Presupposition and Assertion in Dynamic Semantics

Part (II) The Assertion: What Comes First in Dynamic Semantics

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Abstract

This work concerns presupposition in natural language, which is here understood as a mechanism whereby features of an utterance indicate that the speaker assumes certain facts as common knowledge. The thesis is divided into two parts. The first part introduces the basic concepts (assuming some at least good undergraduate level knowledge of syntax, semantics and logic, but no specific knowledge of presupposition theory), and critically reviews previous theories of presupposition. Various of the approaches discussed appeal to scope ambiguity (e.g. Russell), to Gricean argumentation (e.g. Atlas, Gazdar, Kempson, van der Sandt and Wilson), to partiality and trivalence (e.g. Burton-Roberts, Strawson, Seuren and Link), to an extra presuppositional dimension of evaluation (e.g. Herzberger and Karttunen and Peters), to a mechanism allowing presuppositions to be canceled (e.g. Gazdar, Mercer and van der Sandt), to sentence internal dynamics (e.g. Karttunen, Heim, van der Sandt, Zeevat), and to accommodation, the process whereby linguistic contexts are augmented to ensure that presuppositions are accepted (e.g. in theories of Heim, van der Sandt, Fauconnier and Zeevat). The application of these methods is discussed in detail.

In the second part of the thesis, the Context Change Potential (CCP) model of presupposition, due primarily to Karttunen and Heim, is formally elaborated and modified within a propositional dynamic logic, a quantificational dynamic logic, and within three sorted type theory. It is shown that the definitions of connectives and quantifiers can be motivated independently of the phenomenon of presupposition by consideration of the semantics of anaphora and epistemic modality, and that these independently motivated definitions provide a solution to the projection problem for presupposition. It is argued that with regard to the interaction between presupposition and quantification the solution is empirically superior to those in competing accounts. A semantics which integrates dynamic treatments of quantification, anaphora, modality and presupposition is then specified for a fragment of English. The semantics is compositional, and follows the spirit of Montague’s approach although it differs considerably with respect to the details of his analysis. A formal model of global accommodation is proposed which is essentially dependent on world-knowledge and common sense reasoning. It is shown to provide both a general solution to the problem of the informativeness of presuppositions, and a specific solution to a problem within the CCP model, namely its tendency to yield inappropriately weak conditionised presuppositions. It argued that the model provides superior predictions to any purely semantic theory of presupposition, and to any existing theory of accommodation or cancellation.
Dedication

This thesis is dedicated to my parents Allan and Muriel, and my sisters Alexis and Michelle. Without them, semantics would have been meaningless.
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# Contents

1 Introduction .......................................................... 9
   1.1 Overview of Part I ........................................... 10
   1.2 Overview of Part II ......................................... 11

I The Presupposition: A Critical Review of Presupposition Theory 15

2 Presuppositions and How to Spot Them ............................ 16
   2.1 Projection/Heritability ...................................... 19
   2.2 Cancellation/Defeasibility .................................. 21
   2.3 Application and Interpretation of Presupposition Tests .... 24
   2.4 Formal Models of Projection ................................ 26
   2.5 Are Presuppositions Conventional? .......................... 28

3 Multivalence and Partiality ......................................... 37
   3.1 Trivalent Accounts .......................................... 39
   3.2 Two Dimensions ............................................. 45
   3.3 The Need for Pragmatics .................................... 48
      3.3.1 Negation .............................................. 48
      3.3.2 Disjunction .......................................... 49
      3.3.3 Conditionals and Conditionalised Presuppositions .... 51
      3.3.4 Pragmatic Extensions ................................ 56

4 Cancellation and Filtering .......................................... 57
   4.1 Plugs, Holes and Filters .................................... 58
      4.1.1 The First Version of Local Filtering ................. 58
      4.1.2 The Revised version of Local Filtering .............. 59
   4.2 Global Cancellation ......................................... 61
      4.2.1 Cancellation without Implicatures .................... 62
      4.2.2 Adding Implicatures .................................. 63
4.3 Projecting By Default ........................................... 64
4.4 The Pre- in Presupposition ..................................... 66
4.5 Presupposition and Entailment ................................ 69
4.6 Combining Cancellation and Filtration ....................... 73
4.7 Quantification in Cancellation and Filtering Models ........... 76
4.8 Conditional Presuppositions .................................... 77

5 Dynamic Semantics ................................................. 82
5.1 From Projection to Satisfaction .................................. 84
5.2 Context Change Potential ....................................... 88
5.3 Quantification in the CCP model ............................... 90
5.4 Projection from Propositional Complements .................... 92
5.5 Anaphoricity ..................................................... 93

6 Accommodation ...................................................... 101
6.1 Heim and van der Sandt ........................................ 102
6.2 The Cancellationist Heritage ................................. 110
6.3 Accommodation of Unbound Presuppositions ................ 112
6.4 The Partial Match .............................................. 113
6.5 Intermediate Accommodation ................................. 115
6.6 Structural Accommodation and Conditional Presuppositions .... 118
6.7 Anaphora from Accommodated Material ....................... 122
6.8 Alternative Accommodation Strategies ....................... 124
6.8.1 From Amputation to Accommodation ...................... 124
6.8.2 The Transformation from Russell to van der Sandt ....... 125
6.8.3 Accommodation as a Journey through Mental Space ....... 127
6.9 Accommodation and Projection ................................ 131

II The Assertion: What Comes First in Dynamic Semantics .......... 132

7 Two Birds and One Stone ........................................... 133
7.1 Aim ......................................................... 133
7.2 Descriptive versus Explanatory Adequacy ..................... 136
7.3 Hide and Seek with Epistemic Modalities .................... 137
7.4 Update Logic (UL) ............................................. 140
7.5 A Presupposition Operator ..................................... 147

8 A Bit Like English ................................................ 153
8.1 Introduction ..................................................... 153
Chapter 1

Introduction

Russell once commented [Ru46, p.212] that all the significant advances in philosophy have been made in the teeth of opposition from the disciples of Aristotle. The introduction of the notion of presupposition is no exception:

“For manifestly, if Socrates exists, one of the two propositions ‘Socrates is ill’, ‘Socrates is not ill’ is true, and the other false. This is likewise the case if he does not exist; for if he does not exist, to say that he is ill is false, and to say that he is not ill is true.” (From Aristotle’s Categories [Ar50, Ch.10:13b pp. 27-35])

“That the name ‘Kepler’ denotes something is just as much a presupposition for the assertion ‘Kepler died in misery’ as for the contrary assertion.” (From Frege’s On Sense and Meaning [Fr84b, p.168])

Russell, who clearly enjoyed being in the right, in one respect had helped engineer the truth of his own observation. After his criticism in [Ru05] of Frege’s radical departure from Aristotelian dogma, presupposition seems to have been largely forgotten, and when, a half a century later, Strawson rediscovered the notion, Russell was once more on hand to fight a rearguard action.¹ That this action, although fairly successful, did not

¹It has been suggested by Larry Horn that the mediaeval philosopher Petrus Hispanus (possibly the only logician ever to have become pope) should be credited with the first introduction of a notion of presupposition. Horn (noting that the reference differs from an erroneous citation in [Horn85]) pointed out to me the following passage from around line 100 of Petrus Hispanus’ Tractatus Expositiolium, this tract being found in translation (and original) in [Mul45]: “Prima est quod dicitio reduplicativa praeconposit praequod praedicatum inequ aliqii subiecto et denotat quod illud super quod adit immediate sit causa inhaerentiae illius.” The praeconposit/denotat distinction made here does not, however, appear to play a central role in Petrus’ philosophy. A big topic of research three quarters of a millennium ago concerned what was called suppositio, but despite the superficial morphological similarity, this field has no obvious connection with the study of presupposition. Suppositio is related to supponere, meaning to substitute, and this medieval field is more closely connected with paradoxes and problems of
completely halt the further development of the theory of presupposition is attested by Sag and Prince's collected bibliography of works dealing with presupposition in [O&D79], which contains more than three hundred items.

1.1 Overview of Part I

In the fifteen years since the Sag and Prince bibliography was collated, work on presupposition has continued apace, so that the discussion in this thesis does not and could not provide a comprehensive survey of the voluminous existing literature. But the five chapters in Part I of this thesis are devoted to a critical examination of at least a significant part of that literature. It is to be hoped that this part of the thesis will at least provide the mathematically inclined reader with a grasp of empirical and linguistic issues, and the linguistically inclined reader with a grasp of relevant formal methods and logical issues: the aim is thus not to summarise the existing literature, but to make it more accessible to logicians and linguists alike.

The first chapter in Part I is largely introductory, tackling the basic concepts and setting the methodological scenes. In the four chapters thereafter some of the more influential formal attempts to explain presupposition related inferences are outlined and discussed. The grouping of proposals cannot in any sense be standard, because it is not obvious whether there is any standard taxonomy of different theories. The division of theories across chapters 3–6 does not reflect the philosophical interpretation which the original progenitors advocated, but is fixed by my own perspective on the technical apparatus underlying the presuppositions. The division loosely represents four different ways in which the dynamics of the interpretation process is used to explain presupposition projection. Here is a summary (some of the technical terms mentioned in the summary have not yet been introduced):

Chapter 2: “Presuppositions and How to Spot Them” Basic concepts and methods are introduced, and some of the underlying assumptions of presupposition theories are discussed.
Chapter 3: “Multivalence and Partiality” In this chapter models are presented in which the dynamics of the interpretation process plays no role at all. In these purely static theories the possibility of presupposition failure is tied to the presence of extra truth values in a multivalent (or partial) semantics.

Chapter 4: “Cancellation and Filtering” Here we discuss models in which the context of evaluation influences which presuppositions are projected. Such models can involve an *inter-sentential dynamics* or *dynamic pragmatics* since the context of evaluation is modified with each successive utterance, although this intersentential dynamics is generally relatively simple and involves only incrementation with new propositions.

Chapter 5: “Dynamic Semantics” In these models presuppositions constrain the local contexts in which a presupposition trigger is admissible. These theories involve not only incrementation of context with successive sentences, but also *sentence internal* dynamics. For a given context of evaluation for a complex sentence the dynamics of the interpretation process determines what the local contexts of evaluation will be for the parts of the sentence, and given that the parts are only admissible in certain contexts, this in turn determines whether the sentence as a whole is admissible.

Chapter 6: “Accommodation” Accommodation based theories of presupposition allow for a much more sophisticated dynamic pragmatics than in the earlier chapters. This dynamics manifests itself in a process of accommodation. This allows repair or modification of contexts of evaluation so that presuppositions are justified, but the thus modified contexts may be local contexts of evaluation rather than the global context in which the sentence as a whole is interpreted.

1.2 Overview of Part II

In ordinary talk, we presuppose a great deal. That is, we take certain things for granted, assuming that the other conversational participants share our knowledge of them. This can make communication efficient in two ways. Firstly, we can limit our conversational goals to communicating only things we do not believe are already shared knowledge. Secondly, when attempting to meet those goals, we can use the knowledge that we take to be shared in order to help us express what we want to say in a concentrated form. In this thesis I will follow the many of the researchers whose work is discussed in Part One of this thesis by accepting that the structure of language reflects this difference between what we take for granted and what we actually intend to communicate. On this view there are lexical items and syntactic structures which either as part of their
communicative function, or in order to achieve their communicative function, signal that certain propositions have been presupposed by the speaker. In the second part of the thesis I will study how this view should be realised in a formal model of utterance interpretation. I will limit myself to assertive speech acts, and do not make any formal proposals about the interpretation of other types of utterance, such as questions or imperatives. Given this simplifying restriction, utterance meaning may be broken down into two parts, the presupposition and the assertion. The question then arises as to how this dichotomy, between information which a speaker takes as shared and information which a speaker intends to communicate, should be integrated into a theory of meaning.

The last few years have seen a shift of emphasis in the study of semantics. The traditional job of the natural language semanticist involved relating sentences to truth conditions, or to functions from certain contextual parameters to truth conditions. However, much recent work has concentrated on how the process of understanding itself helps to determine the relevant contextual parameters, and thus to determine the truth conditions. To some researchers, truth conditions have become secondary, the primary object of study being the way in which context changes during language processing. In other words, there has been a shift from a static conception of meaning, through a contextually sensitive but still essentially static conception, leading (finally?) to a radically dynamic view. At the same time as this philosophical shift has occurred, there has also been a tendency for semanticists to import formal approaches to modeling dynamics from the discipline of theoretical computer science.

Heim's Context Change Potential (CCP) model of presupposition, which develops ideas of Stalnaker and Karttunen, is a dynamic account of meaning par excellence, but its genesis preceded recent technical advances in dynamic semantics. In Part Two of the thesis, I will elaborate and defend the CCP model. I will show that by taking advantage of recent technical developments, most of the outstanding problems with that model can be overcome, and I will propose a way in which the CCP account of sentence presupposition may be formally combined with an account of the inferential processes which hearers use when determining the assumptions of the speaker. Furthermore, dynamic theories of presupposition, anaphora, quantification and epistemic modality will be integrated into a single compositional grammar fragment. The reason for attempting this integration is twofold. Firstly it is worth providing an integrated theory with wide empirical coverage just to show that this can be done, and thus that the various theories are compatible. Without such integration, the community of researchers in dynamic semantics would be open to the accusation that they shared only an allegiance to jargon, and no deeper common view on the nature of meaning. Secondly, the essential details of

\[^{3}\text{However, in Chapter 11 there will be some discussion of extensions of the formal model which would apply to presuppositional phenomena occurring in speech acts of denial}^\]
the theory of information underlying the CCP model can be motivated independently of the study of presupposition, and this can only be demonstrated within an integrated theory that incorporates both presupposition and various other dynamic phenomena.

The formal developments will take place within the general framework of dynamic semantics which has been laid out by such researchers as van Benthem, Dekker, Groenendijk and Stokhof, and Veltman. In a dynamic semantic account, sentences of formal and natural languages are understood as providing a means of updating the information state of some agent, or as instructions for an agent to use in order to update. Given a simplified formal model of an information state, it is possible to specify exactly which sentences of a precisely defined formal language correspond to which instructions. A notion of logical consequence can then be defined not in terms of truth or falsity of sentences in a model, but in terms of the information which sentences can provide an agent. The relevance of such notions of consequence to natural language may be demonstrated by providing dynamic semantic interpretations for formal languages which form significant fragments of natural language, and showing that the consequence relation accords with empirical data concerning implications between sentences of natural language. This is essentially the path taken in the first four of the six chapters comprising Part II of the thesis, with the remaining two chapters serving to draw some formal and empirical comparisons with the approaches to presupposition discussed in Part One. Here is a summary of what is to come:

Chapter 7: "Two Birds and One Stone" Some simple (purely propositional) dynamic systems are presented. It is shown how the context sensitivity of epistemic modality can be modeled in a propositional dynamic logic, and how this same system can be adapted to account for presupposition data.

Chapter 8: "A Bit Like English" A dynamic semantics for predication, quantification and anaphora within a first-order language, ABLE, is developed.

Chapter 9: "Presupposition and Modality" The ABLE semantics is further refined. The refinement is shown to solve some difficult problems occurring in the interaction between quantification and modality on the one hand, and quantification and presupposition on the other.

Chapter 10: "Let's Get Real" It is shown how the semantics developed for ABLE can be used as the basis of a compositional analysis of English, within a system I refer to as Kinematic Montague Grammar (KMG). It is then demonstrated that an adequate treatment of presupposition accommodation must involve a complex interaction between world knowledge and compositionally derived meaning. A formal model of global accommodation based on an extension to KMG is devel-
oped, and it is shown that this model has the potential to account for empirical observations which are problematic for other theories of presupposition.

Chapter 11: "Connections and Directions" Some technical connections are made between the theory developed here and some of those theories described in Part One of the thesis. These connections are shown to open up some promising directions for further research, suggesting solutions to problems both within the theory developed here and within pre-existing theories of presupposition.

Chapter 12: "Conclusion" Coverage of the model proposed here, as compared with that of other models, is summarised, and a final assertion is made.
Part I

The Presupposition: A Critical Review of Presupposition Theory
Chapter 2

Presuppositions and How to Spot Them

The non-technical sense of the word *presupposition* serves as a good basis for understanding many of the various technical definitions which have been given. Certainly this is true of the notion of presupposition introduced by Frege, according to whom presuppositions are special conditions that must be met in order for a linguistic expression to have a denotation. He maintained that presuppositions constitute an unfortunate imperfection of natural language, since in an ideal language every well-formed string would denote something. The possibility of what we would now call *presupposition failure*, which in a Fregean picture would mean cases when a well-formed expression failed to denote, was repugnant to him. He cites, as well as the 'Kepler' sentence quoted earlier, the following examples (all from Frege [Fr84a]) :

E1 Whoever discovered the elliptic form of the planetary orbits died in misery.

