f) = A(2\piif)^2 \int_{-\infty}^{\infty} V \exp(-2\piift) dt \quad \ldots \ldots (7)$$

The bubble volume $V$ is positive during the interval $(0,T)$ and zero in the intervals $(-\infty,0)$ and $(T,\infty)$. The integral in equation (7) may therefore be replaced by the integral

$$\int_{0}^{T} V \exp(-2\piift) dt.$$

At frequencies small compared to $1/T$ (i.e., $ft<< 1$), the above integral
is, to the first order in \( f \), independent of the frequency. Equation (7) therefore shows that \( g(f) \) is proportional to \( f^2 \). From equation (8), it may be seen that the spectral density \( S(f) \) is proportional to \( f^4 \), the fourth power of the frequency. Therefore, in the low frequency range, the spectra may be expected to rise as \( f^4 \) i.e., at 12 dB per octave. This is in substantial agreement with the experimentally observed rise of 12 to 16 dB per octave.

c. Noise spectral behaviour at high frequencies.

The frequencies considered here are high compared to the reciprocal of the bubble lifetime.

Upon integration by parts, equation (4) may be written

\[
g(f) = A \left[ \frac{p e^{-2\pi if t}}{-2\pi if} \right]_0^T + \frac{1}{2\pi if} \frac{dp}{dt} e^{-2\pi if t} dt \quad \cdots \cdots (8)
\]

Since the pressure \( p \) does not change abruptly, i.e. possesses no discontinuities in \((0,T)\), \( \frac{dp}{dt} \) will be likewise continuous in \((0,T)\). Therefore as \( f \to \infty \), the integral on the right hand side of equation (8) tends to zero. Equation (8) may therefore be written

\[
g(f) = A \left[ \frac{p e^{-2\pi if t}}{-2\pi if} \right]_0^T \quad \cdots \cdots (9)
\]

because the pressure \( p \) is zero in the intervals \((-\infty,0)\) and \((T,\infty)\).

Substitution of the limits in equation (9) gives

\[
g(f) = A \left( \frac{1}{2\pi if} \right) \left[ p(0) - p(T)e^{-2\pi if T} \right] \quad \cdots \cdots (10)
\]

which for \( fT \gg 1 \) reduces to

\[
g(f) = A. \frac{p}{2\pi if} \quad \cdots \cdots (11)
\]

From equations (11) and (3), it may be seen that
the spectral distribution goes as $f^{-2}$, i.e., as -6 dB per octave, at frequencies high compared to that given by the reciprocal of the bubble lifetime. Referring to figs. 25 to 32 it may be seen that the experimental spectra follow, only approximately, the theoretical prediction.

3.5. Pressure pulse amplitude in relation to bubble lifetime and maximum radius.

Simultaneous recording on high speed film of the bubble motion and associated pressure pulse yielded data on the pressure pulse amplitude, maximum bubble radius and the total lifetime of the bubble. The films were analysed on a Lytax projector by methods described in section 5 of Chapter 2. Figures 33 and 34 show the variation of pressure pulse amplitude with maximum radius and lifetime of bubble at atmospheric pressure and at 3.0 psia. The bubbles grew and collapsed at random on the heating surface. Consequently the distance between the bubble and the transducer did not remain constant. By assuming that the pressure varied inversely with the distance, the individual pulse amplitudes were calculated at a point 100 mm distant from the centre of the bubbles. The assumption that the pressure varied inversely with distance is reasonable, because most of the growth and collapse occurred in the low velocity region.

Straight lines were fitted to the data in figures 33 and 34 by the method of least squares. The pressure pulse amplitude varied approximately as the $3/2$ power of the maximum radius and as the $3/4$ power of the bubble lifetime. This would appear to indicate that for a single bubble, the maximum radius is proportional to the square root of bubble lifetime. This may be compared to Zuber's (34) prediction for the maximum radius $R_o$:

$$R_o = N_Ja \sqrt{at/s}$$

$N_J$ is the Jakob Number, $a$ the thermal diffusivity of the liquid and $t$ the time required to reach the radius $R_o$. For the bubbles observed in
this study, the bubble lifetime was always greater than the time, $t$, required to reach the maximum radius $R_o$. Zuber's expression may therefore be expected to give correctly an order of magnitude estimate of the maximum radii of bubbles with the total lifetime greater than the time to reach the maximum radius.

3.6. **Energy content of the noise generated by single bubbles.**

A quantity of interest in the study of single bubble noise is the energy contained in the pressure wave generated by the bubble. The magnitude of this energy may be estimated as follows, assuming that the liquid is incompressible. The energy content of the pressure wave radiated by a bubble is given by the relation

$$ E_s = 4\pi r^2 \int_0^T p u \ dt $$

where $E_s$ is the energy in the sound wave. $p$ is the sound pressure measured at a point distant $r$ from the centre of the bubble and $u$ is the liquid velocity at this point. $T$ is the duration of the pressure pulse. With the assumptions of simple acoustic theory, the relation between $p$ and $u$ is of the form (17),

$$ u = p/\rho C $$

where $\rho$ and $C$ are the density and sonic speed of the liquid respectively. Substitution of equation (13) in equation (12) gives

$$ E_s = (4\pi r^2 / \rho C) \int_0^T p^2 \ dt $$

The total energy in the bubble equals the potential energy at maximum radius when the wall velocity is zero. The potential energy is given by

$$ E_p = (4\pi R_o^3 / 3) p_o $$

where $R_o$ is the maximum radius of the bubble and $p_o$ the bulk liquid pressure.

The energy $E_s$ was evaluated by plotting a graph of $p^2$ against $t$,.
measuring the area under it and substituting in equation (15). The ratio \( \frac{E_s}{E_p} \) was calculated for four pressures 1.0, 3.0, 7.0 and 14.7 psia. Experimental maximum bubble radii were used in the evaluation of \( E_p \). The variation of \( \frac{E_s}{E_p} \) with liquid pressure is shown in fig.35. With increase of liquid pressure, the potential energy of the bubble increases but the energy in the radiated pressure wave decreases. From the numerical values shown in fig.35, it can be seen that only a very small fraction of the bubble energy is radiated as noise.

3.7. Noise generated due to bubble division.

That noise is generated by the division of a bubble into two or more bubbles has been noted during examination of high speed films of subcooled boiling obtained in this study. A pulse was emitted at the instant of bubble division and thereafter the pulse decayed sinusoidally. Plate 4 shows three consecutive frames of boiling at 3.0 psia with a subcooling of 15 deg. C. In frame no.621, are shown, the parent bubble, roughly ellipsoidal in shape, and the oscilloscope trace of the pressure pulse. In frame No.622, the parent bubble has split into two smaller bubbles and the associated pressure pulse can be clearly seen. This pulse is in the form of a decaying sinusoid. In frame No.623, the pulse has fallen to zero. The amplitude of the pressure pulse is of the order of 0.01 psi and its frequency, 45 kHz.

The shape of the pressure pulse indicates that the bubble generates noise by volume pulsations. The driving force for this volume pulsation is the difference in pressure due to the reduced surface tension on the larger parent bubble. An expression for this difference in pressure may be obtained as follows (11):

Let \( R \) be the radius of the larger bubble and \( a \), the radius of the two smaller bubbles, assumed equal in size. Let \( A \) be the ambient liquid pressure far from the bubbles.
The vapour pressure, $P$, near the boundary of the larger bubble is

$$P = A + \frac{2\sigma}{R}$$  \hspace{1cm} (16)$$

and that near the boundary of either of the two smaller bubbles, $P'$, is given by

$$P' = A + \frac{2\sigma}{a}$$  \hspace{1cm} (16')$$

where $\sigma$ is the surface tension of the liquid at the temperature corresponding to the pressure $A$. It is assumed that, during the bubble division, the temperature remains constant.

The pressure difference $P' - P$ is the driving force causing the volume pulsation. From equations (16) and (16'),

$$P' - P = \frac{2\sigma}{R}(\frac{R}{a} - 1)$$  \hspace{1cm} (17)$$

The relation between $a$ and $R$ is

$$2\left(\frac{4}{3\pi a^3}\right) = \frac{4}{3\pi R^3}$$

Therefore

$$\frac{R}{a} = \frac{2}{3}$$

and the pressure difference $(P' - P)$ equals $\frac{2\sigma}{R}(\frac{1}{3} - 1)$.

Therefore the sound pressure at a distance $r$ is given by

$$p_s = (P' - P)\frac{R}{r}$$

which on substitution for $(P' - P)$ becomes

$$p_s = 0.52\frac{\sigma}{r}$$  \hspace{1cm} (18)$$

Comparison with experimental results.

The above theoretical predictions will be compared with the experimental results at 3.0 psia, for the event shown in Plate 4.

The bubble in frame No. 621 of Plate 4 is found to have an equivalent spherical bubble radius of 0.004 ft. Its resonant frequency, given by equation (1), is

$$f = \left(\frac{3\gamma A}{\rho}\right)^{\frac{1}{2}} / 2\pi R_m$$

where $A$ has been written for the liquid pressure $p_\infty$.

Here $\gamma$, the ratio of the specific heats for the gas and vapour within
the bubble will be assumed to be the same as that for air, i.e. 1.4.
Substitution of these values in equation for \( f \) gives
\[
\frac{1}{2} f = 1.23 \text{ kHz}.
\]
This value is an order of magnitude smaller than the frequency measured by experiment, which is 45 kHz. The amplitude of the pressure pulse calculated from equation (18) works out to be 18.8 dynes per sq. cm.
The experimental value is 400 dynes per sq. cm. Again, the theoretical value is an order of magnitude smaller than the experimental result.
The explanation for this poor agreement lies in the frequency response characteristics of the transducer. At the instant of bubble division, the transducer is subjected to a pressure difference which, in its form, resembles an impulse, with a frequency in the region of 100 kHz. The natural frequency of the transducer is very much lower, in the region of 40 kHz, and therefore its response to an impulse would decay at its own natural frequency. There is close agreement between the measured frequency of 45 kHz and the resonant frequency of the transducer.
This was confirmed by measurements at 1.0 psia. At this pressure, the measured frequency of the pulse was roughly 44 kHz which is again very close to the resonant frequency of the transducer.

The main conclusions to be drawn from the investigation are:
1) The spectrum of noise generated by single bubbles rose as \( f^4 \), i.e., as 12 dB per octave at frequencies much lower than that equivalent to the reciprocal of the bubble lifetime.
2) At high frequencies, the spectra diminished at rates varying from 1 to 4 dB per octave.
3) At intermediate frequencies, the spectra remained generally irregular, but in some cases the spectra resembled white noise.
4) The amplitude of the pressure pulses from single bubbles
varies as the $^{3/2}$ power of the maximum radius and as the $^{3/4}$ power of the bubble lifetime.

5) The energy content of the radiated pressure wave, which is a negligible fraction of the maximum energy in the bubble, decreases with increase of ambient liquid pressure.

6) Bubbles emit a pressure pulse upon division into two or more smaller bubbles. The magnitude and frequency of the pressure pulse, obtained from theory, did not agree with experimental values due to the limitations imposed by the frequency response of the pressure transducer.
CHAPTER 4

SOUND EMISSION DURING NUCLEATE BOILING AND ITS VARIATION
WITH DISSOLVED OXYGEN CONTENT OF BULK LIQUID

4.1. Introduction and Survey of literature.

In systems where the occurrence of boiling or cavitation is undesirable, the growth and subsequent collapse of bubbles will have deleterious effects. For example, the rapid increase of voidage due to bubble growth in the coolant channel of a reactor, which if left undetected, may lead to blockage (14). On the other hand, if cavitation should occur, for example in the inlet passages of a pump, the resulting collapse of bubble may cause considerable damage to the vanes of the impeller in a short interval of time. Also, the performance of the pump, as measured by the head rise, will be much reduced. Consequently, the object of most of the studies reported in the literature was to measure the radiated noise when boiling or cavitation occurred. This was done with a view to establishing these characteristic features of the noise spectrum, which, if capable of unambiguous interpretation, could lead to the development of an acoustic method of detecting the onset of boiling or cavitation.

Osborne and Holland (18) have presented noise spectra of subcooled boiling on wires immersed in a river. Essentially free field conditions existed and the noise recorded could be attributed directly to the growth and collapse of bubbles. The water temperature remained constant at 5 deg.C. whereas the heat flux ranged from 390 to 1200 watts per sq. cm. Figure 36 shows the spectra for boiling on a .28 gauge nichrome wire for two values of the heat flux. With increase of heat flux, the frequency of maximum sound intensity shifted to a slightly higher value. For other wires, this effect was not apparent. McLeod (19) showed that an increase in heat flux produced an increase in the power level of the spectrum but the general shape remained unaltered.
Walton (14) also presented similar spectra for boiling of water in a fast reactor channel at several heat fluxes. Fig. 37 shows the data of Walton. An increase of heat flux from 8.0 to 8.7 watts per sq. cm. increased the sound pressure level by approximately 14 per cent.

Frequency spectra for noise generated in saturated boiling has been reported by Schwartz and Siler (16). Boiling occurred on a horizontal stainless steel tube immersed in a pool of water at atmospheric pressure. The spectra showed that most of the sound was radiated in the low frequency range, 25-1000 Hz; and negligible sound was radiated in the range 1-20 kHz. The nature of the sound was that of random noise, with no particular frequency predominating. This behaviour has also been noted in subcooled boiling (14,19). At high frequencies, the spectrum was sustained at approximately -30 dB per octave. This may be compared with the gradual slope of the subcooled boiling spectra, figs. 36 and 37, where, at high frequencies, the spectra diminish roughly at 6 dB per octave.

The intensity of the sound generated in boiling as a function of the heat transfer rate has been investigated by several authors (14,16,18,19,20). Schwartz and Siler (16) presented data on the sound intensity in decibels as a function of the temperature difference between the wall and the bulk liquid (AT) in saturated boiling of water at atmospheric pressure. This functional relationship was of the form

\[ \text{Intensity (dB)} = M + m \log_{10} AT \]  

where \( M \) and \( m \) are empirical constants. This relation was found to be similar to the equation connecting the heat flux to the temperature difference \( AT \) in view of the decibel scale being a logarithmic energy relationship. This may be predicted, as a first order approximation, by the following argument.

The average sound intensity, \( I \), due to the growth of single
single bubbles can be represented as

\[ I \propto \frac{1}{T} \int p^2 \, dt \] ......(2)

where \( p \) is the sound pressure and \( T \), the period of the pressure pulse.

Equation (2) states that the average sound intensity is the area enclosed by the curve \( p^2 \) versus \( t \), divided by the period.

Letting \( \int p^2 \, dt \) equal to \( W \), equation (2) may be written

\[ I \equiv \frac{W}{T} \] ......(3)

Assuming a fairly even distribution of bubbles and active sites, the total sound intensity \( I_N \) is given by

\[ I_N = \frac{fN}{T} \] ......(4)

where \( f \), \( N \) are the frequency of bubble emission and the number of active sites per unit area of heating surface respectively.

Since in general, the heat flux \( q/A = \Delta T^m \) (48), the total number of bubbles per unit time will bear a similar relation to the superheat.

Hence, the total sound intensity \( I_N \) may be written

\[ I_N \equiv \Delta T^m \] ......(5)

where \( m \) is a constant.

Writing equation (5) in the form

\[ \log_{10} I_N/I_0 = m' + n \log_{10} \Delta T \] ......(6)

where \( I_0 \) is the reference intensity, the equivalence of the equations (1) and (3) is established.

Ponter and Haigh (20) studied the sound of boiling water on stainless steel tubes at atmospheric and several sub-atmospheric pressures. The subcooling of the bulk liquid ranged from 20 deg. to 60 deg. C. At atmospheric pressure, the sound intensity increased in a regular manner with an increase in the heat flux. But at low pressures of 1.75 and 4.0 psia, a "sporadic boiling regime" was found to exist in which, at low heat fluxes, intermittent sounds of high intensity were emitted.
No unique relation between the heat flux and sound intensity was observed in this region.

The bulk liquid temperature is another variable which may affect the sound emitted in boiling. In subcooled boiling, the increase in the lifetime of the bubbles due to an increase in liquid temperature may be expected to decrease the frequency at which the sounds are emitted. Osborne (15) presented a set of oscillograms of the sound emitted in subcooled boiling of water at atmospheric pressure. With an increase in the water temperature from 25 deg. C to 80 deg. C, the frequency of the sound decreased from 7 kHz to 1.5 kHz.

There have been numerous investigations dealing with the noise generated in cavitation. Mellen (12) measured the noise produced in water by rotating a metal rod, 2 in. long and 1/16 in. diameter, at 4300 rpm about a transverse axis through its centre. The pressure was atmospheric. The spectrum rose to a peak in the range 1 to 5 kHz and thereafter decayed approximately as the reciprocal of the square of the frequency. Esche (21) studied cavitation noise generated by sound fields at frequencies from 100 Hz up to 3 MHz. The noise spectra were found to be continuous but superimposed with discrete lines of high amplitude at frequencies which were harmonics and sub-harmonics of the fundamental driving frequency. Esche attributed the occurrence of these sub-harmonics to the fluctuations in the volume of the air bubbles taking part in the oscillation.