E2 After the separation of Schleswig-Holstein from Denmark, Prussia and Austria quarreled.

For Frege, the propositions, respectively, that the name 'Kepler' designates something, that somebody discovered the elliptic forms of the planetary orbits, and that Schleswig-Holstein was once separated from Denmark, are not part of the "thoughts expressed" by the above examples, but are presupposed by them. An important aspect of Frege's separation of *sense* and *reference* is his contention that linguistic expressions may have a sense, yet lack a reference. This aspect of the Fregean scheme is often ignored, and, for instance, is absent from Richard Montague's otherwise essentially Fregean PTQ [Mo74b]. Thus for Frege noun-phrases like those underlined in E1–E2 could have failed

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1It is not obvious why Montague chose against fully implementing this part of Frege's theory. The possibility of propositional formulae having sense but no reference would have complicated the (already
Presuppositions and How to Spot Them

to have any reference, at least if the world had been different and there had not been any relevant entities to which they could refer. In this case the sentences containing the noun phrases would themselves have had a sense but no reference, which, given that the Fregean reference of a sentence is the True or the False, would have meant that the sentences did not have a truth value.

Most authors follow the Fregean line of relating presuppositions to assumptions that have been made, thus as concerning either the way in which utterances signal assumptions, or, conversely, the way in which utterances depend upon assumptions to be meaningful. However, some words of caution are in order. It is not the case that all technical uses of the term presupposition involve reference to assumptions. Indeed, if by assumptions we mean the assumptions of some agent, then the notion of an assumption is essentially a pragmatic one, whereas for some theorists presupposition is a purely semantic relation. Thus phenomena that one theorist explains in terms of what is assumed, another may explain without essential reference to assumptions, and yet both theorists may use the term presupposition. It is not even the case that all proponents of pragmatic accounts of presupposition take assumption as a central notion. For instance, Gazdar’s influential theory of presupposition [Gaz79a, Gaz79b] does not involve a commitment to presuppositions being in any sense assumed.

Having mentioned the terms semantic and pragmatic, I must warn the reader that they are bandied about rather freely, and indeed confusingly, in the presupposition literature: I will attempt to clarify.

In a semantic theory presupposition is usually defined as a binary relation between pairs of sentences of a language. What makes this relation semantical is that it is defined or explicated solely in terms of the semantic valuation of the sentences, or in terms of semantical entailment. Thus a definition in terms of semantic valuation might, following one interpretation of Strawson’s work, say that one sentence (semantically) presupposes another if the truth of the second is a condition for the semantic value of the first to be true or false. Other such notions will be explored in Chapter 3 below.

In pragmatic theories the analysis of presupposition involves the attitudes and knowledge of language users. In extreme cases such as Stalnaker’s [St74] account, presupposition is defined without any reference to linguistic form: Stalnaker talks not of the presuppositions of a sentence, but of the speaker’s presuppositions, these being just
those propositions which are taken for granted by a speaker on a given occasion. Other
pragmatic theories are less radical, in that linguistic form still plays an essential role
in the theory. The majority of well-developed pragmatic theories concern the presup-
positions not of a sentence (as in semantic theories) or of a speaker (as in Stalnaker's
theory) but of an utterance. In some theories, utterances are explicated as pairs con-
sisting of a sentence and a linguistic context, and as a result presupposition becomes a
ternary relation, holding between two sentences and a context.\(^2\) In other theories, the
presuppositions of a sentence are seen as conditions that contexts must obey in order
for an utterance of the sentence to be felicitous in that context.\(^3\)

The post-Fregean philosophical study of presupposition has been dominated by an
assumption-based conception, but, given the range of linguistic and philosophical theo-
ries which have been formulated during the last twenty years, such a characterisation is
no longer apt.

Furthermore, saying that presuppositions are not part of what is asserted but of what
is assumed does not in itself provide any practical method of identifying presuppositional
constructions in language, or even of showing that there are any such constructions. If
one theorist argues that a definite description asserts the existence of a (unique) object
satisfying the description, and another theorist maintains that the existence of a relevant
object is not asserted but presupposed, how are we to tell who is right? This issue was
at the heart of the famous Russell-Strawson debate. Neither party could offer a solid
empirical justification of his position, since the debate appeared to hinge on whether
a simple sentence containing an unsatisfied description was false, as Russell claimed,
or meaningless, as Strawson, taking his lead from Frege, maintained. Judgements on
whether sentences are meaningless or false are typically hazy — indeed, it is hard even
to know how to pose to a naive informant the question of whether a given sentence is
meaningless or false — and the debate arguably never reached a satisfactory conclusion.\(^4\)

So what is the defining characteristic of the recent linguistic study of presupposition?
We will see that a large class of lexical items and grammatical constructions, including
those identified as presuppositional by philosophers such as Frege and Strawson, produce

\(^2\) Strawson's account can be seen as the first such theory, although the Frege's sparse remarks on
presupposition are already suggestive. See [St50] and the reconstruction in [So89]. Chapter 4 introduces
a number of such theories, and it is there suggested that (the second version of) the theory in [Kar73]
is the first in which a definition of utterance presupposition is formally realised.

\(^3\) Keenan [Kee71, p. 49] defines pragmatic presupposition as follows: "A sentence pragmatically
presupposes that its context is appropriate." On the other hand Karttunen writes: "Strictly speaking,
it would be meaningless to talk about the pragmatic presuppositions of a sentence. Such locutions are,
however, justified in a secondary sense. A phrase like "the sentence A pragmatically presupposes B" can
be understood as an abbreviation for "whenever A is uttered sincerely, the speaker of A presupposes B"
(i.e. assumes B and believes that his audience assumes B as well.)" [Kar73, pp.169-170]

\(^4\) The main references for this debate are Strawson's [St50, St64], and Russell's [Ru05, Ru57]. Note
that the 1964 Strawson paper is quite conciliatory.
distinctive patterns of inference. It is difficult to find any common strand to current analyses of presupposition, save that they all concern (various parts of) this class.\textsuperscript{5}

2.1 Projection/Heritability

Taking $\models$ for the moment to be classical semantic entailment (also termed \textit{necessitation} or \textit{consequence}), it does not follow from $\phi \models \psi$ that $\neg \phi \models \psi$, and, for an arbitrary choice of $\chi$, it does not follow that $\phi \rightarrow \chi \models \psi$. Neither does it follow that $\diamond \phi \models \psi$, where $\diamond$ is a classical modal possibility operator. These properties of classical entailment mirror the standardly recognised properties of inference between sentences of natural language. For example, Frege's E1, repeated below\textsuperscript{6}, has E3 as one of its implications, and it is no surprise that E3 does not follow from any of E4–E6.

E1' Whoever discovered the elliptic form of the planetary orbits died in misery.

E3 Somebody died in misery.

E4 Whoever discovered the elliptic form of the planetary orbits did not die in misery.

E5 If whoever discovered the elliptic form of the planetary orbits died in misery, he should have kept his mouth shut.

E6 Perhaps whoever discovered the elliptic form of the planetary orbits died in misery.

However, consider E7, which Frege claims to be presupposed by E1. Strikingly, E7 seems to be implied by E1, but also by all of E4–E6. We may say that one implication

\textsuperscript{5}The following list contains many of the constructions that are commonly identified as presuppositional:

- Definite Noun Phrases
- Quantificational Noun Phrases
- Factive verbs
- Factive nouns
- Clefts
- Counterfactual Conditionals
- 'Non-neutral' intonation
- Aspectual verbs
- Aspectual adverbs
- Iterative adverbs
- Sortally Restricted Predicates
- Questions

\textsuperscript{6}When repeating examples I will follow a convention of adding an apostrophe to the original numbering, so that e.g. E1 becomes E1'.

 architects with expertise in the field of psychology.
of E1 is inherited or projected such that it also becomes an implication carried by the complex sentences in E4–E6, whereas another implication of E1 is not inherited in this way.

E7. Somebody discovered the elliptic form of the planetary orbits.

This takes us to the curse and the blessing of modern presupposition theory. Certain implications of sentences are inherited more freely to become implications of complex sentences containing the simple sentences than are other implications, and such implications are called presuppositions. In its guise as curse this observation is called (following Langendoen and Savin [LS71] the presupposition projection problem. The problem is twofold. Firstly we must say exactly when presuppositions are inherited, and secondly we must say why. But the observation is also a blessing, because it provides an objective basis for the claim that there is a distinct presuppositional component to meaning, and a way of identifying presuppositional constructions, a linguistic test for presupposition on a methodological par with, for instance, standard linguistic constituency tests.

To find the presuppositions of a given grammatical construction or lexical item, one must observe which implications of simple sentences are also implications of sentences in which the simple sentence is embedded under negation, under an operator of modal possibility or in the antecedent of a conditional. To be sure, there is nothing sacred about this list of embeddings from which presuppositions tend to be projected, and the list is certainly not exhaustive. The linguist might equally well choose to consider different connectives, such as in E8, or non-assertive speech acts, as with the question in E9 — questions having been considered as test-embeddings for presuppositions by Karttunen — or the imperative in E10. E9 is not a question about whether anybody discovered elliptic form of the planetary orbits, and E10 does not act as a request to guarantee that somebody has discovered the elliptic form of the planetary orbits. Rather, we would take it that an utterer of either of these sentences already held the existence of a discoverer of the elliptic form of the planetary orbits to be beyond doubt. Thus the sentences could be used as evidence that E7 is presupposed by the simple assertive sentences from which E9 and E10 are derived.8

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7 The behaviour of presuppositions in imperatives is discussed by Searle [Sea:69, p. 162].
8 Burton-Roberts suggests the following generalisation of the standard negation test for presuppositions: “Any formula equivalent to a formula that entails either p or its negation, and the negation of any such formula, will inherit the presuppositions of p.” [Bu89b, p.102] Such a generalisation seems problematic. For if we allow that a contradiction entails any sentence, then it follows that a contradiction presupposes everything. But any tautology is standardly equivalent to the negation of a contradiction, so all tautologies must presuppose everything. Further, if a tautology is entailed by any other sentence, it immediately follows that every pair of sentences stands in the relation of presupposition. I fear Burton-Roberts presupposes too much.
Presuppositions and How to Spot Them

E8 Unless whoever discovered the elliptic form of the planetary orbits died in misery, he was punished in the afterlife.

E9 Did whoever discovered the elliptic form of the planetary orbits die in misery?

E10 Ensure that whoever discovered the elliptic form of the planetary orbits dies in misery!

Returning to projection qua problem rather than qua test, it is often forgotten that, from a semantic perspective, the projection problem for presuppositions fits quite naturally into a larger Fregean picture of how language should be analysed. The projection problem for presuppositions is the task of stating and explaining the presuppositions of complex sentences in terms of the presuppositions of their parts. The larger problem, which strictly contains the presupposition projection problem, could naturally be called “the projection problem for meanings”, i.e. the problem of finding the meanings of complex sentences in terms of the meanings of their parts. Of course, this larger problem is conventionally referred to as the problem of compositionality.

2.2 Cancellation/Defeasibility

The projection test is dependent upon a source of data which has been central to semantical enquiry since Aristotle, namely our intuitions concerning which pairs of sentences stand in the relation of implication. But the notion of implication is not always one we can take for granted, especially when presuppositions are involved.

In the first place, and specifically with regard to examples E9 and E10, we must be careful when talking about the implications of a non-assertoric speech act. But if we say that ‘A implies B’ means that any utterance of A indicates that the speaker believes B, then it is safe to say that both E9 and E10 imply E7, but do not imply E3.

A more serious problem is that many of the inferences on which we base the identification of a presupposition have an worryingly will-o-the-wisp character: if you look hard at them, they sometimes disappear. Suppose that we wished to back up Strawson’s position in his debate with Russell by showing that there was an identifiable presuppositional component to the meaning of a sentence containing a definite description, and that this component behaved quite differently from ordinary assertions. We might begin by considering embedding under negation of a simple sentence, e.g. the locus classicus of presupposition theory E11:

E11 The King of France is bald.

E12 The King of France is not bald.

E13 Somebody is bald.
E14 There is a king of France.

Asking naive informants whether E14 follows from E12 produces a mixture of bemused looks, positive and negative replies. Such a result is hard to interpret: it seems plausible the world knowledge of the informants, the fact that they know there is no King of France, is affecting the way the informants answer. To factor out the effect of world knowledge, one may follow Gazdar [Gaz79a] and replace ‘France’ with an unfamiliar country name, such as ‘Buganda’, before re-asking the question. This has a quantitative rather than qualitative effect, increasing the proportion of respondents who will attest that (the Bugand-ised version of) E14 follows from (the Bugand-ised version of) E12. It would be a brave scientist who, on the strength of such results, would assert that the negative sentence entails the existence of an object satisfying the definite description. Even more problematically for one who wished to assert the presence of such an entailment, it is easy to invent contexts where a Bugand-ised version of E12 might be uttered without any commitment to the existence of a Bugandan king:

E15 A: The King of Buganda is bald.
B: Come now A, Buganda is not even a monarchy.
A: OK, I was wrong then. The King of Buganda is not bald.
Perhaps it was the president I was thinking of?

Many presupposition theorists argue that cases like E15 are somehow deviant or exceptional. For instance, it has often been claimed that there are not one, but two negations in English (and presumably other languages), and thus that the occurrence of E12 in the discourse E15 involves a different negation to the more everyday truth conditional negation. Others argue for E15 being exceptional on the basis of its highly marked intonation contours. If E15 is highly marked, then even without explaining why speaker A need not be committed to E14 after E15, it might still be possible to maintain that utterances of E12 with neutral stress (e.g. no deaccenting, no mid-sentence focal stress) imply the existence of an appropriate king.

Others, especially since Kempson [Kem75] and Wilson [Wi75], have taken the defeasibility or cancellability of presuppositions to be one of their defining characteristics. But rather than being disturbed by the tendency of presuppositional implications to disappear in certain contexts, some theorists would regard such behaviour as grist to the mill of one trying to defend the existence of a distinct presuppositional realm of meaning. Presuppositional implications are defeasible, they argue, whereas ordinary entailments are not. This is certainly a justifiable position when one considers in detail the data on which the identification of presuppositions is based, for occasional disappearance of claimed presuppositional implications is not restricted to sentences involving negation. Given any characteristically presuppositional implication, that is, an implication of a
sentence which appears to remain as an implication in various embeddings, it is usually possible to find contexts in which utterance of the complex embedding does not yield the purported presupposition as an implication. For example, the first of the two following monologues contains an embedding of 'the King of Buganda' in the antecedent of a conditional, and the second contains the same definite noun phrase in a question, and yet in neither case would we think that an utterer of the monologue was committed to the existence of a king:

E16 I don't know for sure whether the King of Buganda is alive, although I've heard it rumoured that he is 90 and completely bald. I have only one comment on the issue. If the King of Buganda is bald, then that must be somebody else inspecting the troops.

E17 You maintain that Buganda has a king, and that he's a recent ex of yours. Well, as it happens I saw the Bugandan head of state opening parliament. I'll ask you just one question about him, and if you can answer correctly and without hesitation, I'll grant that you're right. Is the King of Buganda bald?

All of the following examples may be viewed as cases where a presupposition is canceled:

E18 If Mary's married then her husband must be very tolerant.

E19 If Jack has children, then all of Jack's children are bald.
[Kar73]

E20 You say that somebody in this room loves Mary. Well, it isn't John who loves Mary, and it certainly isn't Sandy who loves her, (and so on, everybody in the room being enumerated). So, nobody in the room loves Mary!
Example adapted from [Kee71]

E21 If I realise that I was wrong I'll tell everybody.
Example adapted from [Kar71]

E22 (Teacher to Pupil) I wasn't aware that you were allowed to smoke behind the bicycle sheds!

E23 If the King of France is bald, then I'm a Dutchman: there is no King of France.

E24 If Kennedy knows the war is over, the war is over!
[Gaz79a]

E25 Nobody has yet discovered that protons are influenced by the CIA!
[Gaz79a]
Presuppositions and How to Spot Them

E26 (A man is seen in the park crawling around the bushes and whistling.) If he's lost his dog, that would explain his strange behaviour.
Example adapted from [Kay92]

E27 (Same situation.) He's either lost his dog or he's lost his mind!

E28 (Same situation.) Perhaps he's lost his dog?

Even if the occasional disappearance of claimed presuppositional implications does not force us to give up analysing various implicational properties of language in terms of presuppositions, it at least forces us to be careful when defining tests for presuppositional constructions. It will not do simply to say "a sentence S has a presuppositional implication P if any utterance of S or of various (listable) complex sentences containing S shows that the speaker believes P". It is not easy to give a simple restatement of the identification conditions for presuppositions, one which allows for defeasibility. A rough and ready reformulation might run along the lines of "... utterances of S or of various (listable) complex sentences containing S where the linguistic context provides no relevant information about the speakers attitude towards P, provide evidence that the speaker believes P".

2.3 Application and Interpretation of Presupposition Tests

There are many grammatical constructions which cannot easily be embedded as the tests demand. For example, texts consisting of several sentences cannot be embedded under negation, in the antecedent of a conditional or under a modality. Do we want to conclude from this that texts do not have presuppositions? It seems more natural to remain agnostic, accepting that we cannot directly obtain data about the presuppositions of texts and thus that the consequences of any particular theory regarding the presuppositions of texts are untestable. Similar remarks apply to non-assertoric speech acts. It is sometimes suggested that a wh-question presupposes the existence of an object satisfying the properties predicated of the wh-element, yet such a claim is difficult to test: what is the negation of a question? (‘What is not the negation of a question?’ ?)

For another example, consider presupposing polarity items (discussed by van der Sandt in his thesis): sentences containing the positive polarity item (PPI) still are hard to negate (at least without the feeling of denial, which is just what we want to avoid), and on the other hand a sentence containing the negative polarity item (NPI) any more normally is negative, so that further negation is highly marked.9

9In broad terms, an NPI is a word or phrase that tends to occur only in non-positive contexts (e.g.
The following are all cases where a naive application of presupposition tests yields bogus presuppositions. The (a) sentence of E29, for example implicates the (c) sentence, that Mary did not eat one of the two bananas, which is also entailed by the negation of (a), in (b). The remaining triples are all based on scalar implicatures.

E29
a. Mary ate one of the two bananas
b. Mary didn’t eat one of the two bananas
c. (Mary didn’t eat one of the two bananas)

E30
a. Mary’s total assets are worth $10.
b. Mary’s total assets are not worth $10.
c. Mary’s total assets are not worth $11.

E31
a. Jane submitted one abstract in her entire academic life.
b. Jane did not submit one abstract in her entire academic life.
c. Jane did not submit two abstracts in her entire academic life.

E32
a. It is possible that Bill is happy.
b. It is not possible that Bill is happy.
c. Bill is not necessarily happy.

E33
a. I think Bill is happy.
b. I don’t think Bill is happy.
c. I don’t know Bill is happy.

So called scalar implicatures are thought of as being generated in the presence of a linguistic scale. Such a scale is found whenever two expressions have similar distributional properties, but a simple sentence involving the first is logically stronger than the sentence with the second expression substituted for the first. For instance, ‘know’ might be analysed as higher on a scale than ‘believe’ (which may be notated ‘know’>‘believe’) since E33a is strictly entailed by ‘I know Bill is happy’. Generally, given a scale such that $A < B$ then if $S$ contains $A$, $S$ implicates the negation of $S$ with $A$ replaced by $B$, which we may write ‘Not S[A\B]’\(^\text{10}\), and ‘Not S’ also carries this entailment. Therefore, one

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\(^{10}\)Here, and in future, expressions of the form ‘Not A’ are used to denote the negation of A, it being assumed irrelevant how that negation is realised unless specifically indicated to the contrary.
standard presupposition test indicates that for any such $B > A$ and $S$ containing $A$, $S$ presupposes 'Not $S[A\setminus B]$'.

As a final type of case where tests indicate what might be seen as a bogus presupposition, consider parentheticals, as in the following example drawn from the Guardian newspaper:

E34 Sweden may export synthetic wolf urine — sprayed along roads to keep elk away — to Kuwait for use against camels. — AP

Syntactically, the parenthetical 'sprayed along roads to keep elk away' appears within the scope of the modal operator 'may': presumably it could be analysed as an attributive relative clause acting as adjunct to the NP 'synthetic wolf urine'. One would infer both from a non-modal version of the example (e.g. with 'may export' replaced by 'is exporting'), and from the example as it stands that synthetic wolf urine is sprayed along roads to keep elk away. Thus, according to the embedding-under-modals test (and others can be applied with the same result) this inference should be designated as presuppositional. But many theories associate presuppositions with information which is in the common ground between interlocutors, or assumed by the speaker to be in this common ground. On such an account the anti-elk application of synthetic wolf urine would not appropriately be termed presuppositional, since the writer of the text very likely does not expect readers to have any previous knowledge of the subject. This is not to say that in this case presupposition tests are definitely wrong. Rather, there is no pre-theoretical right and wrong in the matter, and a theoretician has to choose which subsets of phenomena that tests indicate are presuppositional are given a unitary explanation in terms of a theoretical notion of presupposition. Karttunen and Peters [KP77, KP79] have argued that the set of phenomena which tests identify as presuppositional is in fact highly heterogeneous, and indeed that no single sub-group of these phenomena even merits the name presupposition.

2.4 Formal Models of Projection

"...this will suggest ... the appearance of wonderful new 'logical' connectives, and of rules of 'deduction' resembling the prescriptions to be read in The Key of Solomon. Since no one expects that standard logic texts shall ever read like witches' grimoires, this inclines one to dismiss the technical study of presuppositions as a mathematical parlor game." [vF69]

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11 Gazdar [Gaz79a, p.108] attributes this type of example to Andrea Howard.

12 Tests for presupposition might be understood to indicate that footnotes, which can be seen as a notation for parentheticals, are presupposed. For the negation of a sentence containing a footnote reference would usually suggest the author's belief in the footnote just as much as the corresponding positive sentence would. But I would hesitate to say that this footnote (or any other) is presupposed.
For a first glimpse of what is to be expected of a formal presupposition theory, it should be observed that there are some prima facie difficulties involved in attempting to explain presupposition related inferences in terms of classical logic. For instance, a purely semantic characterisation of presupposition may begin with the idea that presuppositions can follow both from a formula and its negation. Indeed, this is often taken as the basis of a formal definition of a presupposition relation between sets of formulae: $\phi$ presupposes $\psi$ iff $\phi \models \psi$ and $\neg \phi \models \neg \psi$. But classically we have that if $\phi \models \psi$ and $\neg \phi \models \psi$, then $\models \psi$. In other words, the above definition together with classically valid patterns of argumentation would yield the unwelcome consequence that only tautologies could be presupposed.

To give another example, since presuppositions are commonly taken to project from the antecedents of conditionals, it might be suggested that if $\phi$ presupposes $\psi$, then $\phi \rightarrow \chi \models \psi$ (for arbitrary $\chi$). If this were allowed, then, on natural assumptions about the presuppositions of definite descriptions, we would have 'If the King of France has a bald head then the King of France has a head' entailing that 'There is a King of France.' Now, it seems reasonable to insist that whatever logic is employed should support an inference from 'The King of France has a bald head' to 'The King of France has a head'. However, a consequence of the logician's beloved deduction theorem is that from $\phi \vdash \psi$ we can conclude $\vdash \phi \rightarrow \psi$. But then 'If the King of France has a bald head then the King of France has a head' is a tautology, and from this a fortiori 'There is a King of France' must also become a tautology. This is an unpalatable consequence. Should we be prepared to accept, for example, the loss of the deduction theorem as one way of breaking down this argument?

Apparent defeasibility of presuppositions suggests abandonment of other classically valid inference patterns. Right monotonicity allows derivation from $\phi \vdash \psi$ that $\phi, \chi \vdash \psi$ (for arbitrary $\chi$). Projection of presuppositions from negated contexts means that 'The King of Buganda is not bald' should normally licence an inference to 'There is a King of Buganda'. However, 'The King of Buganda is not bald. There is no King of Buganda.' apparently does not licence this inference, so maybe right monotonicity should not be valid in a logic for presuppositions.\(^{13}\)

The philosopher's traditional, intentionally oversimplified picture of interpretation

\(^{13}\)Note that Gazdar's solution to the projection problem, the best known theory which takes defeasibility as its basis, preserves right monotonicity, but effectively removes left monotonicity: if previous sentences contradict the presuppositions of a sentence, the presuppositions are not added to the context. Note also that all the above examples concern meta-logical rules of inference, inferences from the validity of one argument pattern to the validity of another, such as are found in Gentzen sequent systems. There has been little systematic study of how presupposition in particular bears on general properties of proof systems. The only discussion I am aware of is in [vF75]. But van Fraassen makes no attempt to connect the systems he presents with natural language. On the other hand Thomason [Th72, Th79] does apply a variant on van Fraassen's supervaluation approach to natural language, but there the discussion lacks the (perhaps excessively rarified) generality of van Fraassen's work.
involves mapping sentences of natural language into similar sentences of classical logic. The above observations suggest that a formal model of presupposition projection problem requires a significant departure from such a picture. There are several different strategies for effecting this departure:

1. Changing the interpretation of Logical form (see e.g. the theories in Chapter 3)
2. Complicating the relation between surface structure and logical form (this is the basis of the neo-Russellian account discussed immediately below)
3. Adding pragmatic mechanisms to act in parallel with semantic interpretation (e.g. the theories in Chapter 4)

Many of the theories that will be described in the coming pages could not be said to adopt just one of these, but are hybrids combining several. In particular, the approach which will be introduced in the second part of this thesis is a hybrid of strategies 1 and 3.

2.5 Are Presuppositions Conventional?

We have seen that theories differ as to whether presuppositions are viewed semantically or pragmatically. Another dimension of variation along which theories can in principle differ concerns whether presuppositions are understood to be conventional or not. By a theory of conventional presupposition I mean one in which the grammar explicitly encodes that a certain subset of constructions are presuppositional, and determines for each such construction with what presuppositions it is associated. These constructions are known as presupposition triggers. Many accounts involve a difference between the presuppositions attached to utterances and those attached conventionally (and independently of utterance context) to the trigger, and this may be reflected terminologically, typically by referring to the presuppositions associated with triggers as elementary, primitive or potential presuppositions. When referring to theories of conventional presupposition, I include such accounts: although in some cases it might be argued that it is inappropriate to call the utterance presupposition itself conventional, the utterance presupposition still originates from a conventional stipulation in the grammar.

A conventional specification of elementary presuppositions is fundamental to all of the formal theories of projection to be discussed in detail in the coming chapters of this thesis. Before ploughing into description and inter-comparison of these theories it is as well to put them into some perspective by asking just how solid their basic foundation is. Must presuppositions be conventionally marked in the grammar? I will now present some accounts which appear to provide, and in some cases perhaps really do provide, the basis of a negative answer to this question.
Scope Ambiguity

Although Russell did not find the concept of presupposition to be useful, his theory of definite descriptions (as well as the updated and extended version of that theory in [Ne90]) provides a serious candidate for a treatment of some phenomena taken by others to be presuppositional in nature. It would be a mistake to overlook Russell's simple and elegant account for the sake of philosophical dogma. Furthermore, in §6.8.2 it will be shown that the Russellian analysis of definites is not so distant from current presupposition theory as some might imagine, so that it is well worth taking the trouble to see how Russell's ideas extend to other phenomena usually taken to be presuppositional. Others have proposed how Russell's program might be extended to account for the problems of modern presupposition theory, including Delacruz [De176], Cresswell [Cr73, pp.168-169] and Grice [Gr81], and the extension I will propose is perhaps most like the first of these.\(^{14}\)

A radically Russellian theory of presupposition might be based around the following tenets:

1. The logical form (LF) of language is homogeneous, in the sense that there are no semantically distinct presuppositional and assertional components.

2. The LF of a sentence may differ markedly from its surface form, since certain expressions can take semantic scope which is very unconstrained with respect to the syntactic domain in which they are realised at surface structure. Call these expressions the free scopers.

3. Call whatever is entailed by all formulae having such an operator at wide scope, the basic assertions of the operator (e.g. the existence of a king should be a basic assertion of the operator corresponding to the noun-phrase 'the king'.) To say that a presupposition is canceled is to say that the embedding at LF of the operator introduced by the presupposition is such that the basic assertion of that operator is not entailed by the LF, and to say that a presupposition is projected is just to say that the basic assertion is entailed.

4. In general a complex sentence will have a large number of LFs. The claim that "presuppositions tend to project" must be explained as a pragmatic preference across alternative LFs, such that one LF will be preferred over another if \textit{ceteris paribus} a free scoping operator has wide scope over a non-free scoping operator in the first LF, but not in the second.

\(^{14}\)Recently, Heim, in [Hei92], has also suggested that scope ambiguity might have a role to play in explaining projection facts, postulating that this might provide the explanation of projection from attitude contexts.
Given that \( t \) is the Russellian description operator *there is one and only one*, the reader will probably recall that according to Russell E12 is subject to a scope ambiguity\(^{15}\). Its meaning can correspond to either of two logical forms:

\[
E35 \quad \exists x[\text{king-of-france}(x)] : \neg(\text{bald}(x))
\]
\[
E36 \quad \neg(\exists x[\text{king-of-france}(x)] : \text{bald}(x))
\]

The first of these readings entails the existence of a unique French King, and is thus comparable to the reading derived in a presuppositional theory where the definite description remains embedded under the negation at LF but where some semantic or pragmatic mechanism allows the presupposed existence of a unique French king to be inherited as a presupposition of the whole sentence. The second reading, in which the definite remains embedded under the negation, of course corresponds to what in a presuppositional theory would be a case of presupposition cancellation.

Given that these two readings are approximately those commonly discussed in the presupposition literature, what is wrong with Russell’s theory? One point of weakness is Russell’s uniqueness restriction. At the very least, it is clear that there are some uses of definites which do not entail that there is a unique satisfier of the description:

\[
E37 \quad \text{At the Françaises-celebres masquerade Napoleon gets a cheap laugh by offering his hat to one of the Kings of France, of whom several are present. The King of France is bald, and soon loses his head.}
\]

Russell's account fails to allow for anaphoricity of presuppositional expressions such as definites. Whether Russell’s uniqueness constraint could be defended against such examples, perhaps by separating anaphoric and non-anaphoric uses of definites, uniqueness effects are far from straightforward, and there is no competing theory which fully accounts for them.\(^{16}\) More to the point, we are interested in whether Russell’s account of definites can serve as the basis of a more general theory of phenomena treated elsewhere as presuppositional. Uniqueness effects are peculiar to a limited set of constructions (definites and clefts), and would presumably not figure as part of the more general theory. On the other hand, there is another idea in Russell’s analysis, that of justifying the presence both of projection and cancellation readings in terms of an underlying scope ambiguity, which could conceivably be of general applicability.

\(^{15}\)The Russellian need not be committed to analysing definites as introducing a scope bearing operator. Instead, a term forming version of the \( t \) may be defined. This is the option preferred, for example, by Grice in [Gr81]. However, I assume an operator treatment.

\(^{16}\)See e.g. Kadmon’s [Kad90] and McCawley’s [McC79] for discussion of uniqueness. Anaphoricity of presuppositional expressions other than definites is discussed in §5.5, below.
Presuppositions and How to Spot Them

For instance, suppose that we worked in a semantic universe sufficiently rich to allow variables to range over propositions (or over individual correlates of propositions). We might then define a language with expressions \([x - \phi](\psi)\), where this has meaning \(\phi\) holds, and under the assignment of \(x\) to the individual correlate of \(\phi\), \(\psi\) holds. Then we might analyse sentences containing a factive verb along the lines of the following examples:\(^{17}\):

E38
- Pooh realises that Eeyore is sad.
- \([x - \text{sad}(e)](\text{realises}(p, x))\)

E39
- Pooh doesn’t realise that Eeyore is sad.
- \([x - \text{sad}(e)](\neg\text{realises}(p, x))\)
- \(\neg(\text{sad}(e)(\text{realises}(p, x)))\)

Once again, both cancellation and projection readings are found for the negative case. Where then, might the weaknesses of such an analysis lie? Most importantly, our neo-Russellian theory cannot be easily stated without introducing a class of presuppositional constructions, even if by another name.\(^{18}\) For given that presuppositions can project from much more deeply embedded constructions than are found in the above examples, and given that projection is to be explained as wide scope, presuppositional constructions will have to be interpreted as a class of operators which can take extra-clausal (and perhaps completely unbounded) scope. Thus in the following example, the factive complement of ‘realise’ (and also the definite description ‘the forest’) must be allowed to take extra-clausal scope, which appears to be forbidden for the scope bearing quantificational NP ‘every animal’:

E40 If every animal in the forest realises that Eeyore is sad, they will organise a party.