The effect of dissolved air content on bubble dynamics and on the sounds emitted has been recognised for a long time. This is reflected in the standard procedure of measuring the dissolved air content of the liquid in cavitation studies; in boiling studies the bulk liquid is generally degassed for a sufficient length of time before measurements are taken. Williams and McNulty (22) found that
for water flowing through a venturi, a decrease in air content lowered the inception pressure for cavitation. Bohn (23) investigated the effect of air content on the noise generated by acoustic cavitation. In his experiments, water at atmospheric pressure was subjected to sound fields of varying magnitude at a constant frequency of 14.6 kHz. The air content was varied from saturation value down to 76 per cent of saturation. Increasing the sound pressure amplitude from 1 to 4 atm., at a fixed air content, resulted in a shift of the noise spectra to higher frequencies, into the range 400-600 kHz. At low air contents (70-80 per cent), the noise spectra revealed a preferential frequency range of bubble oscillation. This behaviour was absent in air-saturated water where the bubbles formed were in resonance with the exciting frequency of 14.6 kHz.

In boiling studies, the term 'degassed' has been applied to describe liquids of minimum air content. Osborne (15) noted that the intensity of sounds emitted from a degassed system was approximately 10-15 dB higher than for boiling of subcooled liquid containing air. In the latter case, the diminished sound intensity was attributed to the "cushioning" of the shock of collapse by the air present in the bubble. Westwater (214) noted that for boiling methanol, the sound of nucleate pool boiling in the degassed liquid was approximately 10 dB higher than that in poorly degassed liquid. Walton (114) recognised that dissolved gas may have an important effect on the sound generated in boiling in that the frequency spectra will show discrete lines corresponding to harmonics of the resonant frequency of the bubbles, but no experiments were reported. In boiling studies there appear to be few experimental investigations directed at noise levels in which dissolved gas content is a variable.

To investigate the effect of dissolved gas content on the sound
generated in boiling, the study reported here was undertaken. The sound generated in nucleate boiling of water at atmospheric pressure has been determined for several values of dissolved oxygen content, ranging from 6.2 to 28 parts per million. The frequency spectrum of the sound for each value of dissolved oxygen content is reported.

4.2. Measurement of dissolved oxygen content of the bulk liquid.

There are several methods for the determination of the dissolved oxygen content of a sample of liquid. The choice of a suitable method depends on many factors such as the quantity of sample available for measurement, the time of measurement and the accuracy desired. The commonly used methods are:

a. The Winkler method
b. Direct reading method and
c. The Polarographic method.

The Winkler method (25) is based on the oxidation of manganous hydroxide to manganic hydroxide in the presence of an alkali, such as potassium hydroxide, by the oxygen dissolved in the sample of water. To the mixture is added potassium iodide and concentrated sulphuric acid resulting in the liberation of iodine. This iodine is titrated against sodium thiosulphate whose normality is known. The end point can be estimated, with reasonable accuracy, using starch as an indicator. As the amount of iodine liberated is proportional to the dissolved oxygen, the amount of thiosulphate used in the titration is a measure of the oxygen content of the water. The dissolved air content, when required, may be obtained from tables giving the proportions of the other atmospheric gases. The method gives results with good accuracy. There are three disadvantages.

Firstly, contamination of the sample by outside air (secondary air) may occur during measurement; secondly to obtain a reading, a relatively
longer interval of time is required compared to other methods and finally a large quantity of sample, of the order of 250 ml, is required for a single measurement.

The Van Slyke gas analyser (25) is an example of the direct reading method. In this apparatus, a measured volume of water is subjected to a Torricellian vacuum and the water is agitated by shaking the sample container. Another method of agitation employed is to spray the sample several times into the vacuum. This enables the dissolved gases to escape. When sufficient time has elapsed to allow the dissolved gases to escape into the vacuum, they are constrained to occupy a known volume by adjusting the level of the mercury reservoir. The partial pressure of the gases is then read on a manometer and their volume at N.T.P. calculated by applying the usual gas laws. The method takes less time, of the order of 10 minutes, than the Winkler method. The main disadvantage of the method is that, to obtain accurate results, the rate of agitation and level of the sample in the container are of critical importance.

The polarographic method consists in applying a continuously increasing negative potential between a cathode immersed in the sample and an anode, and measuring the resulting current by means of a sensitive galvanometer. In the dropping mercury electrode method, the cathode is formed by drops of mercury issuing from a capillary tube into a quiescent pool of mercury which forms the anode. The current increases gradually at first, then rises rapidly to a maximum and thereafter increases only gradually. There is a step of finite size in the current-voltage curve. This step, the wave height, is directly proportional to the concentration of reducible ions in the sample solution. The formation of the wave may be explained as follows (26). An ion is not reduced until the potential between the
drop and the solution reaches a certain value, at which the ions lose their charge to the cathode surface and current flows. The rate of reduction and hence the flow of current, increases rapidly with further small increase of voltage until the concentration of ions in the neighbourhood of the drop becomes negligible compared to that in the solution. When this happens, the current is limited by the rate at which the reducible ions can diffuse across this depleted layer to the drop surface. In the case of oxygen, the reduction occurs in two steps. First, oxygen is converted to hydrogen peroxide requiring a half wave potential of about -0.1 volt. The hydrogen peroxide is then reduced to water at a half wave potential of -0.9 volt.

In the solid electrode method, the anode is usually a noble metal such as platinum or silver. The cathode is usually made of carbon and the required polarising voltage is applied to the electrode from the instrument. The advantage of the solid electrode is that it can be immersed directly in the test liquid and no samples need be withdrawn for measurement. This eliminates the risk of secondary air contamination.

In the present study, a commercially available dissolved gas analyser made by Protech Services Ltd. and a solid electrode was used for the measurement of the dissolved oxygen content. A sketch of the electrode, showing the relevant details, is shown in fig. 40.

The electrode consists of a silver anode and a carbon cathode, connected electrically by an electrolyte which has potassium chloride as a base solution. The electrode is isolated from the sample by a thin ptfe membrane, which is permeable to oxygen and other gases. The polarising voltage of 1.1 volts for oxygen is supplied to the electrode from the instrument, which voltage is maintained when the electrode is in use. The electrode relies on diffusion for its operation. Consequently a flow velocity of approximately 4 feet per minute is to be
maintained when in use. Under operating conditions, the temperature changes are compensated to within ± 1 degree by thermistors built into the instrument. The instrument is calibrated directly in parts per million, with a maximum range of 0-30 parts per million.

Before using the analyser, it was calibrated first in sodium sulphite solution and then in air according to the procedure recommended by the manufacturer. With air calibration, an accuracy of ± 3.5 per cent could be obtained at the lowest value of oxygen content recorded in this investigation. At higher values of the oxygen content, the error decreased to less than 1 per cent.

4.3. Description of Apparatus

a. Experimental set up

A schematic diagram of the experimental set up is shown in figure 38. The main components are

1) The boiling vessel and test section assembly.
2) Measuring cell and pump for recirculation.
3) Oxygen supply.

The boiling vessel and test section as described in Chapter 2, was used in the present experiments. Provision was made in the top plate of the vessel for the connection of an oxygen supply line. The measuring cell, made of Perspex, measured 4 inches in diameter and 3 inches in height. The sample of water, whose dissolved oxygen content was to be measured, was drawn into the cell by gravity through Perspex connections. Two other Perspex connections, one at the top and the other near the base of the cell, allowed the sample to be recirculated into the cell. The oxygen measuring electrode was inserted into the measuring cell through the top cover plate. All connections were made to close tolerances to prevent any atmospheric air from entering the cell. The recirculating pump was of the peristaltic type. Nitrile rubber tubing was used for
connections thus reducing the risk of contamination.

The dissolved oxygen content of the water was varied by bubbling oxygen through it for specified intervals of time. By this method, the oxygen content could be increased to 30 parts per million (ppm) in about two hours. A trap included in the circuit prevented any water from entering the oxygen cylinder.

b. Recording system

The recording system, shown schematically in fig.39, consisted of a pressure transducer, charge amplifier and a Ferrograph tape recorder. The transducer and amplifier have been described in Chapter 2. The frequency response of the tape recorder up to 20 kHz was adequate for the highest frequencies likely to be encountered in this study. The signal to noise ratio was better than 55 dB at maximum gain.

4.4. Experimental Procedure

Prior to the commencement of a test run, the boiling vessel and test surface were cleaned with hot water and detergent and rinsed in deionised water. The test surface was also cleaned with acetone to remove traces of foreign matter. After assembly, the boiling vessel was charged with distilled deionised water.

The oxygen meter was calibrated in accordance with makers' recommendations. The bulk liquid was brought up to the required temperature and its oxygen content measured and recorded. The oxygen supply line was then opened and the gas bubbled through the water until the reading of the oxygen meter reached the nominated value. The supply was then cut off. The voltage across the test surface was increased until boiling commenced. The heat flux was then adjusted to the test value. The gain and treble controls on the tape recorder were adjusted so that the recording level indicator was at normal. When steady conditions were reached, the run was commenced and the following data
taken:

a. Thermocouple emfs of bulk liquid and tube thermocouples.

b. Voltage and current across the test surface.

c. The setting of the amplifier and the tape speed.

d. The tape recorder was switched on and a recording made for
   at least ten seconds, after which the recorder was switched off.

e. The initial and final readings of the counter.

At the end of the test run, the reading of the oxygen meter was
again taken. In all the tests reported here, the readings at the
beginning and the end of the run remained essentially the same. It is
therefore highly unlikely that any secondary air contamination occurred
during the test run.

4.5. Analysis of the Tape Recordings.

To determine the frequency spectra of the boiling sounds, the
tapes were analysed in the Noise Analysis Laboratory of the Dounreay
Experimental Research Establishment. A schematic diagram of the
recording and analysing equipment is shown in fig. 39. The frequency
spectra in the range 20 Hz to 20 kHz were determined as follows. A
length of tape containing the recording for one set of experimental
conditions was spliced to form a loop. This loop was run on the
tape recorder which was connected to a Bruel and Kjaer frequency
analyser. The analyser was coupled to a logarithmic recorder by a
flexible drive from the recorder motor. This ensured coincidence
between the analyser frequency and the recorder chart calibration.
The analyser divides the range 20 Hz to 20 kHz into six subranges.
Starting with the lowest range, 20 to 63 Hz, each range is scanned
automatically in succession. The tape loop was therefore run six
times. The output of the frequency analyser was connected to the
logarithmic recorder and a plot of sound pressure level in decibels
against frequency was obtained on pre-calibrated paper. In this manner, all the tape recordings were analysed and the frequency spectra obtained.

4.6. Results and Discussion.

In figs. 41 to 50 are shown the frequency spectra for the range of parameters studied in this investigation. The dissolved oxygen content ranged from 6.2 to 28 parts per million by weight. All the data were obtained at atmospheric pressure. At each value of the oxygen content, the sound was recorded at two heat fluxes, 76000 and 116000 Btu/hr. per sq. ft.; at each heat flux, data were recorded for bulk liquid subcoolings of 5, 25 and 35 deg.C.

Frequency spectra in relation to heat flux.

Figure 41 shows the spectra of sound at two heat flux values, 76000 and 116000 Btu/hr. per sq. ft. The dissolved oxygen content is 28 parts per million and the subcoolings 5 deg.C. It is seen that increasing the heat flux increases the level of the sound generated. This is in accordance with previous observations (15,19,20).

Frequency spectra in relation to liquid temperature.

Figs. 42, 43 and 44 show the effect of liquid temperature on sound spectra for two values of the dissolved oxygen content. It is seen from the figures that decreasing the liquid temperature (increasing the liquid subcooling) results in a shift of the spectra to higher frequencies. This is again in accordance with previous observations (15,19).

Frequency spectra in relation to dissolved gas content.

Figs. 45 to 50 show the effect of dissolved gas content on the frequency spectra. In the frequency range 1 to 4 kHz, the spectra decay at the rate of roughly 6 dB per octave. But it may be seen that the spectra cut off at a frequency which is usually in the range 10-14 kHz for the experiments reported here. The rate of decay after
The cut-off is very sharp, varying from 33 to 90 dB per octave. It may be seen from the figures that there is no definite correlation between the rate of decay of the spectra and the dissolved oxygen content. Also, the cut-off frequency is not strongly dependent on the oxygen content. These two features are shown separately in fig. 50A. The sharp decay in the noise spectra obtained in this study appears not to have been noted previously in the literature.

One possible explanation for the cut-off is that the transducer is functioning as a filter, i.e. it cuts out all sound with frequencies greater than the cut-off frequency. However, this is highly unlikely in view of the fact that the resonant frequency of the transducer is in the region of 40 kHz, which is more than twice the highest cut-off frequency observed in the present experiments. It seems therefore reasonable to assume that the cut-off of the spectra is a result of the presence of gas within the bubbles. The spectral density is a measure of the energy available in the sound generated. Therefore, it appears that in sound generated by bubbles containing gas, there is little or no energy beyond a frequency of roughly 12-14 kHz. The mechanism whereby this depletion in the energy of the sound is brought about cannot be determined from the experiments reported here.

4.7 Conclusions

From the study reported here the following conclusions may be drawn:

1. An increase in the heat flux increases the level of the spectra of sound in nucleate boiling.

2. A decrease in the bulk liquid temperature results in a shift of the sound spectra to higher frequencies.

3. In the present experiments the presence of dissolved gases in a boiling liquid results in a cut-off of the spectra of sound at frequencies in the range 10-14 kHz. This cut-off frequency was
found to be only weakly dependent on the dissolved gas content. No definite correlation was found between the rate of decay after cut-off and the oxygen content.
CHAPTER 5

THE GROWTH RATE OF VAPOUR BUBBLES FROM A CYLINDRICAL SURFACE IN NUCLEATE BOILING

5.1. Introduction.

There are now available in the literature several theories dealing with bubble growth in a non-uniform temperature field. In the development of these theories, a common feature is the assumption of a linearized temperature profile in the thermal layer next to the heated surface. This assumption has been shown, by experimental measurements, to be very nearly true for boiling on a flat surface at atmospheric pressure where the thermal layer is uniform and of small thickness, of the order of a few thousandths of an inch. However, when the heated surface is a cylindrical tube, as in the present study, the thermal layer is no longer uniform but increases in thickness towards the top of the tube. It therefore seemed desirable to calculate bubble growth rates from such a surface without making assumptions regarding the thickness of the thermal layer. Further, no analytical expressions are used to describe the form of the temperature field in the thermal layer at the instant of bubble growth, but is calculated from the relevant equations.

In the study reported here, bubble growth rates for saturated boiling of water on a cylindrical tube for atmospheric and several sub-atmospheric pressures have been presented. A numerical solution of the governing equations of bubble growth from a cylindrical surface has been obtained. The experimentally determined bubble growth rates are compared with the various theories available in the literature. Before dealing with these, a brief survey of the literature is presented. To provide a ready understanding of the differences in approach, the various theories to be discussed are compared in Table I.
5.2. Literature Survey

Bubble growth in liquids was first analysed for the uniform temperature field. In the analysis of Plesset and Zwick (27), the physical model consists of a spherical vapour bubble growing in a liquid which has negligible viscosity and compressibility. Equilibrium is assumed to exist at the vapour-liquid interface so that the temperature of the vapour is that of the liquid at the interface. The pressure of the vapour within the bubble equals the saturation pressure corresponding to the liquid temperature at the interface.

With the above assumptions, the momentum equation for the liquid was obtained:

\[ R \frac{d^2 R}{dt^2} + \frac{3}{2} \frac{R^2}{R} = (P - P_\infty)/\rho \]  

Here \( P \) is the pressure in the liquid at the bubble boundary and is related to \( P_v \), the pressure of the vapour within the bubble by

\[ P = P_v - 2\sigma/R \]  

where \( \sigma \) is the surface tension of the liquid and \( R \), the bubble radius at time \( t \). The liquid was, initially, at a uniform temperature \( T_0 \) greater than the saturation temperature \( T_{sat} \) at the ambient liquid pressure \( P_\infty \). The liquid superheat was therefore \( (T_0 - T_{sat}) \). The bubble was assumed to grow from an embryonic nucleus of radius \( R_i \). Initially, therefore, one has the relation

\[ P_v = P_\infty + 2\sigma/R_i \]  

where \( P_v \) is the pressure of the vapour within the nucleus.