Of course, there are other operators which are less constrained as to their possible scope than the bulk of quantificational NPs. For instance, neither indefinite NPs nor

\(^{17}\)The structure in E38 is somewhat reminiscent of the type of syntactic analysis suggested by the Kiparsky’s [KK70]. Kempson [Kem75, pp.130–135] provides a similar semantic analysis of factives to that here, tracing her equivalent to the \([x - \phi](\psi)\) construction back to Reichenbach’s fact functions [Reic47].

\(^{18}\)Grice [Gr81, p.280] also comes to the conclusion that, at least for some constructions, it will be difficult to explain the data without postulating conventional marking of presuppositions: “I do not see that it is going to be particularly easy to represent the implication in the case of regret as being one of a conversational kind. It does not look as attractive as the Russellian case.” Here by the “Russellian case” he refers to the Russellian treatment of definite descriptions. Grice then proceeds to suggest marking certain presuppositional expressions using special brackets, and suggests a rewriting operation on logical forms which effectively gives the presuppositions wide scope. He later describes this as “a minimal strengthening of a Russellian pattern of analysis by the addition of a purely syntactical scope device...”
sentential connectives are subject to a constraint on extra-clausal scope. What would make the analysis of presuppositional constructions unusual is that they would have to be given wide scope as a default, for it is well known that the cancellation (i.e. narrow scope) readings only occur in very specialised circumstances, such as when consequences of the wide scope reading are explicitly contradicted. It is not obvious how the preference for wide scope could be explained, but it is at least clear that the explanation would have to make clear why this preference applied to presuppositional constructions and not other scope bearing elements, and thus that some class of presuppositional constructions would have to be distinguishable in the theory.

One final observation on the neo-Russellian theory. If presupposition triggers were a special sort of scope bearing element, then there would presumably often be other readings than just wide and narrow scope with respect to all other operators. For example, E40 should be expected to have a reading where the sadness of Eeyore outscopes 'every animal' but remains within the conditional (i.e. 'If Eeyore is sad and every animal realises it, then they will have a party'). Such readings will be discussed later in the context of what at first might appear to be a quite different style of presupposition theory: see §6.8.2.

Underspecification

Atlas [At77], Kempson [Kem75] and Wilson [Wi75] have all presented theories of presuppositional phenomena which, like the neo-Russellian account above, do not involve the postulation of a semantic division between presuppositions and assertions. Further, these authors were amongst the first to show that sentences involving a factive verb (or definite description) under a negation do not always implicate the truth of the factive complement (or existence of an object satisfying the description), demonstrating that in some contexts of utterance the implication is lacking. Yet all three authors objected to the postulation of Russellian scope ambiguities as an explanation for this instability. Indeed, all three argue that negation in standard (for English) VP-modifier position bears fixed, wide scope semantically. Thus for them 'Jane realises that Bill is happy' simply entails that Bill is happy, and the negation of this sentence 'Jane does not realise that Bill is happy' entails that its not the case both that Bill is happy and that Jane realises it: the negative sentence does not entail that Bill is happy. The occasional surfacing of presuppositional inferences, in this case the inference that Bill is happy, is to be explained not in terms of semantic entailment, but in terms of Gricean argumentation, utilising general principles such as informativeness and relevance.

The three authors differ considerably in the details of their pragmatic analyses. Kempson seems to suggest that definite descriptions are in some sense (syntactically) more complex than indefinites, this complexity consisting in an extra "+[def]" feature.
She constructs an argument to the effect that when a speaker chooses to use a definite over an indefinite a generalised conversational implicature is generated, the hearer having to explain what extra information is signalled by the choice of a definite. But since “the definite article can only be construed as offering extra information if it is used to convey the same information as its obligatory (anaphoric) use” [Kem75, p.178], the hearer concludes that the speaker is using the definite as if it were anaphoric on some previously introduced entity, and the existence of an appropriate entity is inferred. 19

Although Kempson does not invoke any principles of analysis specific to presupposition, and terms the inference to the truth of a presupposition a *conversational implicature*, her theory nonetheless counts as a theory of conventional presupposition on the rather broad definition which I have provided, for what is the [+ def] feature if not a means of conventionally marking presuppositions? In the case of definite descriptions the presence of the special feature can be independently justified as signalling a preference for an anaphoric use, but this explanation does not seem appropriate in the case of other presuppositional classes, such as aspectual verbs and factives. Furthermore, it is not obvious that there would be any descriptive difference between Kempson’s theory and a version of that theory where the [+ def] marked constructions directly triggered a conventional implicature. This is essentially the idea of the cancellation theories of presupposition to be considered in Chapter 4, of which Kempson’s account can be seen as a forerunner.

Wilson’s analysis, in [Wi75], by contrast seems not to be a theory of conventional presupposition. 20 I will exemplify the account with her treatment of negative sentences involving factive verbs (on pp.99-100). This analysis depends on listing a certain set of cases in which the sentence would be semantically correct, and then providing arguments why in various of these cases the sentence would be pragmatically inappropriate. For instance, if ‘Jane does not realise that Bill is happy’ is uttered and the field of alternatives has been narrowed down to the case where Bill is not happy (which she argues entails that Jane is not in the state of realising that Bill is happy), and the case where Bill is happy but Jane does not realise it, then the first case can be ruled out. If the speaker knew that Bill was not happy, then simply uttering ‘Bill is not happy’ would be more perspicuous. The analysis depends heavily on selecting the right cases: the hidden

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19Kempson applies the same analysis to factive verbs, assuming, as mentioned, that at deep structure the propositional complement of a factive verb also involves a [+ def] feature. However, the argument seems to me rather weaker than in the case of definite descriptions, for there the analysis apparently rests on the speaker having the choice of either using a definite or an indefinite. So to construct the same argument for factive verbs one would need choices of semantically similar predicates which lacked the [+ def] feature. In general, such choices appear to be lacking.

20On the other hand, Wilson’s joint account with Sperber [WS79] does involve conventional stipulation, but this stipulation is (a) much finer grained than the standard presupposing/non-presupposing contrast, and (b) not in general attached to specific lexical items but to different grammatical constructions and stress patternings.
premise seems to be that the speaker has complete knowledge of the situation being described, and is not, for instance, in a state of knowing that either the first or second of the above cases holds, but not knowing which.

For Atlas, presuppositional inferences arise in order that negative sentences may be informative, and in order that they may tie in with entities and topics already under discussion in a discourse. Witness the following from [At77]: “The presuppositional understanding of a negative sentence will be logically stronger and more informative than the non-presuppositional one.”[p.150]; “The presuppositional understandings of sentences are logically stronger than the non-presuppositional understandings. Claims about the world are more informative when singular terms designate, predicates have non-null extensions, modifiers are modifiers of something...”[p.152]. I wish to draw out one theme from this analysis which seems relevant to any theory of presupposition: the fact that presuppositions tend to project might be explained in terms of a general preference for logically stronger interpretations over weaker ones. This approach to ordering interpretations I will term “the Atlas method”, it being appropriate that a thus-named scholar should have formulated a preference for brute strength over ignorance. Whereas for Atlas this strengthening is a matter of further specifying a single weak logical form, for the neo-Russellian the strengthening would be a matter of picking the logically stronger of the available readings. The fact that \( \forall x [\text{king-of-france}(x)] : \neg (\text{bald}(x)) \) entails \( \neg (\forall x [\text{king-of-france}(x)] : \text{bald}(x)) \), but not vice versa, would justify choosing the first reading over the second. The first, the wide scope definite reading, of course corresponds to what others would term projection of the definite’s presupposition. I leave it to the reader to consider examples where the alternative scope readings are not ordered by logical entailment, and to establish whether there is indeed any preference for the projection readings in these cases.

The program of showing that there is no need for presuppositions to be conventionally marked in the grammar is perhaps carried to its furthest extent in Atlas and Levinson’s joint work [AL81]. But even here, where the range of presuppositional constructions dealt with is not large, and where there is attention to formal precision, the difficulty of executing the general program satisfactorily is manifest. The main difficulty is that whilst the cornerstone of the program must be essentially Gricean, formalisation of Gricean argumentation is notoriously problematic. Not only are we lacking any generally accepted statement of the Gricean maxims, we are also lacking any generally accepted logic which is able not just to encode those maxims, but also to support the

\[21\text{This is not to say that any evidence has been presented here, or by Atlas for that matter, that there is such a general preference. But it is at least an interesting and not implausible hypothesis that such a preference exists. The possible relevance of logical strength to the ordering of interpretations in a theory of presupposition was pointed out to me by Henk Zeevat, who suggested that it would be an alternative basis for ordering the readings available in van der Sandt’s theory.}\]
sort of reasoning that would be required. If an anti-presuppositionalist claims to have completely eliminated the need for presupposition, but that claim rests on a Gricean account of pragmatics which is still not adequately formalised, then the claim must remain, in part, mere whistling in the wind.

There is no a priori reason to introduce a notion of presupposition into grammar. If a grammar can be developed in which the class of constructions which have been called presuppositional are not distinguished in any special way, but the combination of this grammar and a general theory of utterance interpretation can predict the type of inferences which are commonly thought of as presuppositional, then the notion of presupposition, conceived of as something to be encoded explicitly in grammar, will have been rendered superfluous. However, seen in this way, at least two of the theories considered in this section, Kempson’s theory and the neo-Russellian theory, do not take us any nearer this goal. For these two can be seen as presuppositional theories, in that the class of constructions commonly identified as presuppositional must be distinguished in the grammar. But if Gricean theory could be adequately formalised, and it could be demonstrated that presuppositional inferences arose as mere side effects, that would surely count as a tremendous success. Furthermore, such an explanatory success might not necessarily conflict with the presuppositionalist’s program. For even if conversational principles can explain presuppositional inferences, the possibility remains that what were once conversational inferences have become conventionalised. Thus Geurts [Geu95] discusses the possibility that “what started off as a pragmatic regularity has been encoded in the grammar”, and Grice himself [Gr81, p. 282] says of his own account that it could be “regarded as a conventional regimentation of a particular kind of non-conventional implicature.” So it is at least possible that the type of argumentation developed by Wilson, Atlas and Levinson could be interpreted not as replacing presuppositional theories, but as supplementing them, as providing an account of how linguistic presuppositions came in to being in the first place and providing an interpretation for whatever formal apparatus the presuppositionalist proposes.

As I have indicated, all the remaining theories to be discussed involve presuppositions (occasionally under another name) being conventionally marked in the grammar, some function being utilised which maps simple positive sentences onto a set of propositions called the presuppositions (or elementary/primitive/potential presuppositions) of the sentence. This is not to say that conventional stipulation of presuppositions has been

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22 As I have indicated, Kempson distinguishes presuppositional constructions with a [+ def] feature, and the neo-Russellian seems forced to distinguish a class of free-scopers.

23 One might think that the definition of this function would be the central part of presupposition theory. But in fact most authors either assume such a function, or only define it for a small subset of constructions. Geurts’ thesis [Geu95] contains an illuminating discussion concerning the difficulty of defining a function from simple sentences to their elementary presuppositions.
validated, but it has not been invalidated either.
Chapter 3

Multivalence and Partiality

This chapter concerns a subset of approaches to presupposition which follow the first of the options mentioned in §2.4, namely modifying not the logical form itself, but the interpretation of that logical form. In general this refinement may concern the interpretation of objects of any syntactic category, but I will concentrate on the meaning of sentence level units, or, when looking at artificial languages, on the interpretation of formulae rather than of terms. The subset of approaches now to be discussed are those in which the interpretation of a formula defines not only a set of worlds such that when interpreted relative to one of these worlds the formula is true (call this set $T$), and a set where it is false ($F$), but also a set where its presuppositions are satisfied ($P$) and a set where they are not ($\neg$).

There are three standard ways in which this redefinition is achieved. Firstly, there is trivalent semantics in which the Boolean domain of truth values $\{t, f\}$ may be extended to include a third value $\ast$, such that the $T$, $F$ and $N$ worlds are those where the formula has the value $t$, $f$ and $\ast$ respectively, and $P = T \cup F$. Secondly, there is partial semantics. Here the domain of truth values is allowed to remain Boolean, but the interpretation function is partialised, such that for a given formula $T$ is the set relative to which the valuation produces $t$, $F$ is that against which the valuation produces $f$, $P$ is still the union of $T$ and $F$, but now the set $N$ is not a set relative to which the formula is given some particular valuation or valuations, but rather it is the set of worlds against which the valuation function is not defined for the formula. Thirdly there are two dimensional systems, where the valuation is split into two parts, or dimensions, each of the two sub-valuations being boolean. There is some variation in how the split is made, but the approaches I will describe make a split between a presuppositional and an assertional sub-value. For the assertional sub-value $T$ is the set of worlds where the formula has value $t$, and $F$ is the remaining set where the formula has the value $f$, and for the

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1Some might prefer to read models where I write worlds.
presuppositional sub-valuation, $P$ is the set of worlds where the formula has value $t$, and $N$ is the remaining set where the formula has the value $f$.

If the trivalent, partial and two-dimensional accounts differ as to the precise refinement from classical interpretation which they utilise, they none the less share a basic approach to presupposition projection:

1. Presuppositions are constraints on the range of worlds/models against which we are able to evaluate the truth or falsity of predications and other semantic operations, or against which this evaluation is legitimate.

2. If these constraints are not met, semantic undefinedness, or illegitimacy of the truth-value, results.

3. Presupposition projection facts associated with a given operator are explained compositionally, in terms of the relation between the definedness/legitimacy of that operator and the definedness/legitimacy of its arguments in some model, and this relation is recoverable from the semantics of the operator alone.

For the purposes of the following discussion, partial and trivalent semantics will be collapsed. This is possible because the discussion is restricted to systems where the connectives are defined truth functionally. Truth functionality is taken to mean that, for any compound formula the only information needed for evaluation relative to some world is (1) the semantics of the head connective, and (2) for each argument whether there is a valuation in the given world, and, if so, what that valuation is. Given such a restriction, from a technical point of view all systems which are presented as trivalent could be presented as partial, and vice versa, whilst maintaining extensionally identical relations of consequence and presupposition. I will firstly consider trivalent systems.

\[\text{This restriction to truth functional systems does exclude one important method of supplying partial interpretations, namely the supervaluation semantics developed by van Fraassen. See [vF69, vF75, Th72, Th79]. One advantage of the supervaluation approach is that it allows a logic, say classical first order logic, to be partialised such that logical validities remain intact. (Note that classical validities are also maintained in the two dimensional approaches which are discussed below.) I was once horrified to hear a group of presupposition theorists arguing bitterly about whether the treatment of presupposition should use a partial or a trivalent logic. There may be philosophical significance to the decision between partial and trivalent systems, and it may be that there are applications (like the treatment of the semantical paradoxes) where it really makes a difference whether the semantical universe contains only two values for the extension of a proposition or is in some way richer. But it seems unlikely that the decision to use a partial or trivalent logic has significant empirical consequences regarding presupposition projection. In general, relevant aspects of a model of presupposition projection presented in terms of either a trivalent logic or a partial logic are straightforwardly reformulable in terms of the other with no consequences for the treatment of presupposition data. See, for example, Karttunen's discussion of van Fraassen in [Kar73]. However, in saying this I am possibly taking for granted what I take to be the conventional use of the term partial logic by logicians (see e.g. [Blam89]), whereby, for instance, versions of both Kleene's strong and weak systems are sometimes referred to as partial logics. Seuren [Seu85, Seu90a] offers an alternative characterisation whereby only Kleene's weak system (Bochvar's internal system) would count.}\]
then two dimensional systems, and then discuss some of the general advantages and disadvantages, showing why most contemporary proponents of such approaches accept that presuppositional data cannot be explained in purely semantic terms, but require some additional pragmatic component.

3.1 Trivalent Accounts

In a trivalent logic, where the semantic valuation of a formula $\phi$ with respect to a model $M$ (here written $[\phi]_M$) may take any of the three semantic values, typically thought of as true, false and undefined ($t, f, \ast$), presupposition may be defined as follows:

**Definition D1 (Strawsonian Presupposition)** $\phi$ presupposes $\psi$ iff for all models $M$, if $[\phi]_M \in \{t, f\}$ then $[\psi]_M = t$.