The vapour pressure, \( P_v \), was related to the vapour temperature (which equals the liquid temperature \( T \) at the bubble wall) by a linear function of temperature:

\[ (P_v - P_\infty) = \rho A(T - T_{sat}) \]  

where \( A \) is a constant. The temperature \( T \) changes during bubble growth and its value is determined by the heat transfer due to conduction.
from the surrounding liquid. Plesset and Zwick hypothesized that the
temperature drop, \( T_0 - T \), for heat transfer occurs in a thin liquid
layer, the thermal layer, surrounding the bubble. Using this hypothesis,
the temperature \( T \) was evaluated by a solution to the transient heat
conduction equation in the liquid using a perturbation procedure. This
solution for \( T \) and equations (2), (3) and (4) were substituted in the
momentum equation to yield an expression relating the inertia and surface
tension forces to the temperature difference \((T - T_0)\). Except in the
early stages of growth, the inertia and surface tension forces were
found to be unimportant. Consequently, these two forces were neglected
and a solution obtained for the bubble radius during the later, asymptotic
stage of growth. The expression thus obtained for \( R \) is

\[
R = \frac{2\sqrt{3} k(T_0 - T_{sat})}{h_{fg} \rho_v \sqrt{\alpha \sigma}} \quad \cdots \cdots (5)
\]

Here \( k \) and \( \alpha \) are the thermal conductivity and diffusivity of the liquid
respectively. \( \rho_v \) is the vapour density and \( h_{fg} \), the latent heat of
vapourisation. Equation (5), when compared with experimental data (40A)
gave excellent agreement.

Forster and Zuber (28), in an analysis of bubble growth in uniformly
superheated liquids, also started with the momentum equation and related
the vapour pressure to the temperature \( T \) at the bubble boundary by a
linear function. The temperature \( T \) was then evaluated by considering
the liquid near the bubble wall as a spherically distributed heat sink
and using the Green's function to solve the heat conduction equation.
Using the solution thus obtained, an expression relating the inertia
and surface tension forces to the temperature difference \((T - T_0)\) was
obtained. Again, the inertia and surface tension forces were found to
be important only during the initial growth of the bubble. Neglecting
these two forces, an equation for \( R \), applicable during the asymptotic
phase of bubble growth was obtained:

\[ R = \left( \frac{\pi}{\alpha} \right) \frac{k(T - T_{\text{sat}})}{h_{\text{fg}} \rho_v} t^{\frac{3}{2}} \quad \ldots \quad (6) \]

Equations (5) and (6) are identical except for the constants, \(2(\frac{3}{\pi})^{\frac{3}{2}}\) and \(\frac{1}{3}\), which are nearly equal. Equation (6) was found to give excellent agreement with experimental data.

Scriven (29) assumed, on dimensional grounds, that the bubble radius is given by the relation

\[ R = 2\beta \sqrt{at} \]

and substituted this equation for \(R\) in the heat conduction equation for the liquid. The equation was solved by numerical methods and the constant \(\beta\), called the "growth constant", was evaluated by using the appropriate initial and boundary conditions. Scriven's results for \(R\) were found to be consistent with those of Plesset and Zwick and of Forster and Zuber.

The theories referred to above deal with bubble growth in a uniformly superheated liquid. When a bubble grows on a heated surface, it does so in a temperature field which is non-uniform, the temperature varying from the heater wall temperature to the bulk liquid temperature. In the following, analyses pertaining to bubble growth in non-uniform temperature fields will be discussed.

Griffith (30) presented a computer solution to the problem of bubble growth in a non-uniform temperature field. Surface tension and inertia effects, which are important only in the initial stages of growth, were neglected. For a hemispherical bubble growing on a heated surface, the transient heat conduction equation was combined with a velocity term obtained from the equation of continuity. A heat balance at the bubble wall between the heat transferred from the superheated liquid surrounding the bubble and the latent energy in the vapour formed
yielded an expression for the growth rate $\frac{dR}{dt}$, where $R$ is the bubble radius at time $t$:

$$2\pi R^2 (h_{fg} \rho_v \frac{dR}{dt}) = -k \int_{A} \left( \frac{3T}{3r} \right)_{r=R} \, dA$$

In equation (7), $\rho_v$, $h_{fg}$ represent the density and heat of vapourisation respectively of the vapour. The integrand on the right hand side is the temperature gradient in the liquid (of thermal conductivity $k$) at the bubble wall and $dA$ represent an elemental area of the bubble surface. The heat balance given by equation (7) is a fundamental equation in most of the analyses of bubble growth reported to date, from which, by integration, the bubble radius is obtained as a function of time.

In Griffith's analysis, the temperature distribution in the thin superheated layer (of thickness $b$) was assumed to be linear. Using the thickness of the superheated layer, $b$, as the only length parameter, the transient heat conduction equation and equation (1) were made dimensionless and solved simultaneously by finite difference methods for several values of the Jakob number, ranging from 0.35 to 10.6. The solution yielded bubble radius as a function of time. Important conclusions of this analysis were:–

a) For decreasing values of the Jakob number and increasing pressure, the growth rate decreases.

b) At small values of the Jakob number, the maximum size attained by bubbles is independent of the Jakob number but is proportional to the thickness of the superheated layer.

c) The average growth velocity of bubbles with a small maximum size is greater than those with a larger maximum size, for the same value of the Jakob number.

A limitation of Griffith's analysis is the difficulty of predicting accurately the thickness of the superheated layer. Further, the collapse
region, where a high degree of turbulence exists, was not studied on account of the heat conduction equation, as used by Griffith, being applicable only to a laminar flow field.

Bankoff and Mikesell (31), in an analysis of boiling from a heated surface, considered the case of a bubble growing in a liquid layer which has initially either a linear or an exponential temperature profile. Again, the inertial and surface tension effects were neglected and only the asymptotic phase of bubble growth where heat transfer effects are controlling was considered. The model used is one of a bubble forming entirely within the thermal layer and which in growing lifts and stretches the thermal layer so that the temperature distribution is radially symmetric.

The linear and exponential profiles were defined in terms of an undetermined constant $M$. With these profiles as initial conditions in turn, Bankoff and Mikesell solved, simultaneously, the one dimensional transient heat conduction equation and the equation of energy balance across the vapour-liquid interface, equation (7). The bubble radii thus obtained were then compared with experimental data for saturated and subcooled nucleate boiling. The value of $M$ was determined by fitting the expression for bubble radius to one experimental point. In saturated boiling, the agreement between theory and experiment was good, but in subcooled boiling the deviation between theory and experiment was large. The authors attributed this deviation to inaccuracies in the definition of the temperature profiles and to the high degree of turbulence which exists during bubble collapse.

Skinner and Bankoff (32), in an analysis which was similar in many respects to that of Bankoff and Mikesell, used an exponential profile for the initial temperature in the superheated layer surrounding the bubble. This was of the form
\( T(r,0) = T_\infty + (T_\text{W} - T_\infty) \exp(-r^6 / \delta^6) \quad \ldots (8) \)

In equation (8), \( T(r,t) \) is the temperature in the liquid at a distance \( r \) at time \( t \); \( T_\text{W} \) and \( T_\infty \) are the surface and bulk liquid temperatures respectively. \( \delta \) is the thickness of the superheated layer. With this initial condition, the one-dimensional heat conduction equation and the energy balance equation were solved to obtain the radius as a function of time. However, no comparison with experimental results was reported.

Savic and Gosnell (33) made a theoretical study of the growth of a bubble in a non-uniform temperature field by means of a perturbation technique. The bubble was assumed to be hemispherical in shape. The transient heat conduction equation was solved by the method of separation of variables with the assumption that the product of the bubble radius, \( R \), and the growth velocity, \( \dot{R} \), were constant. Both this product \( R \dot{R} \) and the temperature \( T \) in the liquid were written in the form of power series:

\[
R \dot{R} = A + A'R + A''R^2 \quad \ldots (9)
\]

and

\[
T = d(s,\phi) + A'R f(s,\phi) + A''R^2 k(s,\phi) \quad \ldots (10)
\]

where \( A, A', A'' \) are constants and \( d, f, k \) are functions of the non-dimensional radial coordinate \( s \) and the polar angle \( \phi \), only. Substitution of equations (9) and (10) into the heat conduction equation yielded, after some calculation, expressions for the constants \( A, A', A'' \) and the functions \( d, f, k \). Substitution of these values into the expression for \( T \) yielded the temperature profile in the liquid surrounding the bubble. The temperature profiles were reported for several values of the polar angle \( \phi \) and a non-dimensional bubble radius. To make the bubble radius \( R \) dimensionless, it was divided by the thermal layer thickness, \( b \), a device also used by Griffith. For a value of 0.5 for this non-dimensional bubble radius and polar angles of zero and 30 deg.
the slope of the temperature profile at the bubble surface was negative. 
This indicated that when the bubble radius was half the superheated layer thickness, condensation occurs at the top of the bubble even though evaporation may have been taking place at the foot of the bubble.

The bubble radius $R$ may be determined by substituting the values of $A, A', A''$ in the expression for $R^2$ and integrating the resulting equation. This gives, for the bubble radius $R$, the equation

$$R = \frac{1}{2A''}(\frac{1}{2} \tan \frac{1}{2} t - A')$$

where $N = (4AA'' - A'^2)^\frac{3}{2}$. No comparison with experimental results was reported. Presumably this was because for the Jakob numbers studied by Savic and Cosnell, the discriminant, $N$, in the equation for $R$ turns out to be negative.

Zuber (34) modified the expression for bubble growth given by Forster and Zuber (28) to include the effect of a non-uniform temperature field. Zuber's model is shown in fig.51. In (a), the microscopic nucleus and the initial temperature distribution in the superheated layer of thickness $\delta_h$ are shown. In (b), the nucleus has grown and the temperature in the liquid at the bubble surface falls to a value equal to the temperature of the vapour in the bubble. The temperature increases up to a short distance $\delta$ of the bubble surface and thereafter decays to the temperature of the bulk liquid. The bubble grows by evaporation of the liquid within the region of thickness $\delta$. Beyond this region $\delta$, energy flows to the liquid.

The equation for the bubble growth rate, $dR/dt$, developed by Zuber is of the form

$$h_f g \rho_v \frac{dR}{dt} = \frac{\pi}{2}(k(T_w - T_o)/\sqrt{\text{sat}} - q'')$$

The first term on the right hand side of equation (11) represents the energy transferred from the liquid to the bubble across the bubble wall; $q''$ is the energy flowing towards the bulk liquid. On the basis that the presence of the bubble does not appreciably alter the temperature
distribution beyond a distance $\delta$, Zuber equated $q''$ to $q_w$, the heat flux at the solid surface. The expression for the bubble growth rate is then

$$h_f g \rho_v \frac{dR}{dt} = \frac{\pi}{2} \left[ k(T_w - T_{sat}) \right]^{\frac{1}{2}} - q_w \right] \quad \cdots (12)$$

By integration of equation (12), an expression for the bubble radius $R$ was obtained, which when compared with experimental values gave good agreement during the growth phase but not during collapse. The theory was found to predict a much slower collapse than actually occurs. This disagreement between theory and experiment was attributed to the high degree of turbulence in collapse.

Han and Graham (35) modified Zuber's analysis in two important respects. Firstly, the heat transferred to the bubble through its base, which was neglected by Zuber, was accounted for simply by the inclusion of a term which is a product of the base area of the bubble, $A_{ba}$, and the heat flux at the wall, $q_w$. Secondly, the term $q''$ in equation (11) was evaluated by means of a solution to the transient heat conduction equation for a "slab". The initial temperature distribution in the thermal layer, assumed linear, was described by the relation

$$T(x,0) = T_w - q_w \frac{x}{k} \quad \cdots (13)$$

where $T(x,t)$ is the temperature in the liquid at distance $x$ at time $t$. The model for bubble growth is that, when a bubble forms within the thermal layer, the latter is lifted and stretched, and the temperature drops to the vapour temperature. This is stated by the condition

$$T(\delta,t) = T_{sat}$$

where $\delta$ is the thickness of the thermal layer and $T_{sat}$ represents the saturation temperature of the vapour. The thermal layer, bounded on one side by the bubble, and on the other by the liquid at bulk temperature, was treated as a 'slab' whose surface temperature is suddenly reduced to a lower, constant vapour temperature. With the curvature of the bubble
surface neglected, the one-dimensional heat conduction equation was solved to obtain the temperature gradient at the bubble surface. The heat transferred to the bubble from the thermal layer, $q_s$, was then calculated using the Fourier conduction law,

$$q_s = -k\frac{\partial T}{\partial x} \| z = \delta$$

The bubble growth rate was then determined by a heat balance involving the heat flux through the base of the bubble, the heat transmitted through the spherical surface of the bubble and the heat of vapourisation in the vapour formed. This resulted in the expression,

$$4\pi R^2 h_{lg} \rho_v \frac{dR}{dt} = A_{ba} q_{ba} + C A_q q_s ....(14)$$

where $A_q$ is the surface area of the bubble and $C$ is a correction factor to account for the kind of approximation used to calculate the value of $q_s$. Thus for a flat film approximation, the value of $C$ is 1 whereas for a spherical film, $C$ is $\pi/2$.

Hsu and Graham evaluated the ratio $A_{ba}/A_q$ from the experiments of Bashforth and Adams (35). To simplify equation (14), a representative value of 0.25 was used for the ratio $A_{ba}/A_q$, applicable to bubbles with radii larger than 1 mm. The resulting equation for growth rate $dR/dt$ was integrated to yield the bubble radius as a function of time.

The bubble radii were then compared with experimental data, presented by Hsu and Graham, for water boiling at atmospheric pressure with a subcooling of 18 deg.F. and found to give good agreement. As in Griffith's analysis, a limitation of the analysis of Hsu and Graham is the dependence of the bubble radius on the thermal layer thickness $\delta$, accurate values of which are difficult to predict.

Han and Griffith (36) evaluated the terms in equation (14) differently from Hsu and Graham. The heat transferred through the base of the bubble, $q_{ba}$, was evaluated by considering the heat transfer coefficient, $h_v$, between the heating surface and the steam bubble through
its base area. The base and surface areas, \( A_{ba}, A_s \), were written as fractions of the surface area of a spherical bubble, \( 4\pi R^2 \), by the introduction of base and surface factors, \( \phi_b, \phi_s \), respectively. The curvature factor \( C \) was retained. The resulting equation was of the form

\[
\dot{\phi}_v (4\pi R^2) \frac{\rho_v}{\rho} \frac{dR}{dt} = \phi_b (4\pi R^2) \left( T - T_{sat} \right) + C \phi_s (4\pi R^2) \delta \cdots (15)
\]

where \( \phi_v \) is a volume factor and \( R \), the bubble radius. The three factors \( \phi_v, \phi_s, \phi_b \) were evaluated in terms of the contact angle \( \delta \).

Bubble radii computed by the integration of equation (15) agreed within a few percent of experimental data for water boiling at atmospheric pressure with a subcooling of 7 deg.F. Lack of data regarding the heat transfer coefficient, \( h_v \), is a limitation of Han and Griffith's analysis. Another limitation is the dependence of the bubble radii, computed from equation (15), on the thermal layer thickness.

In all the theories reviewed up to now, the bubble is assumed to grow by evaporation of the liquid at the curved surface of the bubble (the bubble wall). In the literature, there has been proposed another mechanism to account for bubble growth in a liquid. In this alternative mechanism, termed "micro-layer vapourisation", bubble growth occurs by the evaporation of a thin liquid layer assumed to exist between the bubble base and the heated wall. Some experiments have produced evidence in support of the existence of the micro-layer (37). Based on this mechanism of micro-layer vapourisation Cooper (38) presented an analysis of bubble growth for a hemispherical bubble growing on a heated wall with the bulk liquid being nearly at saturation temperature. The assumptions of Cooper's analysis are:

a) Rate of evaporation on the curved surface of the bubble is negligible compared to rate of evaporation from micro-layer. This assumption was based on the observations of Cooper and Lloyd (37), who noted
that for toluene bubbles, the rate of growth was nearly equal to the rate of micro-layer vapourisation, provided the bulk liquid was nearly at saturation temperature.

b) The initial superheat temperature difference \( (T_w - T_{sat}) \) is small compared to the fluctuations in the wall temperature during bubble growth. This assumption is satisfied when bubble growth occurs on a solid surface of high thermal conductivity.

c) The inertia forces in the liquid are large compared to the surface tension forces. As long as this assumption is satisfied, the bubble tends to remain hemispherical in shape. When the surface tension forces exceed the inertia forces, the bubble tends to assume a spherical shape. The equations of the analysis are applicable to hemispherical bubble growth only.

d) Inertia forces are large compared to gravitational and viscous forces. This assumption is common to most bubble growth theories and permits the neglect of gravity and viscosity of the liquid.

With the above assumptions, an equation was derived for the bubble radius \( R \) at time \( t \):

\[
R = 2.5 \left( \frac{N_{Ja}}{Pr_L^{\frac{1}{3}}} \right) (\alpha t)^{\frac{1}{2}}
\]

where \( N_{Ja} \) is the Jakob number, \( Pr_L \) the Prandtl number of the liquid and \( \alpha \) the thermal diffusivity of the liquid. Equation (16) when compared with experimental data gave good agreement during the early stages of growth. Thereafter, the experimental data showed a marked change to slower growth which equation (16) did not predict. This was probably due to breakdown of assumption (c). The surface tension forces would then tend to round off the bubble toward a spherical shape. Although Cooper's analysis fails beyond the point when assumption (c) breaks down, it can predict approximately when the breakdown occurs. The bubble radius at this point was found to be sensitive to system pressure.
For sub-atmospheric pressures, it was found to be several millimetres. At atmospheric pressure, it was only 1 mm, leading to the general conclusion that the contribution to bubble growth of micro-layer vapourisation decreases with increase of pressure.

The assumption that equilibrium exists between liquid and vapour at the bubble wall is common to all the bubble growth theories discussed so far. This assumption implies that the temperature of the vapour in the bubble equals the temperature of the liquid at the bubble wall. Further, the pressure of the vapour equals the saturation pressure corresponding to the liquid temperature at the bubble wall. Recently a few theories on bubble growth have appeared in which non-equilibrium effects have been examined. Bornhorst and Hatsopoulos (39) analysed the growth of a spherical vapour bubble in a uniformly superheated liquid taking into account not only surface tension and inertia forces but also the effects of non-equilibrium at the liquid-vapour interface. Starting with the momentum equation for radially symmetric flow of an incompressible liquid, an expression was derived for the excess, $\Delta P$, of pressure of the vapour in the bubble over that of the liquid at infinity (the extended Rayleigh equation). The temperature of the liquid at the interface, $T_{li}$, was determined by a solution of the heat conduction equation using an integral technique. An interesting feature in the solution was the use of a second order polynomial to represent the temperature distribution in the liquid near the bubble. Finally, an expression for the bubble wall velocity, $\dot{V}$, was derived, in terms of $\Delta P$ and the liquid temperature at the interface, using the methods of irreversible thermodynamics. It is here that the analysis of Bornhorst and Hatsopoulos differs from analyses presented earlier. (In the latter the Clausius-Clapeyron equation, in which equilibrium between the two phases - vapour and liquid - is assumed, was used to relate $\Delta P$ and $T_{li}$).
Substitution of the expressions for $\Delta P$ and $T_{fl}$ in the equation for $\bar{R}$ yielded an equation relating $\bar{R}$ to the bubble radius $R$. This equation was numerically solved and the results were presented in the form of graphs of $\bar{R}$ against $R$, for various liquids. It was shown that non-equilibrium effects could be important at low pressures.

Theofanous et al. (40) reported an analysis of bubble growth in a uniformly superheated liquid, in which results from kinetic theory were used to examine the non-equilibrium effects. The vapour density was allowed to vary with time, an additional refinement not considered in other theories.

In the analysis of Theofanous et al., the net mass flux, $W$, across the bubble wall was determined from kinetic theory:

$$W = c\left(\frac{M}{4\pi G}\right)^{\frac{1}{2}} \left[\frac{P_L}{T_L^{\frac{3}{2}}} - \frac{P_v}{T_v^{\frac{3}{2}}}\right]$$

where $c$ is the accommodation coefficient, $M$ the molecular weight, $G$ the gas constant, $P_L$ is the pressure of the liquid at the bubble wall and $T_L$ is the saturation temperature corresponding to the liquid pressure $P_L$. $P_v$ is the saturation pressure at the vapour temperature $T_v$. Since the bubble grows due to the net mass flux $W$, the continuity equation for the vapour is given by

$$\frac{d}{dt}(\rho_v R^3) = W$$

$\rho_v$ was replaced by the expression $P_v/G T_v$. Equation (18) then relates $P_v, T_v$ and the bubble radius $R$. Another equation relating the same variables was obtained by a vapour energy balance. This states that the rate of change of internal energy of vapour equals the energy content of the mass flux $W$ entering the bubble minus the rate at which work is done by the expanding bubble.

The solution of the heat conduction equation for the liquid, using a parabolic temperature profile in the thermal layer, yielded a differential
equation relating $R \frac{T_L - T_0}{T_L - T_a}$ and $\delta$ where $(T_L - T_a)$ and $\delta$ are the superheat in the liquid and thickness of the thermal layer respectively. A final boundary condition was imposed by equating the rate of heat conduction through the thermal layer to the rate of latent energy increase due to the net mass flux $W$:

$$2k(T_L - T_0)/\delta = W h_{fg}$$

The set of four equations thus obtained together with the momentum equation for the liquid were solved numerically to obtain the bubble radius $R$ as a function of time. Computed bubble growth and collapse rates were presented for sodium at 1 atm, and for water at 1.5 psia. The numerical solutions showed that for high superheat temperature differences, the assumption of a constant vapour density results in considerable error. Further, comparison of theoretical solutions with experimental data for water and liquid nitrogen showed good agreement, for values of $c$, the accommodation coefficient, in the region of 0.01. For the same experimental data, the theories which neglect non-equilibrium effects appear to deviate considerably from the experimental curves.

Turning to experimental investigations, bubble growth rates in uniformly superheated liquids were reported by Dergarabedian (40A), who heated distilled water in a clean Perspex beaker by means of two 250 W infra-red lamps. Superheat temperature differences of up to 5.3 deg C were obtained and the bubbles formed were photographed at speeds of 1000 frames per second. The experimental data agreed within a few percent of the bubble growth theory of Plesset and Zwick (27).

There have been numerous experimental investigations of bubble growth in non-uniform temperature fields. Ellicn (41) studied nucleate boiling of water and carbon tetrachloride at atmospheric pressure from a flat stainless steel strip, $\frac{1}{2}$ in x $\frac{3}{4}$ in x 0.0043 in thick. The
temperature of the water ranged from saturation temperature down to a subcooling of 150 deg. F. Bubble growth and collapse rates for water were presented for heat fluxes ranging from 31 percent to 63 percent of peak heat flux. Ellion's data showed that for water, both the maximum bubble radius and lifetime decreased with increase of heat flux. A detailed description is given in reference (41).

Staniszewski (42) reported bubble growth rates in pool boiling of distilled water and methyl alcohol at several pressures, from 1 atm. to 40 psia. The heat flux ranged from 15 to 80 percent of peak heat flux. When the growth curves are written in the form

\[ R = A t^m \] (20)

the exponent \( m \) varied from a value of 0.5 to 1 early in the growth period, with a subsequent decrease to a value of roughly \( \frac{1}{3} \). In addition, the growth rate was found to be independent of system pressure due allowance being made for statistical scatter in the data.

Experimental bubble growth rates at sub-atmospheric pressures were reported by Patten (43), who studied saturated nucleate pool boiling of distilled water on an electrically heated nichrome wire, 0.018 in diameter and 6 in long, for system pressures ranging from 1 atm. down to 0.681 psia. The maximum bubble diameter increased by an order of magnitude with decrease of pressure over the range studied.

Garg (44) also studied the nucleate pool boiling of distilled water at sub-atmospheric pressures, specifically, 1.0, 3.0 and 7.0 psia. The heating surface was a 0.125 in dia. stainless steel tube, 6 in long; the heat flux ranged from 7 to 20 percent of the critical value. Garg found that if the growth curves are represented by equation (20), the values of \( A \) and \( m \) both decrease with increase of system pressure, \( A \) decreasing from 1.12 to 0.02 and \( m \) from 0.75 to 0.325.

Recently Cole and Shulman (45) presented data of bubble growth rates
for nucleate pool boiling of water and several other liquids. The heating surface was a flat zirconium ribbon (3 in x 4 in x 0.01 in.), electrically-heated and supported between copper terminals in a Pyrex glass vessel. The system pressure ranged from 1.0 psia to 7 psia for water; the heat flux varied from 12000 to 21000 Btu per hr per sq. ft. and the Jakob number from 37.7 to 792. Comparison of the experimental data with the theories reviewed in the previous pages showed considerable disagreement, especially at high Jakob numbers. However it was found that the \( \frac{1}{3} \) relation best described the experimental growth curves. This led the authors to propose a correlation of the form

\[
D = 5 \, N_{Ja} \left( a t \right)^{\frac{1}{3}}
\]

where \( D \) is the bubble diameter, \( N_{Ja} \) the Jakob number, \( a \) the thermal diffusivity of the liquid and \( t \), the time.

An experimental investigation, specifically designed to test the several theories of bubble growth presented previously, has been reported by Schrock and Perdick (46) who studied photographically the nucleate boiling of water at pressures of atmospheric and above. The heating surface was a 2 in by 0.2 in platinum ribbon mounted vertically in a small chamber. By applying a heat flux in the form of a step input and an exponential distribution, the temperature profile in the liquid at the time of bubble nucleation was accurately determined. Change of system pressure, subcooling of bulk liquid and heating schedule permitted various values to be obtained of the superheated layer thickness. Comparison of experimental bubble growth rates with theory showed considerable disagreement. The maximum bubble radius was found to be dependent on the subcooling and the Jakob number, although the former was a weak dependence only.

The theoretical and experimental studies discussed so far appear to indicate that no one growth theory adequately predicts experimental
data at both low and high system pressures. The theories deal with spherical or hemi-spherical bubble growth from flat surfaces. An analytical expression is used to represent the initial temperature distribution in the liquid, prior to growth. Consequently some approximation is involved. Since the present study is concerned with boiling from a cylindrical tube at low pressures (high Jakob numbers) it was believed that an exact numerical solution of the governing equations for bubble growth would be desirable. This approach not only takes into account the variation in the thermal layer thickness round the circumference of the cylinder but also removes the approximation involved in using an analytical expression to describe the initial liquid temperature distribution.

5.3. Theoretical Analysis.

a. The bubble growth model and the derivation of the relevant equations.

The bubble growth model used in the present analysis is that postulated by Hsu (51). Consider a nucleus existing at the mouth of an active cavity. (Here the term cavity is taken to mean a pit or depression on the solid surface). The liquid surrounding the cavity is being heated by the wall. When the liquid temperature exceeds the bubble temperature, evaporation takes place into the bubble and growth begins. The subsequent growth of the bubble is determined by the rate of heat transfer at the bubble wall. The solution of the bubble growth problem therefore involves the determination first of the temperature distribution in the liquid at the instant growth begins and then, of the bubble radius as a function of time.

b. Temperature distribution in the liquid around a heated cylinder.

Following the departure of a bubble, a nucleus is left behind at the active cavity and relatively cool liquid at bulk temperature occupies the space left by the bubble, as indicated in Fig. 52(a). The cavity
radius is \( r_c \) and the nucleus radius and height are \( r_n \) and \( b \) respectively.

The temperature distribution corresponding to fig. 52(a) is shown in fig. 52(b). The departure of the bubble induces a velocity in the liquid. It seems reasonable to assume that this velocity equals the departure velocity of the bubble. During the heating of the liquid by the wall, convection would therefore be an important mode of heat transfer. The dotted line in fig. 52(c) shows the temperature distribution in the liquid after a short interval of time. In fig. 52(d), the temperature profile is such that at the height \( b \), the temperature in the liquid equals the temperature of the vapour in the nucleus. The nucleus begins to grow from this instant. The time taken to obtain this profile is the waiting time and the temperature distribution at the end of the waiting time is the initial temperature distribution for the calculation of the subsequent growth of the bubble.

c. Growth of vapour bubbles from a cylindrical surface.

At the end of the waiting period, the temperature in the liquid is everywhere equal to or greater than the bubble temperature and the growth begins. The surface tension and inertia effects are appreciable only during the early stages of growth and will be neglected. The major portion of the growth of the bubble will be determined by the rate at which energy is transferred across the interface. The temperature distribution in the liquid close to the bubble will therefore influence the growth. The assumptions made in the present analysis are:

1) The liquid is incompressible, infinite in extent and has constant thermodynamic and transport properties. Thermodynamic equilibrium exists between the liquid and vapour across the interface.

2) The bubble remains spherical at all times.

3) Surface tension and inertia effects are negligible.

With these assumptions, the temperature \( T \) in the liquid surrounding the
bubble satisfies the equation

\[ \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial r} = \alpha \left( \frac{\partial^2 T}{\partial r^2} + \frac{2}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\cot \theta}{r^2} \left( \frac{\partial T}{\partial \theta} \right) \right) \]  

\[ \ldots (22) \]

where \( r \) is the radial co-ordinate, \( \theta \) the polar angle and \( t \) the time. \( u \) is the radial velocity in the liquid induced by the bubble growth. If at time \( t \), the bubble radius is \( R \) and its growth velocity \( \dot{R} \), the velocity \( u \) is given by

\[ u = \frac{R^2}{r^2} \frac{\partial R}{\partial t} = \left( R^2 \frac{dR}{dt} / r^2 \right) \]  

\[ \ldots (23) \]

The bubble radius is determined by an energy balance across the interface. The area of a spherical segment of the bubble is given by \( R^2 \sin \theta \, d\theta \, d\phi \) and the heat transfer rate across the bubble wall is

\[ q = 2 \int_0^\pi \int_0^{2\pi} \frac{1}{k} \frac{\partial T}{\partial r} \sin \theta \, d\theta \, d\phi \]  

\[ \ldots (24) \]

where \( k \) is the thermal conductivity of the liquid. This rate of heat transfer equals the rate at which the latent energy of the vapour in the bubble is changing which is given by

\[ \frac{d}{dt} \left( \frac{4}{3} \pi R^3 \rho_v h_{fg} \right) \]

where \( \rho_v \) is the vapour density and \( h_{fg} \), the latent heat of vapourisation. Thus

\[ \frac{d}{dt} \left( \frac{4}{3} \pi R^3 \rho_v h_{fg} \right) = 2 \int_0^\pi \int_0^{2\pi} \frac{1}{k} \frac{\partial T}{\partial r} \sin \theta \, d\theta \, d\phi \]

which may be written

\[ \frac{dR}{dt} = \left( \frac{k}{2} \rho_v h_{fg} \right) \int_0^\pi \int_0^{2\pi} \sin \theta \, d\theta \, d\phi \]  

\[ \ldots (25) \]

The bubble radius \( R \) may be obtained by a simultaneous solution of equations (22) and (25), subject to the appropriate initial and boundary conditions which are given by:

**Initial conditions:**

\[ t = 0 : \quad T(R,\theta,0) = T_w \]

\[ T(r,\theta,0) = T_0 \]

\[ T(r,\theta,0) = f(r,\theta) \]  

\( R < r < \infty \)
where $T_w$ is the wall temperature and $T_\infty$ the bulk liquid temperature.

$f(r, \theta)$ is the temperature distribution in the liquid at the instant of bubble growth.

**Boundary conditions:**

$t > 0; \quad T(R, \theta, t) = T_{\text{sat}}$

$T(\infty, \theta, t) = T_\infty$

where $T_{\text{sat}}$ is the saturation temperature corresponding to the liquid pressure.

To facilitate the solution of equations (22) and (25), the following non-dimensional variables may be defined.

$$U = \frac{T - T_\infty}{T_w - T_\infty}$$

$$V = \frac{R}{R_1}, \quad y = \frac{r - R}{R_1}$$

$$\mu = -\cos \theta \quad \tau = \frac{ct}{R_1^2}$$

where $R_1$ is a suitable radius used to make the bubble radius $R$ dimensionless.

In terms of these variables, equations (22) and (25) may be written,

$$\frac{3U}{\partial \tau} - \frac{dV}{d\tau}(1 - \frac{V^2}{(y + V)^2})\frac{\partial^2 U}{\partial y^2} + \frac{2}{(y + V)^2}\frac{\partial U}{\partial y}$$

$$+ (1 - \mu^2)/(y + V)^2\frac{\partial^2 U}{\partial \mu^2} - \frac{2\mu}{(y + V)^2}\frac{\partial U}{\partial \mu} = \ldots \ldots (26)$$

$$\frac{dV}{d\tau} = \frac{N_{Ja}}{2} \int_{-1}^{+1} \frac{3U}{\partial y} \bigg|_{y=0} \, du$$

where $N_{Ja}$ is the Jakob number. The initial and boundary conditions become

$t = 0; \quad U(\infty, \mu, 0) = 1.0$

$U(\infty, \mu, 0) = 0.0$

$U(\infty, \mu, 0) = f'(y, \mu), \quad V < y < \infty$

and

$t > 0; \quad U(\infty, \mu, 0) = (T_{\text{sat}} - T_\infty)/(T_w - T_\infty)$

$U(\infty, \mu, 0) = 0$

Equations (26) and (27) together with the initial and boundary conditions define the bubble growth problem.

To start the solution, a small value for $V$ is assumed. This value
will not affect the subsequent growth of the bubble. The initial
growth rate \( \frac{dV}{dt} \) is, however, set equal to zero. Under these conditions
equation (26) may be put in finite difference form and solved by the
alternating direction implicit method (52). The details of this solution
are given in Appendix 5. Once the temperature distribution is known, its
slope at the bubble surface is obtained by numerical differentiation.
Application of Simpson's rule to equation (27) yields the non-dimensional
bubble radius \( V \).