Let us assume, for the moment, a Tarskian notion of logical consequence as preservation of truth ($\phi \models \psi$ iff for all models $M$, if $[\phi]_M = t$ then $[\psi]_M = t$) Let us further assume that a negation $\neg$ is available in the formal language which is interpreted classically with respect to classically valued argument formulae, mapping true to false and vice versa, but which preserves undefinedness. This defines a so-called choice negation having the following truth table:

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>$\neg \phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>$f$</td>
</tr>
<tr>
<td>$f$</td>
<td>$t$</td>
</tr>
<tr>
<td>$\ast$</td>
<td>$\ast$</td>
</tr>
</tbody>
</table>

Given these notions of consequence and negation, it is easily shown that the above definition of presupposition is equivalent to one mentioned earlier:

**Definition D2 (Presupposition Via Negation)** $\phi$ presupposes $\psi$ iff $\phi \models \psi$ and $\neg \phi \models \psi$

These, then, are the standard approaches to defining presupposition in three-valued logics. One author who offers a significant deviation from these definitions is Burton-Roberts [Bu89a]. He defines two separate notions of logical consequence, weak consequence, which is just the notion $\models$ above, and strong consequence, which is just the notion $\models$ above, and strong consequence, which will here be

as a gapped/partial logic. This is because he implicitly limits consideration to systems which are truth functional in a stronger sense than is given above, such that a compound formula can only have a value defined if the valuation of all the arguments is defined. On the other hand, Burton-Roberts [Bu89a] offers a system which he claims to have the only true gapped bivalent semantics, and which just happens to contain exactly the connectives in Kleene's strong system! Given this lack of consensus among such forceful rhetoricians as Seuren and Burton-Roberts, it is perhaps unwise to stick one's neck out.
denoted $\models_s$, and is defined by: $\phi \models_s \psi$ iff (1) $\phi \models \psi$, and (2) for all models $M$, if $[\psi]_M = f$ then $[\phi]_M = f$. Thus for one proposition to strongly entail another, the truth of the first must guarantee the truth of the second, and the falsity of the second must guarantee the falsity of the first. Burton-Roberts then suggests that presuppositions are weak consequences which are not strong consequences:

**Definition D3 (Burton-Roberts Presupposition)** $\phi$ presupposes $\psi$ iff $\phi \models \psi$ and $\phi \not\models_s \psi$

This seems an attractive definition, and is certainly not equivalent to the standard definitions above. However, it has some rather odd properties. For example, assuming this definition of presupposition and Burton-Roberts' quite standard notion of conjunction, it turns out that if $\phi$ presupposes $\psi$, then $\phi$ presupposes $\psi \land \phi$. Let us assume that 'The King of France is bald' presupposes 'There is a King of France'. According to Burton-Roberts' definition it must also presuppose 'There is a King of France and he is bald', which seems completely unintuitive. More generally, if $\phi$ presupposes $\psi$ then according to this definition it must also presuppose the conjunction of $\psi$ with any strong consequence of $\phi$.

Moving back to the standard definitions, we can examine the presupposition projection behaviour of various three-valued logics. A simple picture of presupposition

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3 Wilson [Wi75] took a definition of consequence like $\models_s$ as fundamental, and used it as part of her argument against semantic theories of presupposition. In a more technically rigorous discussion, Blamey [Blam89] also suggests that the strong notion should be the basic one.

4 Burton-Robert's system uses Kleene's strong falsity preserving conjunction, whereby a conjunction is true if and only if both conjuncts are true, and false if and only if at least one conjunct is false. The following argument then shows that a proposition must presuppose any conjunction of a presupposition and a strong entailment:

1. Suppose $\phi$ presupposes $\psi$ in Burton-Roberts system
2. Then (a) $\phi \models \psi$, and (b) $\phi \not\models_s \psi$
3. From 2, $[\psi]_M = f$ and $[\phi]_M \neq f$ for some model $M$
4. Suppose $\phi \models_s \chi$
5. By definition of $\models_s$, we have that $\phi \models \chi$
6. By 2(b), 5 and definitions of $\land, \models_s$, it follows that $\phi \models \psi \land \chi$
7. Relative to the same model $M$, where $\psi$ is false, falsity preservation of $\land$ tells us that $\psi \land \chi$ is false
8. Since there is a model (M) where $\phi$ is not false and its weak entailment $\psi \land \chi$ is false, it follows that $\phi \not\models_s \psi \land \chi$
9. Hence $\phi$ must presuppose $\psi \land \chi$ in Burton-Roberts system. Q.E.D.

It should be mentioned that the above is not the only definition of presupposition that Burton-Roberts offers: it seems to be intended as a definition of the elementary presuppositions of a simple positive sentence. Presuppositions of compound sentences are given by a relation of Generalised Presupposition, which I will not discuss here.
projection is what is known as the cumulative hypothesis according to which the set of presuppositions of a complex sentence consists of every single elementary presupposition belonging to any subsentence. As far as the projection behaviour of the logical connectives is concerned, such a theory of projection would be modelled by a trivalent logic in which if any of the arguments of a connective has the value *, then the value of the whole is also *. Assuming that combinations of classical values are still to yield their classical result, this yields the so-called internal Bochvar or weak Kleene connectives:

Definition D4 (The Weak Kleene or Internal Bochvar Connectives)

<table>
<thead>
<tr>
<th>$\phi \land \psi$</th>
<th>$t$</th>
<th>$f$</th>
<th>$*$</th>
<th>$\phi \rightarrow \psi$</th>
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<th>$f$</th>
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<tr>
<td>$\phi \lor \psi$</td>
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<td>$f$</td>
<td>$*$</td>
<td>$\phi$</td>
<td>$\neg \phi$</td>
<td>$t$</td>
<td>$f$</td>
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<tr>
<td>$t$</td>
<td>$t$</td>
<td>$t$</td>
<td>$*$</td>
<td>$t$</td>
<td>$f$</td>
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</table>

A naive version of the cumulative hypothesis, such as is embodied in the definition of Bochvar's internal connectives, is not tenable, in that there are many examples of presuppositions not being projected. Let us consider firstly how this is dealt with in the case that has generated the most controversy over the years, that of negation. In a trivalent semantics, the existence of cases where presuppositions of sentences embedded under a negation are not projected, is normally explained in terms of the existence of

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5The cumulative hypothesis is commonly attributed to Langendoen and Savin. However, their view appears to have been more sophisticated than some have suggested. Regarding examples where a presupposition of the consequent of a conditional does not become an implication of the conditional as a whole, they comment [LS71]pp.58: "A conditional sentence has the property that its presupposition is presupposed in a (possibly imaginary) world in which its antecedent is true... and no mechanism for suspending presuppositions is required." Although the informality of their proposal makes it difficult to evaluate, it is clear that Langendoen and Savin were aware of cases where presuppositions of an embedded sentence are not implications of the whole and did not see them as counterexamples to their theory. Indeed, on a charitable reading (where it is read as a generic about a property holding of worlds which satisfy the antecedent of a conditional) the above quote seems to prefigure the inheritance properties that Karttunen later attributed to conditionals.

6Horn's article [Horn85] provides an excellent overview of treatments of negation and considers cases of presupposition denial at length. For a longer read, his [Horn89] is recommended. Extensive discussion of negation within the context of contemporary trivalent accounts of presupposition is found in the work of Seuren [Seu85, Seu88], and Burton-Roberts [Bu89c, Bu89a]. These latter publications produced considerable debate, to a degree surprising given that Burton-Roberts, though innovative, presents what is essentially a reworking of a quite well worn approach to presupposition. This refreshingly vehement debate provides the definitive modern statements of the alternative positions on negation within trivalent systems: see Horn's [Horn90] and Burton-Roberts' reply [Bu89b], Seuren's [Seu90a] and Burton-Roberts' reply [Bu90], and Seuren and Turner's reviews [Seu90b, Tu92].
a denial operator (here \( \dagger \)) such that when \([\phi]_M = \star\), \([\dagger \phi]_M = t\). Typically the following exclusion (sometimes called weak) negation operator results:

<table>
<thead>
<tr>
<th>( \phi )</th>
<th>( \dagger \phi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t )</td>
<td>( f )</td>
</tr>
<tr>
<td>( f )</td>
<td>( t )</td>
</tr>
<tr>
<td>( \star )</td>
<td>( t )</td>
</tr>
</tbody>
</table>

Since there apparently exist both cases where a negation acts, in Karttunen’s terminology, as a hole to presuppositions (allowing projection) and cases where it acts as what Karttunen called a plug (preventing projection), the defender of a trivalent account of presupposition appears not to have the luxury of choosing between the two negations given above, but seems forced to postulate that negation in natural language is ambiguous between them. Unfortunately, convincing independent evidence for such an ambiguity is lacking, although there may at least be intonational features which mark occurrences of denial negation from other uses, and thus potentially allow the development of a theory as to which of the two meanings a given occurrence of negation corresponds.7

There is a frequently overlooked alternative to postulating a lexical ambiguity, dating back as far as Bochvar’s original papers. Bochvar suggested that apart from the normal mode of assertion there was a second mode which we might term meta-assertion. The meta-assertion of \( \phi \), \( A\phi \), is the proposition that \( \phi \) is true: \([A\phi]_M = t \) if \([\phi]_M = t \) and \([A\phi]_M = f \) otherwise. Bochvar showed how within the combined system consisting of the internal connectives and this assertion operator a second set of external connectives could be defined: for instance the external conjunction of two formulae is just the internal conjunction of the meta-assertion of the two formulae (i.e. \( \phi \wedge^{\text{ext}} \psi \) \(=\) def \( A(\phi) \wedge^{\text{int}} A(\psi) \)), and the external negation of a formula is just the exclusion negation given above, and defined in the extended Bochvar system by \( \dagger \phi \) \(=\) def \( \neg A(\phi) \).8 Thus whilst the possibility of declaring natural language negation to be ambiguous between \( \neg \) and \( \dagger \) exists within

7If the raison d’être of a trivalent denial operator is to be yield truth when predicated of a non-true and non-false proposition, then in principle some choice remains as to how it should behave when predicated of a simply false proposition. Thus the denial operator need not necessarily have the semantics of the exclusion negation, although, to my knowledge, only Seuren has been brave enough to suggest an alternative. Seuren’s preferred vehicle for denial is an operator which maps only \( \star \) onto \( t \), and maps both \( t \) and \( f \) onto \( f \). Seuren has also marshalled considerable empirical evidence that negation is in fact ambiguous, although his main justification for his choice of denial operator is, I think, philosophical.

8External negation, given that it can be defined as \( \neg A(\phi) \) where \( A \) is a sort of truth-operator, has often been taken to model the English paraphrases ‘it is not true that’ and ‘it is not the case that’. Although it may be that occurrence of these extraposed negations is high in cases of presupposition denial — I am not aware of any serious research on the empirical side of this matter — it is certainly neither the case that the construction is used in all instances of presupposition denial, nor that all uses of the construction prevent projection of embedded presuppositions. Thus the use of the term external for
Bochvar's extended system, another possibility would be to translate natural language negation uniformly using ~, but then allow that sometimes the proposition under the negation is itself clad in the meta-assertoric armour of the \( A \)-operator.

There is no technical reason why the Bochvarian meta-assertion operator should be restricted in its occurrence to propositions directly under a negation. Link [Li86] has proposed a model in which in principle any presupposition can be co-asserted, where coassertion, if I understand correctly, essentially amounts to embedding under the \( A \)-operator. Let us term a theory where all occurrences of cancellation are explained away in these terms a floating-\( A \) theory. Such a theory is flexible, since it leaves the same logical possibilities open as in a system with an enormous multiplicity of connectives: for instance if the \( A \) operator can freely occur in any position around a disjunction, then the effects of having the following four disjunctions are available: \( \phi \lor \psi, A(\phi \lor \psi), A(\phi) \lor \psi \) and \( \phi \lor A(\psi) \). It is then necessary to explain why presuppositions only fail to project in certain special cases. Link indicates that pragmatic factors will induce an ordering over the various readings, although he does not formalise this part of the theory. Presumably a default must be invoked that the \( A \) operator only occurs when incoherence would result otherwise, and then with narrowest possible scope. The term incoherence must then be explicaded: perhaps it can be understood as semantic undefinedness in the set of models corresponding to our assumptions about the world. At base then, a floating-\( A \) theory consists of a semantic component generating multiple meanings encoding varying degrees of presupposition projection, and a pragmatic component selecting between these meanings. This selection could, for instance, be based on the Atlas method, the principle of preference for logically stronger readings — see §2.5. In §3.3 we will see that, given an argument in essence due to Soames, the defender of a trivalent account of presupposition might be forced into some version of a floating-\( A \) theory. For the moment let us merely observe that in a floating-\( A \) theory the lexical ambiguity of negation which is common in trivalent theories is replaced by an essentially structural ambiguity, and in this respect is comparable with the Russellian scope-based explanation of projection facts.\(^9\)

So far we have only considered cases where presuppositions of each argument are either definitely projected to become presuppositions of the whole, or definitely not projected. Fittingly, in the land of the included middle, there is a third possibility. The presupposition may, in effect, be modified as it is projected. Such modification occurs with all the binary connectives in Kleene's strong logic:

\[^9\]Horn85, p.1251 provides a similar explication to that above of the relation between theories postulating alternative 3-valued negations and theories involving a Russellian scope ambiguity. See [BK:MSJ] for further discussion of a floating-\( A \) theory.
Multivalence and Partiality

Definition D5 (The Strong Kleene Connectives)

<table>
<thead>
<tr>
<th>( \phi \land \psi )</th>
<th>t</th>
<th>f</th>
<th>*</th>
<th>( \phi \rightarrow \psi )</th>
<th>t</th>
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<td>t</td>
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<td>f</td>
<td>t</td>
<td></td>
<td></td>
</tr>
<tr>
<td>*</td>
<td>t</td>
<td>*</td>
<td>*</td>
<td>*</td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

To see that under this definition it is not in general the case that if \( \phi \) presupposes \( \pi \) then \( \psi \rightarrow \phi \) presupposes \( \pi \), we need only observe that if \( [\psi]_M = f \) then \( [\psi \rightarrow \phi]_M = t \) regardless of the valuation of \( \phi \). Presuppositions of the consequent are weakened, in the sense that in a subset of models, those where the antecedent is false, undefinedness — read presupposition failure — of the consequent is irrelevant to the definedness of the whole. However, in those models where the antecedent is not false, the presuppositions of the consequent are significant, so that presupposition failure of the consequent is sufficient to produce presupposition failure of the whole. The presuppositional properties of the strong Kleene logic may be determined in full by inspection of the truth tables, and may be summed up as follows:

**Fact F1** Under the strong Kleene interpretation, if \( \phi \gg \pi \) then:

1. \( \neg \phi \gg \pi \)
2. \( \phi \land \psi \gg \psi \rightarrow \pi \)
3. \( \psi \land \phi \gg \psi \rightarrow \pi \)
4. \( \phi \rightarrow \psi \gg (\neg \psi) \rightarrow \pi \)
5. \( \psi \rightarrow \phi \gg \psi \rightarrow \pi \)
6. \( \phi \lor \psi \gg (\neg \psi) \rightarrow \pi \)
7. \( \psi \lor \phi \gg (\neg \psi) \rightarrow \pi \)

If models are restricted to those where \( \psi \) is bivalent, these are maximal presuppositions, in the sense that the right hand side represents the logically strongest presupposition, all other presuppositions being entailed by it.

The occurrence of conditionalised presuppositions can be argued for on the basis of examples like the following:

E41 If Jane is married, then her husband is not here.
Given that the consequent of E41 carries the presupposition that Jane has a husband, the implication as a whole is predicted to carry the presupposition that if Jane is married then she has a husband. If we restrict our attention to models in which this natural — one is tempted to say analytic — condition is satisfied, the (logical rendering of the) sentence will always have a classical truth valuation. Thus, appropriately in this case, the presupposition of the consequent is weakened to the point of triviality, and the sentence does not presuppose (nor entail) that Jane is married. We will return to conditionalised presuppositions, which occur in some two dimensional accounts as well as in Strong Kleene, below.

3.2 Two Dimensions

There are no obvious empirical reasons for using more than three truth values in the treatment of presupposition, and thus Occam's razor commonly makes trivalent semantics the preferred basis for a multivalent treatment of presupposition. However, quite apart from the fact that four valued logics are sometimes thought to be technically more elegant than their three valued cousins, the use of four truth values affords theorists the space to pursue a divide and conquer strategy, separating issues of presupposition from those of classical truth and entailment. The idea was developed independently, but in rather different forms, by Herzberger [Her73] and Karttunen and Peters [KP79], Herzberger's formulation having been further developed by Martin [Ma77] and Bergmann [Ber81]. The semantic domain is considered as consisting of two two-valued coordinates (dimensions), which I will call assertion and presupposition. Thus, if the four values are represented using a pair of binary digits, with the first representing the assertion, and the second the presupposition, then, for instance, (0, 1) will mean that the assertion is not satisfied, although the presupposition is.