Since the time available on the computer used for the solution of
equations (26) and (27) was limited, results could be obtained only for
a system pressure of 7.0 psia, for the early period of bubble growth.
However, the experimentally determined bubble growth rates are compared
with existing theory for the case of saturated boiling.

5.4. Results and Discussion.

The experimentally determined bubble growth rates for the pressures
studied in this investigation are shown in figs. 53 to 56. In fig. 54 is
also shown the result obtained from a computer solution of equations
(26) and (27), as discussed in section 5.2.

The experimental bubble diameter-time curves have been compared
with several of the theories available in the literature. In fig. 53,
bubble growth rates at atmospheric pressure are compared with the theories
of Zuber (34) and of Hsu and Graham (35). In addition, some experimental
data of Garg (44) and the empirical correlation of Cole and Shulman (45)
are also shown for comparison.

It can be seen from fig. 53 that although Zuber's expression,
equation (41), does not agree with the present results, it seems to
predict one set of Garg's data favourably. In employing the equation
due to Hsu and Graham, since delay times were not measured, the super-
heated layer thickness \( \delta \) was computed from the relation
\[ \delta = k(T_w - T_{\text{sat}})/q_w \]

As a result, the calculated growth curve will represent only the growth rate of an average bubble. In addition, the correction factor C was taken to be \( \pi/2 \). A similar method was used in using the equation of Han and Griffith. It may be seen from fig. 53 that the equations of Hsu and Graham and of Han and Griffith predict reasonably well the present results. The empirical correlation of Cole and Shulman predicts bubble diameters lower than those obtained here but are in broad agreement with the data of Garg. The micro-layer theory of Cooper agrees with the present results only during the initial stages.

In fig. 54 bubble growth data at 7.0 psia are shown compared with theory. The bubble diameters in the initial stages of growth, obtained by a computer solution of equations (16) and (17) are also shown. This solution appears to predict the initial growth rate satisfactorily, but no conclusions can be drawn regarding agreement over the later stages of growth. Zuber's theory also gives good agreement in the initial growth period but the divergence between theory and experiment becomes increasingly greater during the later stages of growth. The correlation of Cole and Shulman, specifically developed for growth at high Jakob numbers, appears to predict correctly the trend of the data. The theories of Hsu and Graham, Han and Griffith and Cooper give poor agreement with the experimental data.

Figs. 55 and 56 show that, as the Jakob number increases, the Cole and Shulman correlation predicts favourably the experimental growth rates; the divergence between theory and experiment increases until at a Jakob number of 880, theory predicts growth diameters an order of magnitude larger than the experimental results.

5.5. Conclusions.

The conclusions that may be drawn from the study reported here are:
1. For boiling at atmospheric pressure, the bubble growth theories available in the literature predict with reasonable accuracy the experimental values.

2. For sub-atmospheric pressures, the empirical correlation proposed by Cole and Shulman appears to give the best results. The deviation between theory and experiment increases as the Jakob number increases.

3. A computer solution of the governing equations for bubble growth from a cylindrical surface predicts the initial growth rate satisfactorily.
This investigation has established that bubbles generate pressure waves in a liquid. Using relatively unsophisticated equipment, the shape and magnitude of the bubble induced pressure waveforms have been measured and recorded. Using the technique of high speed photography, bubble growth and collapse rates have been recorded for pressures of 1.0, 3.0, 7.0 and 14.7 psia. To relate the pressure in the liquid to the bubble motion, two expressions were derived; the first, in which the liquid was assumed to possess a finite but constant sonic speed, gave satisfactory agreement with experimental results during bubble growth and early collapse. The second expression, in which results from simple acoustic theory were used, gave satisfactory agreement with experiment only during the growth phase. It was concluded that, provided some judicious assumptions are made about the position of the bubble, the bubble characteristics may be determined from the measured pressure waveforms and vice versa.

Since each growing vapour bubble is the source of a pressure wave, the method of recording used in this study may be employed to detect the presence of bubbles either when boiling occurs in an opaque liquid (for example, sodium) or when the region of bubble formation is not accessible to visual examination. Further, it may be possible to determine the frequency of bubble formation (number of bubbles per sec.) by counting the pulses recorded in a given interval of time.

The sound emitted by single bubbles has been recorded at atmospheric and several sub-atmospheric pressures. The frequency spectra showed that the sound is equivalent to white noise in the frequency range 0.1 to 10 kHz.

The effect of dissolved gas on the sounds emitted in nucleate
boiling at atmospheric pressure has been investigated. The frequency spectra show that a sharp reduction of noise level occurs in the frequency range 10-14 kHz and this may be important in the detection of boiling by acoustic methods. As an extension to the work reported here, it may be desirable to investigate the dissolved gas effect at sub-atmospheric pressures also. It has been remarked in the literature that the study of boiling in water at sub-atmospheric pressures is relevant to boiling in liquid metals (21).

In Chapter 5, the experimentally determined bubble growth rates in saturated boiling have been compared with various theories available in the literature. It was found that the theories predict reasonably well the bubble growth rates at atmospheric pressure. At low pressures, however, the empirical correlations seem to give better agreement with experimental results. The divergence between theoretical predictions and experimental values increases with decrease in pressure.
Appendix I

Deduction of outside surface temperature of heating tube.

The tube outside surface temperature was calculated from a knowledge of the inside surface temperature. A 0.0076 in. diameter Chromel-Alumel thermocouple was inserted into the tube from each end. The thermocouples were positioned near the centre of the tube and the ends then sealed with epoxy resin. The inside of the tube is insulated and the thermocouples will read the inside surface temperature of the tube. The temperature difference between the outside and inside surfaces is then calculated as follows.

Let 
\[ r_1 = \text{inside radius of tube, ft} \]
\[ r_0 = \text{outside radius of tube, ft} \]
\[ T_i = \text{inside surface temperature, deg.F} \]
\[ T_o = \text{outside surface temperature, deg.F} \]
\[ q_w = \text{heat flux, Btu per hr. per sq. ft.} \]
\[ q''' = \text{heat generated per unit volume, Btu per hr. per cu.ft.} \]
\[ k = \text{thermal conductivity of tube material, Btu per hr. per ft. deg.F.} \]

The temperature at any radius \( r \) \( (r_1 < r < r_0) \) satisfies the equation

\[
\frac{1}{r} \frac{d}{dr}(r \frac{dT}{dr}) + \frac{q'''}{k} = 0 \quad \ldots (1)
\]

Integrating equation (1),

\[
r \frac{dT}{dr} = -\frac{q'''}{k} \frac{r^2}{2k} + \text{constant} \quad \ldots (2)
\]

The constant is evaluated by substituting in equation (2) conditions at \( r = r_1 \) where \( T = T_1 \) and \( dT/dr = 0 \). This gives

\[
\text{constant} = \frac{q'''}{k} \frac{r_1^2}{2k}
\]

Substitution in equation (2) gives

\[
r \frac{dT}{dr} = \frac{q'''}{2k} \left( r_1^2 - r^2 \right) \quad \ldots (3)
\]

Integrating equation (3) yields

\[
T = \frac{q'''}{2k} \left( r_1^2 \ln r - r^2/2 \right) + \text{constant} \quad \ldots (4)
\]
At \( r = r_1 \), \( T = T_1 \); substitution in equation (4) yields

\[ \text{constant } T_1 = \frac{q'''}{2k(r_1^2 \ln r_1 - r_1^2/2)} \]

Substitution of the above value for the constant in equation (4) gives

\[ T = \left( \frac{q'''}{2k} \right) \ln \frac{r}{r_1} + \frac{q'''}{4k(r_1^2 - r^2)} + T_1 \quad \ldots \ldots \ldots (5) \]

At \( r = r_o \), \( T = T_o \); substitution of these values in equation (5) yields

\[ T_o - T_1 = \frac{q'''}{2k(r_1^2 \ln \frac{r_o}{r_1} - (r_o^2 - r_1^2)/2)} \quad \ldots \ldots \ldots (6) \]

Now, the volumetric heat flux \( q''' \) may be written in the form,

\[ q''' = \left( \frac{q_w}{2\pi r_1} \right) R_0 (r_1^2 - r_o^2) \]

where \( l \) is the length of the tube. Substitution for \( q''' \) in equation (6) gives

\[ T_o - T_1 = \frac{q_w R_0}{k} \left[ \frac{1}{2} + \left( \ln \frac{r_o}{r_1} \right) \frac{r_1^2}{(r_0^2 - r_1^2)} \right] \quad \ldots \ldots \ldots (7) \]

This is the expression for the temperature difference \( (T_1 - T_o) \) between the inside and outside surfaces of the tube. In the present study,

\[ r_o = 0.072 \text{ in} = 0.006 \text{ ft} \]
\[ r_1 = 0.048 \text{ in} = 0.004 \text{ ft} \]
\[ k = 9.68 \text{ Btu per hr. per ft. deg.F} \]

The value of the thermal conductivity \( k \) was obtained from the suppliers of the stainless steel heating tube.

Substitution of the above numerical values in equation (7) gives

\[ T_o - T_1 = 1.08 \times 10^{-4} q_w \quad \ldots \ldots \ldots (8) \]

In the present study, the temperature drop across the tube wall was a maximum of 12.5 deg.F corresponding to a heat flux of 118,000 Btu per hr. per sq. ft. The minimum temperature drop was 1.76 deg.F corresponding to a heat flux of 12800 Btu per hr. per sq. ft.

Estimation of errors in the calculated temperature difference is shown in Appendix 7.
Appendix 2

Coding for solution of equation (16) Chapter 2

To facilitate solution of equation (16) by the Runge-Kutta method, it is put into the form

\[ \frac{d^2 R}{dt^2} = \mathcal{F}(R, \frac{dR}{dt}, t) \]

where the right hand side is given by

\[ \mathcal{F}(R, \frac{dR}{dt}, t) = [(p - p_\infty) \rho - (r^2 \frac{dp}{dt})/\rho C - 2R^2 - R^4 \rho^2 / 2r^3] / R^2 \]

The term \((p - p_\infty)\) is given, as a function of time, by the ordinates of the pressure pulse obtained at each interval of time. These ordinates are read into the store at the beginning of the program. The slope \(d\rho/dt\) is obtained by a three point differentiation formula. The solution is started by giving a small value to the bubble radius and carrying out the integration. After each integration, the computed radius and the growth rate \(dR/dt\) are printed out. The integration is terminated when all the time steps have been covered. The coding, in Fortran IV, is given below.
C DOUBLE RADII. TIME HISTORY. RUNG KUTTA METHOD.
REAL K(2),K2,K3,K4
DIMENSION P(400),V(400),P(400),SLOPE(400),DP(400)
IEND=10
READ(7,10) (P(I),I=1,10,E=2)
II=IEND-1
DC 20 I=2,11,2
P(I)=(P(1+I)+P(I-1))/2.0
20 CONTINUE
C CONVERT DATA TO CMF/MM2
T=59.0/10.0
C T IS THE AMPLIFIER SETTING
DC 22 I=1,10,E=2
P(I)=P(C)/T*154.0
22 CONTINUE
C SET INITIAL VALUES
M=1
R/(P)=2.75
D=17.1
C=0.0601
SLOPE(1)=F=0.0
V(0)=V=SLOPE(P)
C CALCULATE COEFFICIENTS
DC 30 I=1,11
DP(I)=0.5*(P(I+1)+P(I))/2.0
S=P(I+1)+P(I)/2.0
K1=H*Z*V(I)+P(I),V(I)/H,D,DP(I)/2.0
K2=H*Z*(V(I))/2.0+K1/4.0,S,(V(I)+K1)/H,D,DP(I)/2.0
I)/2.0
K3=H*H*Z*V(I+1)/2.0+K1/4.0,S,(V(I)+K2)/H,D
1 DP(I+1)/2.0
K4=H*Z*(V(I))+V(I)*K3*P(I+1)+(V(I)+2.0*K3)/3.0,S
1 D,DP(I)/2.0
P(I+1)=P(I)+V(I)+(K1+K2+K3)/3.0
V(I+1)=V(I)+(K1+2.0*(K2+K3)+K4)/6.0
SLOPE(I+1)=V(I+1)/H
L=I+1
WRITE(2,120)F,L,SLOPE(L)
30 CONTINUE
STOP
120 FORMAT(1H13,2F16.9)
10 FORMAT(10F7.2)
END

FUNCTION Z(A,B,C,D,E)
Z=(98576000.0*B*D*C**2-2.0*A*C**2-A**4*C**2)/
(10**3+2.0))/A**2
RETURN
END
Appendix 3

Determination of bubble radius from simple acoustic theory.

Sample Calculation of bubble radius using equation (19), Chapter 2

The calculation will be performed for saturated boiling at 3.0 psia., bubble no.43Al-1. The pressure pulse for this bubble is shown in fig.3. The pulse is divided into three regions and the time span of each of these regions $t_1$, $t_2$ and $t_3$ are 0.6 m sec, 2 m sec, and 16 m sec respectively. The calculations are presented for each of these regions in turn.

Region OA.

Equation (19) of Chapter 2 may be written in the form

$$\frac{d}{dt}(R^2) = \frac{p_s}{\rho} r$$

$(1)$

In region OA of fig.3, $p_s$ may be written

$$p_s = At$$

$(2)$

where $A$ is the slope of line OA which is given by

$$A = \frac{(0.212 \cdot 144 \cdot 32.2)/0.6 \cdot 10^{-3}}{18.5 \cdot 10^5} \text{ lbm/ft sec}^3$$

Further, the value of $r$ for bubble no.43Al-1 is 0.362 in. and $\rho$, the density of saturated water at 3.0 psia is 61.5 lbm/ft$^3$. Substitution of equation (2) in equation (1) and inserting the values of $r$ and $\rho$ gives

$$\frac{d}{dt}(R^2) = 818 \ t$$

$(3)$

On integration, equation (3) becomes

$$R^2 = 818 \ t^2/2 + M$$

$(4)$

where $M$ is a constant.

At $t = 0$, $R = 0$. Hence $M = 0$. Equation (4) therefore yields,

$$R^2 = 818 \ t^2/2$$

One more integration yields,

$$R^3/3 = 409 \ t^3/3 + M'$$
At $t = 0$, the bubble radius is small and its cube may be neglected. Consequently $M'$ is zero. Hence,

$$R^{3/3} = 409 \frac{t^3}{3}$$

and

$$R = 7.42t$$

At $t = t_1 = 0.6$ m sec,

$$R = 7.42 \times 0.6 \times 10^{-3} = 0.0535 \text{ in.}$$

and the bubble diameter is $2R = 0.107$ in.

Also, $R^2 = 409 t_1^2 = 0.143 \times 10^{-3}$.

$$R^{3/3} = 0.03 \times 10^{-6}.$$  

**Region AB**

In this region, the sound pressure $p_s$ is given by the relation,

$$p_s = A'(t_2 - t)$$

where $A'$ is the slope of the line $AB$. $A' = 5.2 \times 10^5 \text{lbf/ft.sec}^3$.

Equation (1) may then be written

$$\frac{d}{dt}(R^2) = 355(t_2 - t) \quad \cdots \cdots (5)$$

Integration of equation (5) yields

$$R^2 = 355(t_2 t - t^2/2) + M \quad \cdots \cdots (6)$$

at $t = t_1$, $R^2 = 0.143 \times 10^{-3}$. Substitution of these values in equation (6) yields, $M = -0.217 \times 10^{-3}$. Therefore equation (6) may be written,

$$R^2 = 355(t_2 t - t^2/2) - 0.217 \times 10^{-3}$$

One more integration yields,

$$R^{3/3} = 355(t_2 t^2/2 - t^3/6) - 0.217 \times 10^{-3}t + M'$$

Substitution of the value of $R^{3/3}$ at $t = t_1$ in the above equation yields, after simplification, $M' = 0.47 \times 10^{-6}$. Therefore,

$$R^{3/3} = 355(t_2 t^2/2 - t^3/6) - 0.217 \times 10^{-3}t + 0.47 \times 10^{-6}$$

This is the equation for the bubble radius in the region $AB$.

At $t = t_2 = 2.10^{-3}$ sec,

$$R^{3/3} = 0.555 \times 10^{-6}$$

$$R = 1.665 \times 10^{-3} \text{ in.}$$
The bubble diameter is $2R = 0.285$ in.

Also, $R^2 \frac{\partial R}{\partial t} = 355(t_2^2 / 2) - 0.217 \cdot 10^{-3}$

\[ = 0.433 \cdot 10^{-3}. \]

**Region BC**

In this region, the sound pressure $p_s$ is approximated by the relation

\[ p_s = 4s(t_2^2 / t^2 - t_2 / t) \]

where $s$, as indicated in fig.3, equals 0.027 psi.

Inserting equation (7) into equation (1) and integrating gives

\[ R^2 \frac{\partial R}{\partial t} = 0.153(-t_2^2 / t - t_2 \ln t) + M \]

\[ \text{.....(8)} \]

Evaluating the constant $M$ from conditions at $t = t_2$,

\[ M = - 1.1 \cdot 10^{-3}. \]

Inserting the above value of $M$ in equation (8) and integrating gives

\[ \frac{R^3}{3} = 0.153 \int_{t_2}^{t} \left( t_2^2 / t - (t_2 + t) \ln t \right) dt - 1.1 \cdot 10^{-3} t + M' \]

Again evaluating $M'$ using conditions at $t = t_2$, yields

\[ M' = - 5.5 \cdot 10^{-5}. \]

Therefore, the equation for bubble radius in the region BC is

\[ \frac{R^3}{3} = 0.153 \int_{t_2}^{t} \left( t_2^2 / t - (t_2 + t) \ln t \right) dt - 1.1 \cdot 10^{-3} t - 5.5 \cdot 10^{-5}. \]

Values of the bubble diameter calculated using the above equation are set out in the table below.

<table>
<thead>
<tr>
<th>$t$, m sec.</th>
<th>$\ln t$</th>
<th>bubble dia., in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.0</td>
<td>-5.82</td>
<td>0.352</td>
</tr>
<tr>
<td>4.0</td>
<td>-5.52</td>
<td>0.393</td>
</tr>
<tr>
<td>6.0</td>
<td>-5.12</td>
<td>0.458</td>
</tr>
<tr>
<td>8.0</td>
<td>-4.84</td>
<td>0.495</td>
</tr>
<tr>
<td>10.0</td>
<td>-4.61</td>
<td>0.522</td>
</tr>
<tr>
<td>12.0</td>
<td>-4.42</td>
<td>0.550</td>
</tr>
<tr>
<td>14.0</td>
<td>-4.30</td>
<td>0.575</td>
</tr>
<tr>
<td>16.0</td>
<td>-4.15</td>
<td>0.590</td>
</tr>
</tbody>
</table>
Appendix 4

Coding for determination of frequency spectra of sound due to single bubbles.

The program given below illustrates the determination of the frequency spectrum for boiling at 3.0 psia, subcooling 15 deg.C. The pressure ordinates determined from the pressure pulse are read in at the beginning of the program. These ordinates are then used to determine the Fourier transform of the pressure pulse \( p(r,t) \). Simpson's rule is used in the integration. The frequency range covered is 20 Hz - 100 kHz. The spectral density at each frequency is then determined using equation (3) of Chapter 3. The program is written in Fortran IV.
POWER DENSITY SPECTRUM OF BUBBLE NOISE

DIMENSION P(200), SCF(200)
IEND=137
PERIOD(7,30) (P(I), I=3, IEND, 2)
WRITE(2,99)
11=IEND-1
F(1)=0.1
DC 31 I=2, I1, 2
P(I)=(P(I+1)+P(I-1))/2.0
31 CONTINUE
DT=0.0001
AI=0.0
BI=0.0
AAI=0.0
BBI=0.0
TDT=2.0*DT
W=20.0
DC 66 J=1, 32
T=T+TDT
AI=AI+P(I)*CCS(2.0*3.14159*W*T)
AAI=AAI+P(I)*SIN(2.0*3.14159*W*T)
46 CONTINUE
DC 56 J=2, 11, 2
T=T+TDT
BI=BI+P(I)*CCS(2.0*3.14159*W*T)
BBI=BBI+P(I)*SIN(2.0*3.14159*W*T)
50 CONTINUE
F1=DT*(4.0*AI+2.0*BI)/3.0
G1=DT*(4.0*AAI+2.0*BBI)/3.0
SCF(J)=F1**2+G1**2
WRITE(2,100) J, F1, G1, W, SCF(J)
W=W+20.0
IF(W=100) 66, 65, 51
51 W=W+80.0
IF(W=1000.0) 60, 60, 52
52 W=W+900.0
IF(W=10000.0) 60, 60, 53
53 W=W+9000.0
IF(W=100000.0) 60, 60, 60
60 CONTINUE
54 DC 79 J=1, 32
POWER=10.0*ALCG10(SCF(J))
WRITE(2,200) J, POWER
70 CONTINUE
STOP
30 FORMAT(10F7.2)
99 FORMAT(54,POWER DENSITY SPECTRUM:BOILING AT 3 PSI:
& SUBCOOLED 15C)
100 FORMAT(1H,2HJ=",I3,2X,3HFI=",F10.5,2X,3HGI=",F10.5,
2X,2HF=",F9.1,5X,4HSCF=",F10.4)
200 FORMAT(1H,2HJ=",I4,10X,6HP=",F10.3,2HDB)
END
Appendix 5

I. Temperature distribution in the liquid surrounding a heated cylinder.

In the present analysis, the bubble is assumed to originate from the top of the cylinder. As discussed in section 5.3 b of Chapter 5, the velocity induced in the liquid by bubble departure, \( v_\infty \), is assumed equal to the bubble departure velocity and denoted by \( v_\infty \). Let the radius of the cylinder be \( a \) and \( v_r \) and \( v_\theta \), the radial and tangential components of the velocity over the cylinder. The temperature \( T \) in the liquid satisfies the equation

\[
\frac{\partial T}{\partial t} + \frac{v_r}{r} \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} = \alpha \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} \right] \quad \ldots \ldots \ldots (1)
\]

where \( v_r \), \( v_\theta \) are given by the relations

\[
v_r = v_\infty \left( 1 - a^2/r^2 \right) \cos \theta \quad \ldots \ldots \ldots (2)
\]

\[
v_\theta = v_\infty \left( 1 + a^2/r^2 \right) \sin \theta \quad \ldots \ldots \ldots (3)
\]

To make equation (1) non-dimensional, the following dimensionless variables are defined:

\[
U = \frac{T - T_\infty}{T_M - T_\infty} \quad \tau = \frac{at}{a^2} \quad y' = \frac{r}{a} \quad \text{and} \quad \mu = \cos \theta.
\]

Using the above variables, equations (1), (2) and (3) may be combined to yield the following relation for the dimensionless temperature \( U \):

\[
\frac{3U}{3\tau} = \frac{3^2U}{3y'^2} + Z \frac{3U}{3y'} - (P + \frac{1}{y'^2}) \frac{3U}{3\mu} + Q \frac{3^2U}{3\mu^2} \quad \ldots \ldots \ldots (4)
\]

where

\[
Z = \frac{1}{y'} \frac{3U}{3y'} + \frac{1}{y'^2 - 1} \frac{\mu u v_\infty a}{a}
\]

\[
P = (1 - \mu^2) \frac{v_\infty a}{y'^2 + 1}
\]

\[
Q = (1 - \mu^2) / y'^2
\]

The initial conditions are:

\[
t = 0; \quad U(1, \theta, 0) = 1, \quad y' = 1
\]

\[
U(y', \theta, t) = 0, \quad y' > 1 \quad \text{for} \ 0 < \theta < \pi
\]

The boundary conditions are:
\( t > 0; \ U(y', \theta, \ t) = 1, \ y' = 0 \)
\( U(y', \theta, t) = 0, \ y' \to \infty \ \text{for} \ 0 \leq \theta \leq \pi \)

The second boundary condition implies that at a large distance from the cylinder, the temperature is equal to the bulk liquid temperature. The physical system and the transformed system for the present analysis is shown in fig.57 which is self-explanatory. Solution of equation (4) subject to the initial and boundary conditions yields the temperature distribution in the liquid.

Equation (4) may be put in finite difference form and solved by the Alternating Direction Implicit (ADI) method (52). A second difference formula is used for the second order derivatives. For the first order space derivative, a simple mean of the forward and backward difference quotients is used. For the time derivative a forward difference quotient is used. Each time step is divided into two half steps. In the first half step, the derivatives in the \( \mu \)-direction are written in explicit form and those in the \( y' \)-direction, in implicit form. For the next half time step, these representations are interchanged, with the calculated values of the temperature in the \( y' \)-direction being used. The process is continued until all the steps have been covered.

Fig.58 shows the region of integration which has been divided into a number of grids. The total number of steps in the \( y' \) and \( \mu \) directions are I and J respectively. If \( \Delta y' \) and \( \Delta \mu \) are the increments in each direction, then \( AD = I \Delta y' \) and \( AB = J \Delta \mu \). The second boundary condition requires that the temperature \( U \) be zero as \( y' \to \infty \). For the numerical scheme, it is only necessary to take \( y' \) sufficiently large such that the temperature is zero there and beyond it. This value of \( y' \) is given, in fig.58, by the distance between the lines \( AB \) and \( DE \). The temperature at a nodal point is denoted by \( U_{i,j,n} \). \( i, j, n \) are integers and refer to the particular step in the \( y', \mu \) and time directions respectively. The
initial temperature distribution is known. The temperature at the various nodal points for the next time step are calculated by the following equations.

For the first half time step:
\[
\begin{align*}
(U_{i,j,n+\frac{1}{2}} - U_{i,j,n})/\Delta t/2 &= (U_{i-1,j,n} - 2U_{i,j,n+\frac{1}{2}} + U_{i+1,j,n+\frac{1}{2}})/\Delta y^2 \\
+ \Delta t (U_{i+1,j,n+\frac{1}{2}} - U_{i-1,j,n+\frac{1}{2}})/2\Delta y' \\
- (U_{i,j+1,n} - U_{i,j-1,n})(P + \mu j/y_1^2)/2\Delta y \\
+ \Delta t (U_{i,j+1,n} - 2U_{i,j,n} + U_{i,j+1,n})/\Delta y^2 \\
\end{align*}
\]

For the second half time step:
\[
\begin{align*}
(U_{i,j,n+1} - U_{i,j,n+\frac{1}{2}})/\Delta t/2 &= (U_{i-1,j,n+\frac{1}{2}} - 2U_{i,j,n+\frac{1}{2}} + U_{i+1,j,n+\frac{1}{2}})/\Delta y^2 \\
+ \Delta t (U_{i+1,j,n+\frac{1}{2}} - U_{i-1,j,n+\frac{1}{2}})/2\Delta y' \\
- (U_{i,j+1,n+1} - U_{i,j-1,n+1})(P + \mu j/y_1^2)/2\Delta y \\
+ \Delta t (U_{i,j+1,n+1} - 2U_{i,j,n+1} + U_{i,j+1,n+1})/\Delta y^2 \\
\end{align*}
\]

Here \( \Delta t \) is the increment in time.

Let \( \Delta t/\Delta y^2 = \Delta t/\Delta y'^2 = \lambda \)

Equations (5) and (6) may be written in the form
\[
\begin{align*}
-(1 - Z \Delta y'/2)U_{i-1,j,n+\frac{1}{2}} + (1+\lambda\Delta y'/2)U_{i,j,n+\frac{1}{2}} - (1+2\Delta y'/2\mu U_{i+1,j,n+\frac{1}{2}} \\
= \lambda/2(Q + (P + \mu j/y_1^2)\Delta y'/2)U_{i,j-1,n} + (1 - \lambda\mu)U_{i,j,n} \\
+ \lambda/2(Q - (P + \mu j/y_1^2)\Delta y'/2)U_{i,j+1,n} \\
\end{align*}
\]

and
\[
\begin{align*}
-(Q + (P + \mu j/y_1^2)\Delta y'/2)U_{i,j-1,n+1} + (1 + \lambda\mu)U_{i,j,n+1} - (Q - (P + \mu j/y_1^2)\Delta y'/2) \\
x U_{i,j+1,n+1} \\
= \lambda/2(1 - Z\Delta y'/2)U_{i-1,j,n} + (1 - \lambda\mu)U_{i,j,n+1} \\
+ \lambda/2(1 + Z\Delta y'/2)U_{i+1,j,n+1} \\
\end{align*}
\]

Equations (7) and (8) form a set of simultaneous equations for the temperatures at each time step. They are tri-diagonal in form and may be
solved by the Gauss elimination method. The choice of the number of steps in the $y$ and $\mu$ directions depends on the magnitude of the term $\nabla a/a$. For large values of this term, small steps in the $\mu$ and $y'$ directions are necessary. The choice of the number of steps is also dictated by the storage capacity of the computer. In this investigation, it was estimated that the largest value of $\nabla a/a$ would occur at 1.0 psia. At this pressure, the departure velocity of the bubbles is roughly 1 ft. per sec. (44). The value of $(\nabla a/a)$ at saturation conditions at 1.0 psia works out to be roughly 2000. Initially 10 steps in the $y'$ direction and 50 in the $\mu$ direction were tried. It was found, in the computer print out, that small negative values of the temperature were obtained in regions beyond the thermal layer, which is physically impossible. Increasing the number of steps in the $y'$ and $\mu$ directions to 15 and 100 respectively gave correct results.

A suitable computer program was developed and the equations solved to obtain the temperatures at the nodal points. Fig. 59 shows the temperature profiles for saturated boiling of water at 1.0 psia. For comparison, the profile neglecting the effect of convection and assuming a wholly radial temperature gradient is also shown in fig. 59.

II. Bubble growth from a heated cylindrical surface.

The governing equations are equations (26) and (27) of Chapter 5.

$$\frac{3U}{3\tau} - \frac{dV}{d\tau} \left[ 1 - \frac{\sqrt{y + V}}{y + V} \right] \frac{3U}{3y} = \frac{3^2U}{3y^2} + 2(\gamma + V) \frac{3U}{3y}$$

$$+(1 - \mu^2)(\gamma + V) \frac{3^2U}{3\mu^2} - 2\mu(\gamma + V) \frac{3U}{3\mu} \ldots \ldots (9)$$

$$\frac{dV}{d\tau} = \frac{N^2}{4a^2} \left\{ \begin{array}{c} +1 \\ -1 \end{array} \right\} \left[ (\frac{3U}{3y}) \right]_{y=0} \frac{d\mu}{d\tau}$$

$$\ldots \ldots (10)$$

The solution of equation (9) may be performed by writing it in finite difference form and integrating it by the ADI method described in the previous section. In finite difference form equation (9) may be written
for the first half time step

\[
\frac{(U_{i,j,n+\frac{1}{2}} - U_{i,j,n})}{\Delta t/2} = \frac{(U_{i-1,j,n+\frac{1}{2}} - 2U_{i,j,n+\frac{1}{2}} + U_{i+1,j,n+\frac{1}{2}})}{\Delta y^2} + \frac{2}{Y_n + V_n}(U_{i+1,j,n+\frac{1}{2}} - U_{i-1,j,n+\frac{1}{2}}) / 2\Delta y \\
+ \frac{1 - \mu_j^2}{(Y_1 + V_n)^2}(U_{i,j-1,n} - 2U_{i,j,n}) / \Delta u^2 \\
- 2\frac{U_{j}/Y_i V_n}{(Y_1 + V_n)^2}(U_{i,j+1,n} - U_{i,j-1,n}) / 2\Delta u \\
+ \frac{\theta_n}{1 - V_n^2/Y_1 n^2}(U_{i+1,j,n+\frac{1}{2}} - U_{i-1,j,n+\frac{1}{2}}) / 2\Delta y
\]

where \( \theta_n = (dV/dr) \).

Putting \( X_1 = Y_1 + V_n, \quad SQX_1 = (Y_1 + V_n)^2 \)

\[
1 - \mu_j^2 = P_j, \quad [1 - V_n^2/Y_1 n^2] \theta_n = Q_{i,n}
\]

\[
\Delta t/\Delta y^2 = \Delta t/\Delta u^2 = M
\]

equation (11) may be written for the first half time step

\[
-M/2(1 - \Delta y/X_1 - Q_1 \Delta y/2)U_{i-1,j,n+\frac{1}{2}} + (1 + M)U_{i,j,n+\frac{1}{2}} - M/2(1 + \Delta y/X_1 + Q_1 \Delta y/2)U_{i+1,j,n+\frac{1}{2}}
\]

\[
= M/2(P_j/SQX_1 + \Delta u \mu_j/SQX_1)U_{i,j-1,n} + (1 - M.P_j/SQX_1)U_{i,j,n}
\]

\[
+ M/2(P_j/SQX_1 - \Delta u \mu_j/SQX_1)U_{i,j+1,n}
\]

For the next half time step, equation (9) may be written

\[
-M/2(P_j/SQX_1 + \mu_j \Delta u/SQX_1)U_{i,j-1,n+1} + (1 + M.P_j/SQX_1)U_{i,j,n+1}
\]

\[
= M/2(1 - \Delta y/X_1 - Q_1 \Delta y/2)U_{i-1,j,n+\frac{1}{2}} + (1 + M)U_{i,j,n+\frac{1}{2}} + M/2(1 + \Delta y/X_1 + Q_1 \Delta y/2)U_{i+1,j,n+\frac{1}{2}}
\]

The initial temperature at the instant of bubble growth is known from the computer solution of equations (7) and (8) presented in the previous section. The radius of the bubble and the growth rate at the instant of growth are also known. The subsequent growth of the bubble is
determined as follows. The temperature distribution at the end of a full time step is determined by solving equations (12) and (13). The slope of the temperature profile at the bubble boundary, \( \frac{\partial U}{\partial y} \), is then calculated using a three point differentiation formula for each value of \( y \). Integration by Simpson's rule yields a value of the integral on the right hand side of equation (10). After this integral is calculated for two half steps, a further application of Simpson's rule yields the non-dimensional bubble radius \( V \), for the first time step. This calculated value of \( V \) is used in the calculations for the second time step. The solution is continued in this manner until all the time steps have been covered.