Treating a four valued semantics as consisting of two boolean coordinates allows for a straightforward introduction of the tools of classical logic to study an essentially non-classical system, and this enabled Karttunen and Peters to provide compositionally derived two-dimensional interpretations for a fragment of English using the classical IL of Montague (familiarity with which I assume). To illustrate the approach, let us suppose

---

10 Cooper [Co83] presents an interesting empirical justification for the use of a fourth value, suggesting that whilst the third value is used to represent presupposition failure, a fourth value is required to signal acts of presupposition denial. This idea, which enables Cooper to give some explanation of cancellation effects without postulating an ambiguity of negation (or other operators) has not, to my knowledge, been taken up elsewhere.

11 What are here called assertion and presupposition are for Herzberger correspondence and bivalence, and for Karttunen and Peters entailment and conventional implicature. The theories differ considerably in philosophical motivation, in that whilst Herzberger's could be reasonably termed a semantic account, Karttunen and Peters' is not presented as such. However, the fact that Karttunen and Peters give a pragmatic explication of their second dimension of evaluation is irrelevant to most of the technicalities.
that expressions of English are associated with two translation functions, \( A \) and \( P \). \( A \) maps expressions to IL formulae representing its assertion, and \( P \) likewise maps to an IL representation of the presupposition. Given that the assertion and presupposition of an expression are assumed by Karttunen and Peters to have identical IL types, and that for English sentences this type is that of truth values, the two dimensional interpretation of a sentence \( S \) relative to an IL model \( M \) and assignment \( g \) will be \( \langle [A(S)]_{M,g}, [P(S)]_{M,g} \rangle \).

Now we might associate with conditionals, for instance, the following translation rule pair:

\[
\begin{align*}
A(\text{if } S_1 \text{ then } S_2) &= A(S_1) \rightarrow A(S_2) \\
P(\text{if } S_1 \text{ then } S_2) &= P(S_1) \land P(S_2)
\end{align*}
\]

This particular rule pair, defines a notion of implication comparable with the Bochvar internal implication. If we associate the value \( (1, 1) \) with \( t \), \( (0, 1) \) with \( f \), and the remaining two values both with \( * \), then a sentence 'if \( S_1 \) then \( S_2 \)' will take the value \( * \) just in case either \( S_1 \) or \( S_2 \) takes this value, and otherwise will take the standard classical value.\(^\text{12}\)

The same approach is extendible to other types. Let us suppose that a sentence of the form 'The guest \( Xs \)' involves the assertion of the existence of a guest with property \( X \) and presupposition of the uniqueness of the guest, and that a sentence of the form 'y curtsied' carries the assertion that \( y \) performed the appropriate physical movement, and the presupposition that \( y \) is female. Then assuming appropriate basic translations, constants \textit{guest}, \textit{curtsied} and \textit{female}, and meaning postulates guaranteeing that, for instance, the constant \textit{curtsied} stands in the correct relation to other constants relevant to the physical act of curtseying, part of the derivation of the meaning of the sentence 'The guest curtsied' might run — departing somewhat from Karttunen and Peters’ original system — as follows:

\(^{12}\)This two dimensional version of Bochvar’s internal implication is found in the first systems proposed in [Her73]. Note that the other Bochvar internal connectives can be defined similarly, such that in each case the assertion is defined entirely in terms of the assertion of the arguments, and the presupposition is defined entirely in terms of the presuppositions of the arguments. This yields what is termed (following Jankowski) a cross-product logic. However, both Herzberger and Karttunen and Peters also define operators for which this property does not hold. For instance, the two dimensional version of Bochvar’s assertion operator considered by Herzberger, thought of as a semantics for the English 'it is the case that' locution, could be defined:

\[
\begin{align*}
A(\text{it is the case that } S) &= A(S) \land P(S) \\
P(\text{it is the case that } S) &= T
\end{align*}
\]

Here the assertion is defined in terms of both the assertion and presupposition of its argument.
\[ A(\text{the guest}) = \lambda X [\exists y \text{guest}(y) \land X(y)] \]
\[ P(\text{the guest}) = \lambda X [\exists y \text{guest}(y) \land \forall z [\text{guest}(z) \rightarrow x = z] \land X(y)] \]
\[ A(\text{curtsied}) = \text{curtsied} \]
\[ P(\text{curtsied}) = \text{female} \]
\[ A(\text{the guest curtsied}) = A(\text{the guest}) \cdot A(\text{curtsied}) \]
\[ = \lambda X [\exists y [\text{guest}(y) \land X(y)]](\text{curtsied}) \]
\[ = \exists y [\text{guest}(y) \land \text{curtsied}(y)] \]
\[ P(\text{the guest curtsied}) = P(\text{the guest}) \cdot P(\text{curtsied}) \]
\[ = \lambda X [\exists y [\text{guest}(y) \land \forall z [\text{guest}(z) \rightarrow x = z] \land X(y)]](\text{female}) \]
\[ = \exists y [\text{guest}(y) \land \forall z [\text{guest}(z) \rightarrow x = z] \land \text{female}(y)] \]

Thus we derive the assertion that a guest curtsied, and the presupposition that there is exactly one guest and that guest is female. The approach seems quite general, but Karttunen and Peters observe, in a by now infamous footnote, that there is a problem associated with their interpretation of existentially quantified sentences. According to their theory, a sentence of the form \( \exists n X Ys \) carries the assertion that an individual in the assertional extension of \( X \) has the property given by the assertional component of \( Y \). Further, the sentence carries the presuppositions (1) that some individual is in the presuppositional extension of \( X \), and (2) that some individual in the assertional extension of \( X \) is in the presuppositional extension of \( Y \). What might be referred to as the binding problem is that there is no link between the variables bound in the assertion and in the presupposition. In particular, there is no guarantee that any entity satisfies both the assertional and the presuppositional requirements. Let us see why this is problematic for the sentence ‘Somebody curtsied’:

\[ A(\text{somebody}) = \lambda X [\exists y \text{person}(y) \land X(y)] \]
\[ P(\text{somebody}) = \lambda X [\exists y \text{person}(y) \land X(y)] \]
\[ A(\text{curtsied}) = \text{curtsied} \]
\[ P(\text{curtsied}) = \text{female} \]
\[ A(\text{somebody curtsied}) = A(\text{somebody}) \cdot A(\text{curtsied}) \]
\[ = \lambda X [\exists y \text{person}(y) \land X(y)](\text{curtsied}) \]
\[ = \exists y [\text{person}(y) \land \text{curtsied}(y)] \]
\[ P(\text{somebody curtsied}) = P(\text{somebody}) \cdot P(\text{curtsied}) \]
Thus the sentence is given the assertion that somebody performed the physical act of curtseying, and the presupposition that somebody is female. Crucially, this interpretation fails to enforce the common-sensical constraint that the person who curtseyed is female. One possible fix would amount to making all presuppositions also assertions, which is standard in some of the accounts to be considered in the next chapter. In fact, as will be discussed there, there is a separate reason to make presuppositions also part of the asserted content, for without this one cannot easily explain why although presuppositions are commonly defeasible, presuppositions of simple positive sentences are not. If the presupposition is also part of the assertion, then the reason for this indefeasibility has nothing to do with the presuppositional dimension itself, but derives from the fact that one cannot ordinarily deny one's own assertions, or make assertions which one knows to be false.

3.3 The Need for Pragmatics

More effort has gone into the development of partial and multivalent solutions to the problems of presupposition theory than into any other general approach. It is thus striking that even the treatment of basic logical connectives in this paradigm remains troublesome.

3.3.1 Negation

In many multivalent and partial treatments multiple homophonous negations are posited, even though postulation of a lexical ambiguity of negation is, if defensible (see Seuren's [Seu85]), never the less controversial. Further, the problems associated with cancellation in sentences involving negation are paralleled by cancellation cases involving other connectives. Witness the following example (related to one discussed in [Kem75, p.93]) which, although it involves no explicit negation, manifests cancellation behaviour which would be typical of a simple negative sentence:

E23' If the King of France is bald, then I'm a Dutchman: there is no King of France!

The theorist who explains cancellation in negative sentences by postulating multiple negations would seem to be led by such examples in the unattractive direction of postulating multiple homophonous conditionals. A further difficulty with the multiple negations story is that if a cancellation negation is posited in some sentence, then all
presuppositions will be blocked, and not only those which the discourse explicitly determines to be problematic. But it seems to me that in the following example, whilst the presupposition that there is a King of France is blocked, the presupposition that the addressee has a son is not:

E42 The King of France didn’t give your son the 

Royaume Medaille d'Honneur. France is not a monarchy, and there is no such award.

Thus either of the following continuations seem natural, and in both cases the main NP (‘he’ or ‘Johnny’) can be understood as coreferential with the just mentioned son.

E43 Besides, he’s only three years old!
E44 Besides, Johnny is only three years old!

3.3.2 Disjunction

Apart from negation, disjunction turns out to be particularly resistant to analysis in terms of multivalent semantics. Although disjunction is also problematic in other approaches, the difficulties are particularly clear cut for multivalent logics based on a standard semantic definition of presupposition. The trouble (c.f. [So79], on which the current discussion leans heavily) is that we can quite easily provide an exhaustive listing of all the connectives that manifest the basic logic of disjunction in a given system, and we can quite easily show that no single definition of the connective would predict all the cases of projection and cancellation which are found. I will consider the following examples from the point of view of a trivalent system:

E45 Either the King of Buganda is now opening parliament, or the Mayor of Nozdrovia hasn’t arrived yet.
E46 Either the King of Buganda is now opening parliament, or else the person who told me Buganda is a monarchy was wrong.
E47 Either the person who told me Buganda is a monarchy was wrong, or else the King of Buganda is now opening parliament.
E48 Either the King of Buganda is now opening parliament, or the President of Buganda is conducting the ceremony.

13One aspect of these examples which I will not consider in detail is the presence of the word ‘either’. As Prince has pointed out [Pri78, p.372], the presence of this word is essential to the felicity of many examples where a presupposition triggered in a disjunction is canceled. Prince conjectures that the ‘either’ acts as a signal to the hearer to “delay attribution” of information in the disjuncts, which she suggests may lead to the presuppositions not being regarded as beliefs of the speaker.
In E45 presuppositions of both disjuncts appear to project, and in a trivalent system with standard semantic definition of presupposition, this would naturally be explained by assuming that whenever either disjunct has the value $\ast$, the whole disjunction also has this value. However, it seems that in a case where the left disjunct of E46 has the value $\ast$, the whole disjunction will in fact be true, the truth of the right disjunct in such a case apparently being sufficient to guarantee this. A mirror argument can be applied in the case of E47, suggesting that whenever the left disjunct of a disjunction is true, the whole disjunction should be true. We are left with the following truth table, which is in fact Kleene’s strong disjunction:

<table>
<thead>
<tr>
<th>$\phi \lor \psi$</th>
<th>$\psi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$</td>
<td>$t$</td>
</tr>
<tr>
<td>$t$</td>
<td>$t$</td>
</tr>
<tr>
<td>$f$</td>
<td>$t$</td>
</tr>
<tr>
<td>$\ast$</td>
<td>$t$</td>
</tr>
</tbody>
</table>

Although under this semantics it is not the case that presuppositions uniformly project, we are at least left with weakened presuppositions from both disjuncts. As mentioned above, if $\phi$ presupposes $\psi$, then $\phi \lor \chi$ presupposes $\neg \chi \rightarrow \psi$, where $\rightarrow$ is the strong Kleene implication, and similarly for the other disjunct.

But now consider sentence E48, which is of a type first considered by Hausser [Ha76]. Here the disjuncts carry conflicting presuppositions: if there is a King of Buganda, then there is no president, and vice versa. Let us accept that the Bugandan head of state is either a president or a King, and assume that 'opening parliament' is synonymous with 'conducting the ceremonies'. We can concentrate on two exclusive and exhaustive possibilities: (1) the head of state opened parliament, or (2) the head of state did not open parliament. In case (1), at least one of the disjuncts must be true, and since (under the above strong Kleene interpretation) truth of a disjunct guarantees truth of the disjunction, it must be that the disjunction as a whole is true. In case (2), it can be seen that one of the disjuncts must be false, and the other undefined. In this case the above table tells us that the disjunction as a whole must be undefined. We thus see that E48 can be either true (if the head of state opened parliament) or undefined (if the head of state did not open parliament), but not false. This seems rather odd. For we are then forced to say that the (standard, internal) negation of E48, perhaps E49 or E50, could never be true. This seems blatantly inappropriate.\(^{15}\)

\(^{14}\)See also the discussion in [Gaz79a, p.95].

\(^{15}\)Burton-Roberts [Bu89a, pp.169–170] seems to regard cases of conflicting presuppositions as being unproblematic in his system, which does assume a strong Kleene disjunction. He argues that a case like E48 is always given the values true or false, and is never undefined. But it seems to me that his argument is flawed. He assumes that the disjuncts have a common strong entailment, which is taken
E49 It is not the case that either the King of Buganda is now opening parliament or the President of Buganda is conducting the ceremony.

E50 Neither is the King of Buganda now opening parliament, nor is the President of Buganda conducting the ceremony.

3.3.3 Conditionals and Conditionalised Presuppositions

Presuppositions of conditionals provide yet another battleground. In the strong Kleene system both the antecedent and consequent presuppositions can be said to be weakened in the course of projection. In Karttunen and Peter's system, with respect to which I have not yet discussed the treatment of conditionals, the antecedent presupposition projects unmodified, but the consequent presupposition is weakened just as in the Strong Kleene system.

In the Strong Kleene system with a Strawsonian notion of presupposition, if \( \phi \) presupposes \( \pi \), then \( \phi \rightarrow \psi \) does not automatically presuppose \( \pi \), but does presuppose \( \neg(\psi) \rightarrow \pi \). I know of no empirical evidence in favour of this weakening of the antecedent presupposition in the Strong Kleene system, and am unable to construct any. It does seem odd that if the consequent is true in a model, then the implication as a whole is defined (and true) independently of the definedness of the antecedent. This might be felt to be a weak point of the Strong Kleene system qua logic for presupposition\(^{16}\), since truth of the consequent of a conditional is manifestly not sufficient for a conditional to be bivalent, and in this case might be a proposition something like \( X = 'There is exactly one head of state and that head of state opened parliament' \). Burton-Roberts begins, as above, by dividing into two cases (1) \( X \) is false, and (2) \( X \) is true. But with regard to case (1), Burton-Roberts diverges from the argument above. Since, by assumption, \( X \) is a strong entailment of both disjuncts, and since (by definition of strong entailment) if \( A \) strongly entails \( B \) and \( B \) is false then \( A \) is false, it follows that both disjuncts are false. From this it follows that if \( X \) is false, the disjunction as a whole is false, and not undefined as argued above.

In this way Burton-Roberts avoids the disjunction as a whole from ever being undefined. But crucial to his argument is the premise that both disjuncts strongly entail \( X \). By fiat he is thus declaring firstly that whenever the head of state did not open parliament, the proposition 'The king opened parliament' is false, irrespective of whether there is in fact a king, and secondly that 'The president opened parliament' is false irrespective of whether there is in fact a president. This seems completely unjustified to me. It could well be that the type of argumentation Burton-Roberts develops later in his book, concerning the question of when presupposition bearing elements are truth valueless, could be applied successfully to such cases, but the discussion on pp.169–170 does not settle the point.

\(^{16}\)Kleene, like Łukasiewicz, did not motivate his semantics in terms of linguistic presupposition but in terms of certain issues in the foundations of mathematics. Bochvar's motivation at least concerned the philosophy of language. For him the third value signalled nonsense, but he used this notion to refer to the denotation of a paradoxical sentence, rather than one in which the presuppositions fail. The Strong Kleene connectives are the most commonly utilised by presuppositionalists, for instance being found in the systems of Hauser [Ha76], Seuren [Seu85] (who adds a negation), Burton-Roberts [Bu89a], and Link [Li86] (who uses Blau's system [Blau78], which contains the Kleene connectives as a subsystem). None of these authors, however, simply combine Strong Kleene with a Strawsonian definition of presupposition and no other pragmatic component.
be interpreted as felicitous:

E51 If the Pope's current obsession with water skiing is anything to judge by, then he hasn't much of a future in professional ice hockey.