A computer program developed to solve equations (10), (12) and (13) is shown in Appendix 6. The choice of a value for \( M \), which determines the time step, is influenced by the magnitude of the Jakob number. At low pressures, the Jakob number is large. For example, in saturated boiling of water at 1.0 psia, the Jakob number was calculated to be 880.0 for a superheat temperature difference of 44 deg.F. This would make \( Q \) large and consequently extremely small time steps have to be used in the initial stages of the calculation. If this is not done, round off errors may accumulate and cause a reversal of the driving force. In the initial stages, therefore, a value of \( 5 \times 10^{-5} \) for \( M \) was used. To reduce computation costs and conserve time, provision was made in the program so that as computation progressed, the value of \( M \) was increased to roughly \( 10^{-3} \). In spite of this, it was found that the time to obtain one complete set of results at one pressure was of the order of thirty minutes. Therefore only limited results could be obtained.
APPENDIX 6
DETERMINATION OF BUBBLE GROWTH RATES FROM A CYLINDRICAL SURFACE
CODING FOR SOLUTION OF EQUATIONS (26) AND (27) OF CHAPTER 5

C BUBBLE GROWTH FROM CYLINDRICAL SURFACE.
C PEACEMAN RACHFORD AND METHOD
REAL JA,MU(101)
INTEGER T
DOUBLE PRECISION U(16,101,3),D(16,101),S(16,101),ALFA(16),
1 A(16),ALFAT(16),AT(101),BT(101),CT(101),DV(3),V(3),Y(16)
DIMENSION SOX(16),G(15,5),AA(16),BB(16),DU(16,101),
1 X(16)
WRITE(6,500)
C SET INITIAL TEMPS FROM KNOWN TEMP DISTRIBUTION
DO 10 J=1,101
U(1,J,1)=0.0
DO 10 I=2,16
10 READ(5,499) U(I,J,1)
C GROWTH AT 7 PSIA
DO 11 N=2,3
DO 11 J=1,101
U(1,J,N)=0.0
U(16,J,N)=0.0
11 CONTINUE
DMU=0.02
DY=0.02
RH0=0.00005
JA=83.0
MU(I)=1.0
DO 12 J=2,101
12 MU(J)=MU(J-1)+DMU
V(1)=0.00005
DV(1)=0.0
V(2)=V(1)
Y(1)=0.0
DO 14 I=2,16
14 Y(I)=Y(I-1)+DY
141 L=1
N=2
T=2
144 DO 15 J=1,101,100
ALFA(2)=1.0+RH0
X(2)=Y(2)+V(N-1)
SOX(2)=X(2)**2
G(2,N)=(1.0-V(N-1)**2/SOX(2))*DV(N-1)
D(2,J)=U(2,J,N-1)+RH0*(1.0-DY/X(2)-G(2,N)*DY/2.0)*U(1,J,N)/2.0
S(2,J)=D(2,J)
C(2)=(1.0+DY/X(2)+G(2,N)*DY/2.0)*RH0/2.0
16 DO 17 I=3,16
X(I)=Y(I)+V(N-1)
SOX(I)=X(I)**2
G(I,N)=(1.0-V(N-1)**2/SOX(I))*DV(N-1)
A(I)=(1.0-DY/X(I)-G(I,N)*DY/2.0)*RH0/2.0
D(I,J)=U(I,J,N-1)
C(I)=RH0-A(I)
ALFA(I)=1.0+RH0-A(I)*C(I-1)/ALFA(I-1)
17 S(I,J)=D(I,J)+A(I)*S(I-1,J)/ALFA(I-1)
U(15,J,N) = S(15,J) / ALFA(15)
I = 14
18 U(I,J,N) = (S(I,J) + C(I) * U(I+1,J,N)) / ALFA(I)
I = 1 - 1
IF (I.LT.2) GO TO 19
GO TO 18
19 DU(1,J) = U(2,J,N) / DY
15 CONTINUE
GO TO (115,27), L
115 J = 2
1151 DO 21 J = 2, 15
AA(I) = (1.0 - MU(J)**2 + DMU*MU(J)) * RHO*0.5 / SQX(I)
21 BE(I) = (1.0 - MU(J)**2 - DMU*MU(J)) * RHO*0.5 / SQX(I)
16 D(2,J) = AA(2) * U(2,J-1,N-1) + (1.0 - RHO*(1.0 - MU(J)**2)) / SQX(I)
1 * U(2,J,N-1) + BE(2) * U(2,J+1,N-1)
S(2,J) = D(2,J)
22 DO 23 J = 3, 15
D(I,J) = C(I) * U(I,J-1,N-1) + (1.0 - PHI*(1.0 - NU(J)**2)) / SQX(I)
* U(I,J,N-1) + 0 * E(I) * U(J,J+1,N-1)
23 S(I,J) = D(I,J) + A(I) * E(I) / ALFA(I)
U(15,J,N) = S(15,J) / LFP(15)
I = 1 - 1
IF (I.LT.2) GO TO 25
GO TO 24
24 DU(1,J) = 0.5 * (4.0 * U(1,J,N) - 3.0 * U(2,J,N) - U(3,J,N)) / DY
J = J + 1
IF (J.GT.100) GO TO 26
GO TO 1151
26 SUM = 0.0
DO 265 J = 2, 100, 2
265 SUM = SUM + DU(1,J)
SUMS = 0.0
DO 266 J = 3, 99, 2
266 SUMS = SUMS + DU(1,J)
DV(N) = JA * DMU * (DU(1, 1) + 4.0 * SUM + 2.0 * SUMS + DU(1, 101)) / 3.0
261 L = 2
T = T + 1
N = 3
V(2,J) = V(1)
GO TO 144
27 I = 2
277 CT(2) = RHO*(1.0 - MU(2)**2 - DMU*MU(2)) * 0.5 / SQX(2)
AT(2) = RHO*(1.0 - MU(2)**2 + DMU*MU(2)) * 0.5 / SQX(2)
Z = RHO*(1.0 - DY*X(I)) - G(I,J,N) * DY / 2.0 / 2.0
Z = RHO - Z
D(1,2) = AT(2) * U(I, 1, N) + Z * U(I-1, 2, N-1) + (1.0 + RHO) * 1 * U(I, 2, N-1) + Z * U(I+1, 2, N-1)
S(I, 2) = D(I, 2)
ALFAT(2) = 1.0 + RHO*(1.0 - MU(2)**2) / SQX(I)
28 DO 29 K = 3, 100
CT(K) = RHO*(1.0 - MU(K)**2 - DMU*MU(K)) * 0.5 / SQX(K)
AT(K) = RHO*(1.0 - MU(K)**2 + DMU*MU(K)) * 0.5 / SQX(K)
BT(K) = 1.0 + RHO*(1.0 - MU(K)**2) / SQX(K)
ALFAT(K) = BT(K) - AT(K) * CT(K-1) / ALFAT(K-1)
D(I,K) = Z * U(I-1, K, N-1) + (1.0 + RHO) * U(I, K, N-1) + Z * U(I+1, 1, K)
1 K*N-1)
29 S(I,K) = D(I,K) + AT(K) * S(I,K-1) / ALFAT(K-1)
U(I,100,N)=S(I,100)/ALFAT(100)

C START BACK SUBSTITUTION
K=99
30 U(I,K,N)=S(I,K)/CT(K)*U(I+1,K,N)/ALFAT(K)
   K=K+1
   IF(K.LT.2) GO TO 31
   GO TO 30
   31 I=I+1
   IF(I.GT.15) GO TO 277
   GO TO 266
266 DO 34 J=2,100
34 DU(I,J)=0.5*(4.0*U(I,J,N)-3.0*U(2,J,N)-U(3,J,N))/DY
   SUM=SUM+DU(I,J)
   J=2,100
269 SUMSUMS+DU(I,J)
   SUMS=SUM+SUMS+DU(I,100)
268 SUMS=SUMS+SUMS+DU(I,J)
   DV(N)=J*SUM+4.0*SUM+2.0*SUMS+DU(I,100)/3.0
   DO 269 J=2,100
   DO 268 J=2,100
   SUMS=SUMS+SUMS+DU(I,J)
   DV(N)=J*SUM+4.0*SUM+2.0*SUMS+DU(I,100)/3.0
   C FIND BUBBLE RADIUS BY INTEGRATION
   DT=RHO*DY**2
   V(N)=DT*(DV(N-2)+4.0*DV(N-1)+DV(N))/3.0
   WRITE(6,600) N,T,V(N)
   DO 350 J=1,101
350 IJ(I,J,1)=U(I,J,3)
   V(1)=V(3)
   DV(1)=DV(3)
   C INCREASE TIME STEP AND ALTERNATE IN DIRECTION
   T=T+1
   IF(T.EQ.100) GO TO 109
   GO TO 110
109 RHO=RHO*10.0
110 N=2
   GO TO 144
   99 STOP

500 FORMAT(2F16.9)
500 FORMAT(1H,6HTIME T,10X,13HBUBBLE RADIUS)
600 FORMAT(1H,16,10X,F16.9)
END
Errors in heat flux measurement.

The heat flux is determined from the power supplied to the heating tube, which is given by the equation

\[ \text{Power} = V \times I \]

where \( V \) is the voltage and \( I \) the current. The voltage and current could be measured to an accuracy of 0.125V and 0.25A respectively. The largest error will occur at the smallest values of \( V \) and \( I \). In the present study, the smallest values of \( V \) and \( I \) were 0.675 Volts and 17.0 amps, respectively, corresponding to a heat flux of 12800 Btu per hr. per sq. ft. Thus

Maximum error in the heat flux measurement is

\[ \pm \left( \frac{0.125}{0.675} + \frac{0.25}{17.0} \right) \times 100 = \pm 20.0 \text{ per cent.} \]

At the highest heat flux of 116,000 Btu per hr. per sq. ft., the values of \( V \) and \( I \) were 2.1 volts and 50 amps, respectively. The error at this heat flux works out to be \( \pm 6.5 \) per cent.

Error in Temperature Measurement.

The Chromel-Alumel thermocouples used for temperature measurement were calibrated to within 1 per cent of tabulated values. For these thermocouples,

\[ 1 \text{ mV} = 44 \text{ deg. F.} \]

Therefore the maximum error in calibration = \( \pm 0.44 \) F. The Potentiometer could read to an accuracy of 1 \( \mu \text{V} \) which corresponds to an error of \( \pm 0.044 \) F. The error in the cold junction temperature was \( \pm 0.05 \) F. The maximum error in the temperature measurement was therefore, \( (\pm 0.44 \pm 0.044 \pm 0.05) = \pm 0.534 \) F.

Tube Wall Temperature Drop and Error in measurement of superheat temperature difference:

The tube wall temperature drop is given by equation (8) of Appendix 1. This is of the form
\[ T_0 - T_1 = \frac{N}{q_w}, N = 1.08 \times 10^{-4}. \]

Since the maximum error in \( q_w \) is 20 per cent and that in temperature measurement is 0.534 F, the error in the wall temperature drop can be calculated. This is found to be

\[ \pm (0.2 \times q_w \times 10^{-4} + 0.534) \text{ deg. F}. \]

If \( T_b \) is the bulk liquid temperature, with an error of \( \pm 0.534 \text{ F} \), the superheat temperature difference \( (AT) \) will have an overall error of

\[ \pm (0.2 \times q_w \times 10^{-4} + 0.534 + 0.534) \text{ deg. F}. \]

For example, at a heat flux of 53400 Btu per hr. per sq. ft., the error is

\[ (\pm 0.2 \times 53400 \times 1.08 \times 10^{-4} + 1.068) \text{ deg. F}. \]

which equals \( \pm 2.305 \text{ deg. F}. \)

At a heat flux of 53400 Btu per hr. per sq. ft., the superheat temperature difference is 39.5 deg. F. Thus the error is

\[ \pm \frac{2.305}{39.5} \times 100 = \pm 5.85 \text{ per cent}. \]

At the minimum heat flux of 12800 Btu per hr. per sq. ft., the error in \( AT \) is \( \pm 1.366 \text{ deg. F}. \), which corresponds to an error of \( \pm (1.366/20)100 \)

\[ = \pm 6.840 \text{ per cent}. \]

**Error in bubble size measurement.**

On the film analyser, the bubble diameters could be measured to an accuracy of \( \pm \frac{1}{2} \text{ mm} \). 40 mm on the screen corresponded to 0.547 inch in real size. Thus the error in bubble size measurement is

\[ \pm \frac{0.547}{2} \times \frac{1}{40} \]

which equals \( \pm 0.0058 \text{ in}. \)

The minimum measurable bubble size on the analyser screen was 4 mm which corresponds to a real size of 0.0547 inch. The maximum error in bubble size measurement is therefore

\[ \pm \frac{0.0058}{0.0547} \times 100 \]

which is \( \pm 12.5 \text{ per cent}. \)
This error will decrease as the bubble grows beyond the minimum measurable size. For example, at atmospheric pressure, the bubble departure size was 0.25 in giving an error of less than 3 per cent.

Errors in pressure measurements.

Bulk liquid pressure

The bulk liquid pressures were measured by a mercury manometer reading to ± 0.05 in of mercury. At the lowest pressure of 1.0 psia, the error in pressure measurement is therefore

\[ \pm (0.05) (0.49) (100) \text{ per cent} \]

which is 2.45 per cent. At atmospheric pressure, the error reduces to 0.16 per cent.

Bubble induced pressures.

The accuracy of the transducer was ± 1 per cent. When pressure pulses were measured on film, the accuracy was ± 1 mm. Since 1 mm on the analyser screen corresponded to 0.47 mV, the error in pressure pulse measurement is therefore ± 0.47 mV. For the lowest pressure pulse amplitude of 9.3 mV, the error is therefore ± (0.093 + 0.47) which is 6.2 per cent. This error decreases to ± 1.1 per cent for a pressure pulse amplitude of 115 mV, the highest observed in this study.
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47. Henley, J.J. and Hummel, R.L. A Third Factor in Boiling Nucleation. 
I&EC Fundns. 1967, Vol. 6, No. 4, pp. 803-808.


<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Assumptions</th>
<th>Model of Bubble Growth</th>
<th>Method of Analysis and Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plesest</td>
<td>Radially symmetric growth and Viscosity, inertia, surface tension small. Equilibrium at interface.</td>
<td>Growth due to heat transfer through thin thermal layer.</td>
<td>Perturbation Technique used to find bubble radius.</td>
</tr>
<tr>
<td>Scriven</td>
<td>Spherically symmetric growth Viscosity, inertia, surface tension small. Equilibrium at interface.</td>
<td>Heat transfer controlled growth.</td>
<td>$R = 28(\alpha t)^{\frac{1}{2}}$ assumed; heat conduction equation solved numerically to find $\beta$.</td>
</tr>
</tbody>
</table>

TABLE I

BUBBLE GROWTH RATES IN LIQUIDS

SUMMARY OF THEORETICAL ANALYSES DISCUSSED IN CHAPTER 5
<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Assumptions</th>
<th>Assumed Initial Temperature Profile</th>
<th>Method of analysis</th>
</tr>
</thead>
</table>
### TABLE I continued

**b. Growth in Non-Uniform Temperature Fields.**

<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Assumptions</th>
<th>Assumed Initial Temperature Profile</th>
<th>Method of Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>7. Savic and Gosnell (33)</td>
<td>Hemispherical bubble on flat surface; inertia, surface tension and viscosity neglected. Equilibrium across interface.</td>
<td>Linear profile, similar to that used by Griffith.</td>
<td>Both the product $R \tilde{T}$ and temperature $T$ expressed as power series. Heat conduction equation solved by perturbation technique.</td>
</tr>
<tr>
<td>8. Zuber (34)</td>
<td>Spherical bubble with heat transfer controlled growth.</td>
<td>Arbitrary initial profile.</td>
<td>Modification of Forster-Zuber theory to include heat transfer to bulk liquid from thermal layer.</td>
</tr>
<tr>
<td>Method of Analysis</td>
<td>Assumed Initial Temperature Profile</td>
<td>Growth in Non-Uniform Temperature Fields</td>
<td></td>
</tr>
<tr>
<td>--------------------</td>
<td>------------------------------------</td>
<td>--------------------------------------</td>
<td></td>
</tr>
<tr>
<td>Thermal layer thickness obtained.</td>
<td>Linear temperature profile.</td>
<td>Spherically symmetric growth.</td>
<td></td>
</tr>
<tr>
<td>Expression for waiting time found.</td>
<td>Model for growth as in (9)</td>
<td>Griffith equilibrium, viscosity and surface tension neglected. Nucleus on heated surface.</td>
<td></td>
</tr>
<tr>
<td>Pleasent-Zwick relation modified to include non-uniform profile.</td>
<td>Growth due to inner-layer evaporation.</td>
<td>Hemisphere bubble on flat surface, bulk liquid at saturation temperature. Inertia forces large compared to surface tension. Viscous forces and gravity forces.</td>
<td></td>
</tr>
</tbody>
</table>

**TABLE I continued**

10. Han

11. Cooper

\[ R = 2.5 \left( \frac{\beta}{Pr_L} \right)^{1/3} \]
<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Theory Description</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theofanous et al. (40)</td>
<td>Spherically symmetric growth in uniformly superheated liquid. Vapour density varies with time.</td>
<td>Simultaneous computer solution of vapour and liquid continuity and energy equations together with momentum equation for liquid. Results for sodium and water.</td>
</tr>
</tbody>
</table>
FIG. 1. PRESSURE DISTRIBUTIONS DURING THE COLLAPSE OF AN EMPTY CAVITY IN AN INCOMPRESSIBLE LIQUID (2).

FIG. 2. PRESSURE WAVEFORM FROM SPARK IGNITED GAS BUBBLES (9).
- Calculated waveform
+ Measured waveform
FIG. 3: PRESSURE WAVEFORM FOR SATURATED BOILING AT 3.0 PSIA.
BUBBLE NO. 43A1-1
$t_1 = 0.6$ MSEC., $t_2 = 2$ MSEC., $t_3 = 16.0$ MSEC.
S = 6 MV.
1. Boiling Vessel
2. Heating Tube
3. Bulk Liquid Heater
4. Kistler Transducer
5. Bulk Liquid & Tube Thermocouples
6. Glass Window

FIG. 4. BOILING VESSEL ASSEMBLY
FIG. 6. SCHEMATIC DIAGRAM OF EXPERIMENTAL APPARATUS

1. Boiling Vessel
2. Bulk Liquid Heater
3. Heating Tube
4. Transducer
5. Water-cooled Condenser
6. Collecting Vessel
7. Air-admittance Valve
8. Vacuum Pump
9. Needle Valve
1 timing light generator
2 amplifier
3 Hycam camera
4 boiling vessel

Plate 1. EXPERIMENTAL APPARATUS
Fig. 7. Power Circuit

Fig. 8. Recording system for waveform measurement

1. Boiling Vessel
2. Pressure Transducer
   KISTLER TYPE 4108
3. Charge Amplifier
   KISTLER TYPE 566
4. High Speed Camera
   Nycam type K2034E, 400ft.
5. Objective Lens
   Cosmicar 50mm F1.4
6. Oscilloscope Lens
   Pentax 50mm F1.8
7. Solartron Oscilloscope
8. Low-Loss Co-ax Cable
Fig. 9. Heat flux vs. Temperature difference in nucleate boiling of water at atmospheric pressure.
Plate 2. PRESSURE TRANSIENT

AT 3.0 psia, Subcooling 15°C
Fig. 10. Bubble diameter vs. time and pressure waveforms for saturated boiling at atmospheric pressure.

Computed curves, Eq. (16).

<table>
<thead>
<tr>
<th>BUBBLE NO.</th>
<th>EXPERIMENTAL VALUES</th>
</tr>
</thead>
<tbody>
<tr>
<td>64-1</td>
<td>y = 0.65 in.</td>
</tr>
<tr>
<td>65-1</td>
<td></td>
</tr>
<tr>
<td>66-1</td>
<td>y = 0.615 in.</td>
</tr>
<tr>
<td>67-1</td>
<td></td>
</tr>
</tbody>
</table>
Fig. 11. Bubble Diameter vs. Time and Pressure waveforms for Saturated Boiling at 7.0 psia.

Computed curves, Eq. (16)

- BUBBLE NO. 39-1
- BUBBLE NO. 41-1
- BUBBLE NO. 42-1
- BUBBLE NO. 43-1

Experimental values:

\[
\begin{align*}
T &= 0.83 \text{ in.} \\
y &= 0.62 \text{ in.}
\end{align*}
\]
Fig. 12. Bubble Diameter vs. Time for Saturated Boiling at 30 psia.

Computed Curves, eq. (16)

- Bubble No. 43A1-1
- Bubble No. 42A1
- Bubble No. 43A1-2

Experimental Values
Fig. 13. Bubble Diameter vs. Time and Pressure Waveforms for Saturated Boiling at 1.0 psia.

- Computed curves, eq. (16)
- Bubble No. 94-1
- Bubble No. 94-1
- Experimental Values
Fig. 14. Bubble Diameter vs. Time for Subcooled Boiling at Atmospheric Pressure. Subcooling = 5 deg C.

Computed curves, Eq. (16)

- Bubble No. 70-1
- Bubble No. 73-1
- Bubble No. 75-1

Experimental values
Fig. 15. Bubble Diameter vs. Time and Pressure waveform for subcooled boiling at 7.0 psia, subcooling: 5 deg. C

- Computed curve, Eq. (16)
- Bubble No. 96-1 - Experimental values
  \( r = 0.83 \) in.

**Diagram Description:**
- The graph illustrates the relationship between bubble diameter, time, and pressure for subcooled boiling at 7.0 psia with a subcooling of 5 deg. C.
- The computed curve and experimental data points are shown.
- Bubble diameter values are indicated at various time intervals, with pressure measurements on the right axis.
- The graph is a visual representation of the dynamics of boiling under these conditions.
Fig. 16. Bubble Diameter vs. Time for Subcooled Boiling at 30 psia. Subcooling: 5 deg.C.

Computed Curves, Eq. (16)

- Bubble No. 52-1
- Bubble No. 56-1
- Bubble No. 57-1

Experimental Values
Fig. 17. Bubble Diameter vs. Time and Pressure waveform for subcooled boiling at 1.0 psia. Subcooling: 5 deg. C

- Computed Curve, Eq. (16)
- Bubble no. 95-1 - Experimental values
  \( r = 0.77 \text{ in.} \)
Fig. 18. Bubble Diameter vs. Time for Subcooled Boiling at Atmospheric Pressure. Subcooling: 10 deg. C.

- Computed Curves, Eq. (16)
- △ Bubble No. 77-1
- ○ Bubble No. 78-1

Experimental Values.
Fig. 20. Bubble Diameter vs. Time for Subcooled Boiling at Atmospheric Pressure. Subcooling: 15 deg. C.

Computed curves, Eq. (16)

- Bubble No. 83-1 | Experimental Values
- Bubble No. 84-1
Fig. 21. Bubble Diameter vs. Time for subcooled boiling at 7.0 psia.
Subcooling: 15 deg. C.

- Computed Curves, Eq.(16)
- Bubble No. 97-1 Experimental Values
Fig. 22. Bubble Diameter vs. Time and Pressure waveforms for subcooled boiling at 3.0 psia, subcooling: 15 deg. C.

Computed curves, Eq. (16):

- Bubble No. 24+1
- Bubble No. 25+1
- Bubble No. 29-1

Experimental values:

\[ y = 0.835 \text{ in}, \quad 0 = 0.73 \text{ in} \]
Fig. 23. Bubble Diameter vs. Time for Subcooled Boiling at Atmospheric Pressure. Subcooling: 20 deg. C.

- Computed Curves, Eq. (16)
  - Bubble No. 4-1
  - Bubble No. 38-1
  - Bubble No. 7-1

Experimental Values
Fig. 24: Comparison of Theoretical and Experimental Bubble Growth and Collapse Rates.

- Simple Acoustic theory, Eq. (19)
- Computed curve, Eq. (16)
- Rayleigh theory, Eq. (24)

○ Bubble No. 7-1, Atm. Pressure, Subcooling 20 deg. C.

△ Bubble No. 26-1, 3.0 psia, Subcooling 15 deg. C.

★ Bubble No. 64-1, Atm. Pressure, Saturated Boiling.
Plate 3. BUBBLE INDUCED PRESSURE PULSE IN SUBCOOLED BOILING AT 3 PSIA

X axis 1 div = 5 msec

Y axis 1 div = 10 mV  T: pulse period
FIG. 25. NOISE SPECTRUM OF SINGLE BUBBLES. PRESSURE: 1 ATM. SUBCOOLING: 10 DEG.C.
BUBBLE NO. 80-1
FIG. 26. NOISE SPECTRUM OF SINGLE BUBBLES. PRESSURE: 1 ATM.
SUBCOOLING: 15 DEG. C. BUBBLE NO. 82-1.
Fig. 28. Noise spectrum of single bubbles. Saturated boiling at 1.0 psia. Bubble No. 89-1.
Fig. 29. Noise spectrum of single bubbles, saturated boiling at 2.0 psia.
Bubble No. 28-A-5.
FIG. 34. NOISE SPECTRUM OF SINGLE BUBBLES. SATURATED BOILING AT 7.05 PSIA.

BUBBLE NO. 44-1.
Fig. 33. Pressure pulse amplitude as function of bubble maximum radius.

- o Atm. pr. 5 deg. c. subcooling; slope: 1.58
- v Atm. pr. 15 deg. c. subcooling; slope: 1.64
- △ 3.0 Psia. 5 deg. c. subcooling; slope: 1.68
Fig. 34. Pressure pulse amplitude as function of bubble lifetime.
- o Atm. pr. 5 deg. c. Subcooling; Slope : 0.747
- v Atm. pr. 15 deg. c. Subcooling; Slope : 0.767
- △ 3.0 psi. 5 deg. c. Subcooling; Slope : 0.77
Fig. 35. Effect of liquid pressure on energy content in the sound radiated from single bubbles.

$E_s =$ Energy in the sound pressure wave due to bubble.

$E_p =$ Potential energy of bubble at maximum radius.
Plate 4. NOISE DUE TO BUBBLE DIVISION.

BOILING AT 3 PSIA

Frame speed: 2880
FIG. 36. FREQUENCY SPECTRA FOR NO. 28 WIRE. SUBCOOLING: 95 DEG.C.

FIG. 37. FREQUENCY SPECTRA OF BOILING SOUNDS IN PFR WATER MODEL(4)
A: 7.5 W/cm²; B: 8.0 W/cm²; C: 8.7 W/cm²; D: 13.8 W/cm²; WATER
TEMP.: 60 DEG.C.
**Fig. 38. Schematic Diagram of Experimental Apparatus**

1. Boiling Vessel
2. Transducer
3. Measuring cell
4. Oxygen Electrode
5. Oxygen Meter
6. Peristaltic Pump
7. Return Lines
8. Moisture Trap
9. Oxygen Cylinder

**Fig. 39. Schematic Diagram of Recording and Analysing Equipment**
Fig. 40. Electrode for oxygen content measurement

1. Air vent
2. Electrode housing
3. Nut
4. Silver anode
5. Electrolyte
6. Membrane tube
7. Carbon cathode
Fig. 41. Effect of Heat Flux on Sound Generated in Nucleate Boiling of Water. Pressure: 4 140 in.; Subcooling: 5°C.

Oxygen content: 26 ppm; Heat Flux: A: 76 x 10^6 Btu/hr-ft^2; B: 16 x 10^6 Btu/hr-ft^2.
Fig. 42. Effect of liquid temperature on sound generated in nucleate boiling of water. Pressure: 1 atm.

Oxygen content: 12.2 ppm, heat flux: 76 × 10^3 Btu/hr ft²

Subcooling: A: 5°C, B: 25°C, C: 35°C
Fig. 43. Effect of liquid temperature on sound generated in nucleate boiling of water. Pressure: 1 atm.; oxygen content: 18 ppm; heat flux: \( 76 \times 10^3 \) Btu/hr ft²; subcooling: A: 5°C; B: 25°C; C: 35°C.
**Figure 44. Effect of Liquid Temperature on Sound generated in Nucleate Boiling of Water.**

- **Pressure**: 1 atm.
- **Oxygen content**: 18 ppm.
- **Heat Flux**: $116 \times 10^3$ Btu/hr*ft$^2$.
- **Subcooling**:
  - A: 5°C
  - B: 25°C
  - C: 35°C
Figure 45. Frequency spectra of sound generated in nucleate boiling. Pressure: 1 atm.; heat flux: 76 x 10^3 Btu/hr ft^2; subcooling: 5°C; oxygen content: A: 6.2 ppm; B: 12.2 ppm.
Fig. 45 continued. Oxygen content: C: 18 ppm; D: 21 ppm; E: 28 ppm.
Fig. 46. Frequency Spectra of Sound generated in Nucleate Boiling. Pressure: 1 atm.; Heat Flux: $76 \times 10^3$ Btu/hr ft$^2$; Subcooling: 25°C; Oxygen Content: A: 6.2 ppm; B: 12.2 ppm.
Figure 50. Frequency spectra of sound generated in nucleate boiling. Pressure: 1 atm, heat flux: 146 x 10^3 Btu/hr ft^2. Subcooling: 35°C, oxygen content: A: 0.2%, B: 0.3%.
Fig. 50a. Effect of dissolved oxygen content on cut-off frequency and rate of decay of spectra in nucleate boiling of water at atmospheric pressure.
Fig. 51. Bubble growth model used by Zuber (34)
(a) Initial temperature profile. (b) Temperature profile following growth of nucleus.
\( T_w \): Wall temperature; \( T_b \): Bulk liquid temperature.

Fig. 52. Bubble initiation model of Hsu (51)
(a) Nucleation cavity
(b) Initial temperature profile
(c) Temperature profile with time increasing
(d) Temperature profile at end of waiting time
Fig. 53. Bubble growth rates in saturated boiling of water
Pressure: 1 atm., Heat flux: $5.1 \times 10^4$ Btu/hr*ft.$^2$

$T_w - T_{sat} = 39.5 \degree F$; $N_{ja} = 65.5$

- $\circ, \times, \square$: Experimental values, present study
- $\sigma, \alpha, \psi$: Experimental values, Garg (9)
- Zuber's Theory, Eq. (6)
- Hsu and Graham, Eq. (8)
- Cole and Shulman, Eq. (11)
- Han and Griffith, Eq. (7)
- Cooper, Eq. (10)
Fig. 54. Bubble growth rates in saturated boiling of water.
Pressure = 701 psia. Heat flux = $2.7 \times 10^4$ Btu/hr ft².
$T_{sat} = 400°F$; $N_A = 132.0$

\[ \begin{align*}
\text{o, A, D, x} & \quad \text{Experimental values, present study} \\
\text{v, o} & \quad \text{Experimental values, Garg (44)} \\
\text{---} & \quad \text{Zuber's theory, Eq. (6)} \\
\text{---} & \quad \text{Hsu and Graham, Eq. (8)} \\
\text{---} & \quad \text{Cole and Shulman, Eq. (11)} \\
\text{---} & \quad \text{Computer solution, Eq. (16)} \\
\text{---} & \quad \text{Han and Griffith, Eq. (9)} \\
\text{---} & \quad \text{Cooper, Eq. (10)}
\end{align*} \]
Fig. 55. Bubble growth rates in saturated boiling of water.
Pressure: 3,000 psi, Heat flux: 1.6 x 10^6 Btu/h ft^2, \( T_{vub} = 35.5^\circ F \),
\( N_2 = 256.0 \)

- \( \delta, \Delta, x \) Experimental values, present study.
- \( y, \sigma \) Experimental values, Garg (44)
- Zuber's Theory, Eq. (6)
- Hsu and Graham, Eq. (8)
- Cole and Shulman, Eq. (11)
- Han and Griffith, Eq. (9)
- Cooper, Eq. (10)
FIG. 56. BUBBLE GROWTH RATES IN SATURATED BOILING OF WATER
PRESSURE: 1.0 psia. HEAT FLUX: 5,310 ft-lb/hr ft².
T_w - T_sat = 44.0°F. N/J = 880,000.

- Experimental values, present study
- Experimental values, Garg (44)
- Zuber's theory, Eq. (6)
- Hsu and Graham, Eq. (8)
- Cole and Shulman, Eq. (11)
- Han and Griffith, Eq. (9)
- Cooper, Eq. (10)
Fig. 57. Co-ordinate systems for determination of temperature profile around cylinder
(a) Physical system  (b) Transformed system

Fig. 58. Numerical scheme for the solution of equations (16) and (17)
Fig. 59. Temperature distribution around heated tube in water. 
Pressure: 7.0 psia, $\theta$: 0 deg.
- Distribution including convection
- Distribution neglecting convection