Here the consequent is true, but I would hesitate to judge the conditional as a whole as true. I certainly would not infer that if the pope has a future in professional ice hockey then he is obsessed with water skiing. However, I don't know that this is a knock down argument against the Strong Kleene treatment of presupposition per se. The difficulty of understanding material implication as representing natural language conditionals are well known, and conditionals with known-to-be-true consequents are generally odd. Strong Kleene extends material implication to a third value so as to maintain what Kleene took to be the basic intuitions of the material implication itself. As Andreas Schöter (p.c.) has pointed out to me, that Strong Kleene predicts E51 to be true independently of whether the pope is currently obsessed with water skiing might best be seen as a reflex of the non-presuppositional problems facing the material implication.

Regarding the weakening of the consequent presupposition, there has been considerably more controversy. Both the Strong Kleene system and Karttunen and Peters' system make the prediction that if $\phi$ presupposes $\pi$, then $\psi \rightarrow \phi$ does not automatically presuppose $\pi$, but does presuppose $(-\psi) \rightarrow \pi$. The examples which have caused controversy are of two basic types, those where the antecedent seems unrelated to the presupposition of the consequent, and those where, under certain assumptions which may be taken to restrict the relevant models of evaluation, the antecedent entails the presupposition of the consequent. An example of the first type is the following:

E52 If I go to London, my sister will pick me up at the airport.

A hearer would be expected to infer from an utterance of this sentence that the speaker has a sister, but the Strong Kleene and Karttunen and Peters systems predict only a weaker conditionalised presupposition, namely that if the speaker goes to London then the speaker has a sister. Karttunen and Peters recognised this problem, and informally suggested a pragmatic line of solution. Although Gazdar [Gaz79a, p.115] suggests a number of examples where a superficial examination seems to indicate that Karttunen and Peters' solution does not work, and where, in Gazdar's words "remarkably zany predictions" result, all of these examples are dealt with by Soames [So82, pp. 542-543]. I will not detail Karttunen and Peters' informal solution here, but point the reader to Soames discussion in [So82] and to the more critical evaluation in Geurts' dissertation [Geu95]. However, later in this thesis, a formal solution will be given to the problem of conditional presuppositions in examples like E52.
Regarding the second type of example, those where the antecedent entails the presupposition of the consequent, I am afraid the standard of argumentation in the literature has sometimes been disappointing. Otherwise excellent critiques of Karttunen and Peters’ system by Gazdar [Gaz79a] and van der Sandt [vdS88] are marred by the presentation of supposed counter-examples to conditionalised presuppositions, but examples in which the conditionalised presupposition is blatantly irrelevant. Consider Gazdar’s E53 (which is not discussed by Soames) and van der Sandt’s similar E54:

E53 If John murdered his father, then he probably regrets killing him, but if he killed him accidentally, then he probably doesn’t regret having killed him.

E54 If John murdered his wife, he will be glad that she is dead, but if she took those pills herself . . .

A hearer of examples E53 and E54 would typically infer that a close relative of John (father or wife, respectively) is dead. Such a presupposition (or, in the first case, a slightly stronger presupposition) is triggered in the consequent of the first conditional in each example, but this presupposition is weakened in the Strong Kleene and Karttunen and Peters’ systems to a trivial proposition that can be glossed ‘If John murdered relative X then relative X is dead’, and there is no prediction of any non-trivial presupposition. Are these counterexamples? Not at all. The inference to relative X being dead has absolutely nothing to do with the factive in the consequent of the respective conditionals. In the following examples the consequents have been replaced with non-presupposing clauses, but in each case the inference to relative X being dead seems just as clear as with the original cases:

E55 If John murdered his father, then he’ll go to prison, but if he killed him accidentally, then he could inherit a fortune.

E56 If John murdered his wife, he will go to prison, but if she took those pills herself . . .

It is clear that the presupposition, if that is what it is, arises not in the consequent of the conditional, but in the antecedent. It is presumably linked with the contrastive stress that one would expect to find in an utterance of these examples. Further, I have nothing to say about the inference to X’s death, except that it manifestly has nothing to do with the issue of conditionalised presuppositions, and that the reader who wishes to know where the inference does come from should look to an account of the interaction between presupposition and stress/topicality. The relevant literature stretches back to Strawson [St64], who suggested that reference failure only produced truth-valuelessness in case the presupposition was topical, and includes accounts (like Strawson’s) of how topicality affects presupposition projection, accounts of the presuppositions generated
by sentence stress, and accounts which conflate presupposition and topicality. See, for instance, Wilson and Sperber’s [WS79], Reinhart’s [Rein82], Sgall Hajičová and Benešová’s [SHB73], Sgall’s [Sg:MS], or Horn’s [Horn86].

Stress is quite obviously central to the analysis of another purported counterexample. Soames [So82, p.497] gives E57 as an example which backs up Karttunen and Peters’ predictions. In this example, the cleft in the consequent carries a presupposition that the problem has been solved, but the weakened presupposition, that if someone at the conference solved the problem then the problem has been solved, is trivial.

E57 If someone at the conference solved the problem, it was Julius who solved it.

Van der Sandt has a different opinion. He maintains [vdS88, p.159] that E57 has an interpretation where the presupposition of the consequent is preserved unmodified, observing that “one way to achieve this [interpretation] is to read at the conference with contrastive stress”, which he notates as in E58. Van der Sandt backs up his claim that the presupposition is preserved by noting that that the continuation in E59 “is completely natural, and clearly presupposition preserving”:

E58 If someone AT THE CONFERENCE solved the problem, it was Julius who solved it.

E59 If someone AT THE CONFERENCE solved the problem, it was Julius who solved it, but if it was solved at the Nijmegen Institute of Technology, it certainly wasn’t Julius.

But why should we accept the legitimacy of adding stress? It seems to me that the stressed antecedent itself tends to produce an inference to the problem having been solved, whether one wants to call this presuppositional or not. As far as I can tell, E60 and E61 below, in which the cleft has been removed, are most plausibly uttered in situations where it has been established that the problem has been solved, and the only remaining questions are where it was solved and by who. These examples suggest that the problem has been solved to just the extent that E58 and E59 do, which shows that the conditionalisation of the consequent presupposition is, once again, completely irrelevant.

E60 If someone AT THE CONFERENCE solved the problem, then JULIUS solved it.

E61 If someone AT THE CONFERENCE solved the problem, then JULIUS solved it, but if it was solved at the Nijmegen Institute of Technology, it certainly wasn’t Julius.

17 What I have to say about the interaction between topicality and presupposition, which is more the statement of a problem than the suggestion of any formal solution, can be found in [Bea94b].
Van der Sandt seems to assume that stress does not determine which interpretations are available, but merely helps us find readings which are there anyway. But why is it that in the following example, E62, I cannot find any 'interpretation' at all whereby it is established that the problem is solved? The simplest answer would be that in this example, unlike in E58, the antecedent does not itself carry a presupposition that the problem is solved, and the presupposition of the consequent is, as predicted in the Strong Kleene and Karttunen and Peters' models, weakened to the point of triviality.

E62  If someone at the conference SOLVED/(FINALLY solved) the problem, it was Julius who solved it.

Conditionalised presuppositions are problematic, but, I know of no convincing general arguments against them. The discussion above, combined with the Soames discussion in [So82], dispenses with all purported counterexamples with which I am familiar. In the Strong Kleene and Karttunen and Peters' systems, such presuppositions arise not only from conditionals themselves, but also from disjunction and conjunction. Disjunction is discussed above. Regarding weakening of presuppositions in conjunctions, I think that the weakening of presuppositions on the right hand side is appropriate, but I am sceptical about weakening of presuppositions on the left, this latter weakening being found in Strong Kleene, but not in Karttunen and Peters' system. In the following two examples another member of the family falls foul of John. The question is whether in each case it is presupposed that John's mother is dead, this being, let us assume, triggered by the factive regrets:

E63  John killed his mother but regrets that she's dead.
E64  John regrets that his mother is dead, but he killed her.

We can answer this question by embedding the sentences in the antecedent of a conditional, which produces the following pair:

E65  If John killed his mother but regrets that she's dead, he'll give himself up.
E66  If John regrets that his mother is dead, but he killed her, he'll give himself up.

I believe that that E65 does not suggest (without stress on killed, c.f. the discussion above) that John's mother is dead, but that E66 does indicate this. If this is right, then the weakening of the left conjunct in Strong Kleene is incorrect, but the weakening of the right is justifiable. I will return to the issue of conditionalised presuppositions repeatedly throughout this dissertation.

\(^{18}\text{We will see some more examples later where weakening of a presupposition on the right hand side of a conjunction seems appropriate, e.g. E107 in Chapter 4.}\)
having multiple (propositional) realizations, which a connective is interpreted, and in this way reasons to each connective

Kaplan [1989] argues that process-sentential considerations can influence the way in which a sentence's propositional content is understood. He also provides essential conditions for realizing a lexical item and enabling realization of a functional expression. Further, he supposes that a meaning of a sentence embeds a strong interpretive system (within an additional a set of additional a set of additional

Poincare-Roberts [Bug9a] discuss a meta-linguistic use of negation within a negation-similar to that which is below called accommodation-a general theory of discourse interpretation. Further, he supposes that a meaning of a sentence embeds a strong interpretive system (within an additional a set of additional

In addition, in some of the work presupposition generation

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Karttunen and Peters [KP79] assume that conversational implicature will strengthen

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Given such difficulties facing multivalent and partial accounts, it is not surprising that

3.3.4 Pragmatic Extensions

Multivalence and Partiality
Chapter 4

Cancellation and Filtering

The theories to be discussed in this chapter have two things in common. Firstly, they are, in a sense, the only true projection theories: the set of presuppositions associated with the utterance of a complex sentence is a subset of the set of elementary presuppositions of that sentence. We can thus say that these theories define (relative to a context) a projection function which determines for each elementary presupposition whether it is projected or not. Secondly, this projection function is context sensitive. Thus whereas in the theories discussed in the previous chapter presupposition was understood as a binary relation between sentences (or formulae), the theories to be discussed now involve definitions of presupposition as a three place relation between a pair of sentence and a context of evaluation. Alternatively, if an utterance is defined as a pair of a sentence (or set of sentences) and a linguistic context, then presupposition becomes a two place relation between an utterance and a sentence.

What are termed here cancellation and filtering are usually regarded as opposing approaches to the treatment of presupposition. However, the two are closely related variations on a single theme:

1. The grammar and lexicon together encode a way of calculating for each simple sentence a set of potential presuppositions, each of which is a proposition.

2. The set of presuppositions of a complex sentence is a subset of the union of the potential presupposition sets of the simple subsentences. Call this subset the projection set.

3. The calculation of the projection set is sensitive to linguistic context (conceived of as a set of sentences), and relies on one or both of the following two strategies:

   **Local filtering** For each subsentence \( S \) consisting of an operator embedding further subsentences as arguments, \( S \) not only carries its own potential presup-
positions, but also inherits a subset of the potential presuppositions of the arguments.

Global cancellation Pragmatic principles determine a function from tuples consisting of the context, the set of potential presuppositions, the assertive content of the sentence, and (except in the version in [vdS88]) a set of Gricean implicatures of the sentence, to that subset of the potential presuppositions which is projected.

I will review the filtering theory of Karttunen, and then discuss the cancellation accounts of Gazdar, Mercer and van der Sandt. Finally it will be shown that although filtering and cancellation accounts can be combined into a theory which is descriptively superior to any of the individual filtering or cancellation accounts, there remain serious problems.

4.1 Plugs, Holes and Filters

Karttunen [Kar73] introduced the following taxonomy (pp.178):

Plugs: predicates which block off all the presupposition of the complement sentence [examples include 'say', 'mention', 'tell, ask'];

Holes: predicates which let all the presuppositions of the complement sentence become presuppositions of the matrix sentence [examples include 'know', 'regret', 'understand', 'be possible', 'not'];

Filters: predicates which, under certain conditions, cancel some of the presuppositions of the arguments [examples include if-then, 'either-or', 'and'].

4.1.1 The First Version of Local Filtering

Assume we are given a function $\pi$ mapping simple sentences or complex constructions onto sets of potential presuppositions, and some taxonomic division of sentential predicates (apart from conditionals, disjunctions and conjunctions) into the classes of plugs and holes. Representing the set of presuppositions associated with a sentence 'A' as $P(A)$, the first version of Karttunen's filtering conditions in his 1973 paper may be stated recursively as follows:
Definition D6 (Karttunen '73, Sentence Presuppositions)

1. \( P(S) = \pi(S) \) for simple sentences \( S \).

2. \( P(S') = P(S) \cup \pi(S') \) where \( S' \) consists of a hole-predicate embedding a sentence \( S \).

3. \( P(S') = \pi(S') \) where \( S' \) consists of a plug-predicate embedding embedding any further sentence.

4. If \( S \) is 'If \( A \) then \( B \)' or '\( A \) and \( B' \) then \( P(S) = P(A) \cup \{ p \in P(B) \mid A \nleq p \} \).

5. If \( S \) is 'Either \( A \) or \( B' \) then \( P(S) = P(A) \cup \{ p \in P(B) \mid \neg A \nleq p \} \).

Let us see how these definitions apply to an example. In E67, A, B and C are the substrings marked by the relevant brackets:

E67 If \([A\] Jane and her husband are going on holiday], then \([B\] its possible that \([C\) their neighbours don't realise that Jane and her husband are going on holiday]]

1. Set \( D = \)'their neighbours realise that Jane and her husband are going on holiday'.
   Now \( P(A) = \pi(A) = \{ \text{Jane has a husband}\}, \) and \( P(D) = \pi(D) = \{ \text{Jane and her husband have neighbours}, \text{'Jane and her husband are going on holiday'}\} \).

2. Since 'not' and 'its possible that' are both holes and introduce no further potential presuppositions, \( P(B) = P(C) = P(D) \).

3. So \( P( \text{if } A \text{ then } B') = \{ \text{'Jane has a husband'} \} \cup \{ p \in P(B) \mid \text{'Jane and her husband are going on holiday'} \nleq p \} = \{ \text{'Jane has a husband'}, \text{Jane and her husband have neighbours'}\} \).

Thus E67 is predicted to presuppose that Jane is married and that her and her husband have neighbours. The potential presupposition that Jane and her husband are going on holiday (triggered in the consequent) is filtered out.

4.1.2 The Revised version of Local Filtering

Karttunen notes the following counterexample to the first version of his filtering conditions. Suppose that Fred thinks Mormons wear special 'holy underwear', and he suspects that Geraldine is a Mormon. One voyeuristic night Fred catches sight of Geraldine wearing an ordinary bra and panties, and exclaims:

E68 Either \([A\) Geraldine is not a Mormon] or \([B\) she has given up wearing holy underwear].
Karttunen maintains that the presupposition from the second disjunct (that Geraldine used to wear holy underwear) ought to be filtered out, but is not because it is not entailed by the negation of 'Geraldine is not a Mormon.' He suggests that the problem should be repaired by relaxing the filtering conditions to take into account the propositions holding in the context previous to the utterance. Thus rather than canceling presuppositions which are entailed by the negation of the first disjunct alone, he suggests canceling of presuppositions which are entailed by a combination of some set of propositions holding in the context of utterance and the negation of the first disjunct. Similar modifications for the other connectives result in the following revised filtering rule, where F is some possibly null set of "assumed facts" (a notion he never makes fully explicit):

Definition \( D7 \) (Karttunen '73, Utterance Presuppositions)

1. \( P(S) = \pi(S) \) for simple sentences \( S \).
2. \( P(S') = P(S) \cup \pi(S') \) where \( S' \) is a syntactic construction containing a hole-predicate embedding a sentence \( S \).
3. \( P(S') = \pi(S') \) where \( S' \) is a syntactic construction containing a plug-predicate embedding any further sentence.
4. If \( S \) is 'If \( A \) then \( B \)' or '\( A \) and \( B \)' then \( P(S) = P(A) \cup \{ p \in P(B) \mid \{ F \cup \{ A \} \} \not\models p \} \).
5. If \( S \) is 'Either \( A \) or \( B \)' then \( P(A) = P(A) \cup \{ p \in P(B) \mid \{ F \cup \{ \neg A \} \} \not\models p \} \).

Here is how the new definition applies to E68:

1. \( P(A) = \emptyset \), and \( P(B) = \{ \text{'Geraldine has worn holy underwear'} \} \).
2. Set the context \( \sigma = \{ \text{'All Mormons have for some time worn holy underwear'} \} \).
3. Assuming that the negation of 'Geraldine is not a Mormon' is just 'Geraldine is a Mormon':
   \( P(\text{'either \( A \) or \( B \)'}) = \emptyset \cup \{ p \in P(B) \mid \{ \text{'All Mormons have for some time worn holy underwear'}, \text{'Geraldine is a Mormon'} \} \not\models p \} = \emptyset \). Thus no presupposition is predicted.

Karttunen's revised filtering conditions probably constitute the first formal definition of presupposition which concerns the presuppositions of utterances rather than sentences, although the philosophical remarks of Strawson (and arguably of Frege) had also pointed to the relevance of context. However in Karttunen's original formulation it is unclear whether the "(possibly null) set of assumed facts", is relative to some particular occasion of utterance or existentially quantified over all occasions. It seems natural to assume the former, especially in the light of Karttunen's later work [Kar74], which will be discussed in the next chapter.
4.2 Global Cancellation

The model presented by Gazdar in [Gaz79a], like Karttunen's revised filtering model, is context sensitive, provides an account of the presuppositions of utterances rather than sentences, and predicts the presuppositions of an utterance to be a subset of the potential presuppositions of the component sentences. Unlike Karttunen's model, the presuppositions are not calculated by bottom-up filtering but by a global cancellation mechanism. All the potential presuppositions of component sentences are collected together into one set, and from that set are removed any members which conflict with (1) propositions in the previous context, (2) the entailments of the utterance, (3) various implicatures associated with the utterance, or (4) each other. Those potential presuppositions surviving this tough selection process go on to become full presuppositions of the utterance.

The basic idea that something cannot be presupposed if that would conflict with implicatures of the utterance is already found in Stalnaker's discussion of Karttunen's full-factive/semi-factive distinction [St74, pp.207–210]. Further, Soames proposed independently of Gazdar that defeat by implicature should be the central notion of a theory of presupposition projection: "A speaker who utters a truth-functional compound, question or epistemic modal indicates that he is presupposing all of the presuppositions of its constituents unless he conversationally implicates (or explicitly states) otherwise." [So79, p.653]. Kempson [Kem75] and Wilson [Wi75] (as discussed earlier) had both recognised that conversational factors determine whether or not a presupposition is projected, although their general strategy was of trying to find implicature-based explanations of all cases where presuppositions do project, rather than assuming by default that they project and only seeking implicature-based explanations of cases where presuppositions are canceled.

Gazdar's theory of presupposition, however, provides the first formalisation of this type of account. It is set within a dynamic model of meaning, in which discourse contexts — sets of propositions — are progressively updated with the information in succeeding utterances. Note that the dynamism is found only at the level of texts, and does not extend downwards to the interpretation of the constituents of sentences. In this respect Gazdar's model contrasts with the accounts of presupposition proposed by Karttunen [Kar74] and Heim [Hei83a], as well as with the accounts of anaphora proposed by Kamp [Kam81], Heim [Hei82, Hei83b] and Groenendijk and Stokhof [GS91a], all of which employ dynamic interpretation at the subsentence level.

Central to Gazdar's model is his notion of *satisfiable incrementation*. The satisfiable incrementation of a context X with a set Y of propositions is just the original context plus all those propositions in Y which cannot introduce inconsistency, where a proposition *y cannot introduce inconsistency* just in case all consistent subsets of $X \cup Y$ are still
consistent after addition of \( y \). The following definition (almost identical to Gazdar’s) results:

**Definition D8 (Consistency, Satisfiable Incrementation)**

\[
\text{cons}(X) \text{ iff } X \not\models \bot \\
X \cup \{ y \} = X \cup \{ y \} \subseteq (X \cup Y) \cup \{ y \} \cup \{ z \} \}
\]

For example, if \( X = \{ p, q \} \) and \( Y = \{ -p, r, s, -s \} \), with all atomic formulae assumed logically independent, then \( X \cup Y = \{ p, q, r \} \). The proposition \(-p\) cannot be added because it is inconsistent with \( X \), \( s \) cannot be added because there are consistent subsets of \( X \cup Y \) (e.g. \( \{ p, q, -s \} \)) which become inconsistent when \( s \) is added to them, and similarly for \(-s\).

### 4.2.1 Cancellation without Implicatures

Gazdar is concerned with reasoning about the hearer’s knowledge of the speaker, and for that reason the propositions in a Gazdarian context are formulae of an epistemic logic, in fact Hintikka’s logic of knowledge and belief [Hi62]. For the moment, let us simplify by ignoring implicatures. In that case updating a context \( \sigma \) with the information conveyed by some utterance of a sentence \( S \) proceeds as follows: firstly the proposition that the speaker knows what is asserted by \( S \) to be true is added to the context, and the resulting context is satisfiably incremented with the potential presuppositions. Thus if the assertion is \( \alpha \), and the set of potential presuppositions of the sentence is \( \pi \) (obtained by taking the union of the potential presuppositions of all the constituents), then the final context is just \( \sigma' = (\sigma \cup \{ K(\alpha) \}) \cup \pi \).

All the potential presuppositions are also assumed to be prefaced with a \( K \) operator: we will have more to say about this assumption later. The actual presuppositions of the utterance are just those members of \( \pi \) which survive in \( \sigma' \). Consider E69:

E69 Mary doesn’t KNOW that Bill is happy: he isn’t.

1. Suppose the initial context \( \sigma = \emptyset \). Take the assertion \( \alpha = \neg(K_m(happy(b)) \land \neg happy(b)) \), and the set of potential presuppositions \( \pi = \{ K(happy(b)) \} \).

2. The update of \( \sigma \) with E69 is given by:

\[
\sigma \cup \{ K(\alpha) \} \cup \pi = \{ K(\neg(K_m(happy(b)) \land \neg happy(b))) \} \cup \{ K(happy(b)) \}.
\]

\[\text{Note that because } K \phi \models \phi \text{ is valid in Hintikka’s logic, the context } \sigma' \text{ will entail the assertion } \alpha, \text{ although this is not added explicitly.}\]
3. Since the assertion $\alpha$ entails $K(\neg \text{happy}(b))$, and this is inconsistent with the only potential presupposition, the potential presupposition is not added, and is not a presupposition of E69 in this context (or, for that matter, in any other context).

4.2.2 Adding Implicatures

A striking aspect of Gazdar's theory is that the same mechanism is used for implicature as for presupposition, in both cases the notion of satisfiable incrementation being central. A sentence is associated with potential implicatures of various sorts, as well as potential presuppositions, and the actual implicatures of an utterance are just those potential implicatures which survive satisfiable incrementation. Crucially, the context is updated with the implicatures before the presuppositions, and this has the effect that implicatures can cancel presuppositions. I will not discuss implicature in general, and I will not distinguish between different types of implicature as Gazdar (following Grice) does. Rather I will focus on one type of conversational implicature, that arising from hypothetical statements to the effect that the speaker does not know the hypothesis to be true and does not know it to be false. This type of implicature, arising from the presence of an embedded clause which is not entailed by the whole sentence, is known as a clausal implicature. For example, the sentence 'If Mary is sleeping then Fred is boring' carries potential (conversational) implicatures $\{\neg K \text{sleeping}(m), \neg K \text{sleeping}(m), \neg K \text{boring}(f), \neg K \text{boring}(f)\}$.

**Definition D9 (Gazdar's Context Incrementation)** If sentence $S$ has semantic content $\alpha$, potential implicatures $\iota$, and potential presuppositions $\pi$, then the update of a context $\sigma$ with $S$ is:

$$\sigma' = (\sigma \cup \{K(\alpha)\}) \cup \iota \cup \pi.$$  

Let us consider the treatment of an example:

**E70** If Mary is sleeping then Fred is annoyed that she is sleeping.

1. Set the context $\sigma = \emptyset$,
   set the asserted content $\alpha = \text{sleeping}(m) \rightarrow \text{annoyed}(f, \text{sleeping}(m))$,
   set the potential implicatures $\iota = \{\neg K \text{sleeping}(m), \neg K \text{sleeping}(m), \neg K \text{boring}(f), \neg K \text{boring}(f)\}$,
   and set the potential presuppositions $\pi = \{K \text{sleeping}(m)\}$.

2. All the potential implicatures survive satisfiable incrementation, so:
   $$(\sigma \cup \{K(\alpha)\}) \cup \iota = \{K(\alpha)\} \cup \iota.$$  

3. Since the only potential presupposition is inconsistent with the implicature $\neg K \text{sleeping}(m)$, which has now been added to the context, the potential presupposition does not survive satisfiable incrementation, and is predicted not to be a presupposition of E70 in this context.
4.3 Projecting By Default

One might say that in Gazdar's model whilst Hintikka's logic of Knowledge and Belief plays centre stage, the performance is kept running smoothly only by considerable off-stage direction. This is not to say that Gazdar's model is not thoroughly formalised: it is, but that formalisation is defined at a meta-level. For instance, the notions of incrementation and satisfiable incrementation, although well defined and linguistically motivated, are not expressed in any logical theory. This is not in itself problematic for the linguist, but a logician or computational linguist might find such devices technically ad hoc, and wonder if the notions which Gazdar utilises could be located in a more general (computational) framework, a general logic of information interchange. Part of the motivation for Mercer's theory of presupposition [Me87, Me92] seems to be to express within a logic some of the machinery that in Gazdar's theory is extra-logical. In particular, Mercer takes Gazdar's insight that presuppositions normally project, and are only canceled as a result of conflict with context or implicatures, and formalises that by explicitly encoding Gazdar's potential presuppositions as default inference rules within Reiter's Default Logic. Mercer assumes that such a rule is associated with each presupposition trigger, so for instance the rule associated with the lexical item 'regret' runs:

\[-(\text{regret}(x, \phi)) \land LF(\text{regret}(x, \phi)) : \phi\]

This can be glossed as: if a theory (a set of propositions closed under ordinary first order logical consequence) includes the proposition that \(x\) does not regret \(\phi\), and it includes the proposition that the Logical Form of an utterance contains (arbitrarily deeply embedded) the formula \(\text{regret}(x, \phi)\), and \(\phi\) is consistent with the theory, then, by default, the theory should be extended by addition of \(\phi\) (and further classical logical closure).

By and large, Mercer's theory makes much the same predictions as Gazdar's. As in Gazdar's system, presuppositions triggered in the lexicon become integrated in a hearer's information state (i.e. projected) only by default, and, as in Gazdar's system, the main factors controlling that integration are consistency with asserted facts and consistency with implicatures. One difference is that Mercer does not attempt to prioritise implicatures over presuppositions, presumably because Reiter's default logic does not allow for prioritisation of different defaults (although related non-monotonic logics do allow such prioritisation). As a result Mercer cannot treat conversational implicatures as defaults, since that would result in conversational implicatures and presuppositions mutually canceling each other, contrary to Gazdar's evidence that implicatures cancel presuppositions. Instead, Mercer treats implicatures not as being defaults, but as be-
Cancellation and Filtering

ing indefeasible. This has the effect that they are able to cancel presuppositions, but, given that such implicatures clearly are defeasible\(^2\), this is not a desirable move. However, I take this as a purely technical problem, and not in itself reflective of any new philosophical considerations. A more serious objection to Mercer's theory is simply that the improvements over Gazdar's theory are quite marginal, and one certainly could not claim mathematical beauty as a justification for the introduction of default logic, given that its model theory can best be described as "work in progress", and its proof theory involves a computationally awkward fixed-point construction.

In one respect Mercer's theory deviates significantly from Gazdar's. Karttunen, in [Kar74], describes one of the goals of his work as that of avoiding a theory of part-time presupposition. Yet Gazdar provided just that, in the sense that in his account unwanted presuppositions simply vanish. In a Mercer type account, on the other hand, presuppositions, which are default rules, could be allowed to remain in the context whatever other pragmatic or semantic information is available. Projection or cancellation do not correlate with maintenance or removal of the presuppositional rule, but with the logical validity or invalidity of certain inferences associated with the application of the rule. Mercer identifies a philosophical difference between his theory and Gazdar's, in that Mercer does not even see his account of the projection data as a theory of projection, but as a theory of certain types of inference. In essence this is an inference which allows a hearer to select, from amongst the set of models which satisfy an uttered sentence, a subset of models which are pragmatically preferred. In this respect Mercer's theory may be thought of as a proof-theoretic version of the less formalised theories of [Kem75, Wi75], except that Mercer does assume that presuppositional constructions are conventionally marked as presuppositional in the lexicon, whereas it is not clear that Wilson would wish to accept this.

A consequence of the fact that in Mercer's theory presuppositional rules remain in the context is that presuppositional inferences which would be licensed by a one sentence text could be induced to disappear when additional text is added afterwards. For instance the first sentence of E71 in isolation would be predicted to licence an inference to the existence of a son, but the complete text does not licence this inference.

\(^2\)For instance, in a *modus ponens* argument '\(\phi\) and if \(\phi\) then \(\psi\), so \(\psi\)', a clausal implicature \(-K\phi\) is canceled.

\(^3\)Time permitting, I should have liked to discuss here some other recent and independently developed proposals due to Marcu [Ma94], Morreau [Morr95], Gervas [Ger95], and Schöter [Schö:MS]. All of these accounts specifically target the defeasibility of presuppositions but have, to my taste, a clearer model theoretic interpretation than Mercer's, and as such perhaps capture the Kempson/Wilson intuitions even better. However, all of these theories differ from Kempson's and Wilson's accounts significantly, in that Kempson and Wilson wish to find conversational principles which explain cases of projection, whereas all recent cancellationist theories take projection as the norm, and only try to explain away cases of cancellation.
E71 Jane definitely doesn't beat her son: she's not the violent type. Besides, I'm not even sure that she has a son.

On the other hand in Gazdar's theory, where sentences are processed one at a time, a decision has to be reached at the end of processing the first sentence as to whether the proposition that Jane has a son should vanish into thin air, or be added irretrievably to the context. The theory predicts that the proposition will be added, so that the is text incorrectly predicted to be contradictory. This is not to say that this is a fatal flaw with Gazdar's theory. One could imagine modifying the theory such that speech acts rather than sentences were processed as units, and claiming that E71, although it consists of more than one sentence, comprises only a single speech act. On the other hand, one could also see the stability of presuppositional rules, and the instability of presuppositional inferences, in Mercer's theory as a disadvantage. For if a presuppositional inference follows from a sentence, it is very strange for the utterer to deny the conclusion of that inference a dozen sentences (or several speech turns) later. It seems that if you are going to deny the presuppositions of part of your utterance, you must do it as soon as reasonably possible. Clearly Mercer's theory would need some modification to account for this.

4.4 The Pre- in Presupposition

In what sense is Gazdar's theory an account of 'presupposition'? I do not mean to suggest that it does not provide an account of presuppositional data. I merely mean that the account does not bear any relation to the fairly intuitive notion of presuppositions as previous assumptions. Indeed, since presuppositions are the last things to be added in Gazdar's definition of update, perhaps it would be more natural to call them post-suppositions.4

My own predilection, as will hopefully become clear in the course of this thesis, is for a theory that takes the pre in presupposition seriously, and which in some way associates presuppositions specifically with the initial context in which a sentence is semantically interpreted, or with the initialisation of that context. To me, at least, the major achievement of the theory first presented in van der Sandt's thesis [vdS82], which only appeared in English somewhat later in [vdS88], is that it does succeed in reconciling ideas from Gazdar's cancellation account with what I take to be the intuitive notion of

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4Given the almost identical way in which presuppositions and implicatures are treated in Gazdar's model, one might think presuppositions could better be labeled as a subtype of implicature. This would be in tune with Karttunen and Peters' suggestion [KP77, KP79] that presuppositions in fact comprise an assortment of different implicatures, largely conventional implicatures. It is notable that Karttunen and Peters arguments have singularly failed to effect any general change in use of terminology, if such was their aim.
presupposition. I will term van der Sandt's 1982/87 account his *cancellation* theory, to distinguish it from his later DRT-based theory, to which we will turn shortly.

One crucial but disarmingly simple insight could be said to drive van der Sandt's cancellation theory. Suppose a sentence $S$ can be coherently uttered in a context $\sigma$, and that one of the constituents of $S$ carries a potential presupposition expressible using the sentence $P$. If in $\sigma$ the text made up of $P$ followed by $S$ is coherent, then utterances of $S$ in $\sigma$ will carry the presupposition $P$, i.e. $P$ is projected, and otherwise $P$ is canceled (see [vdS88, pp.185–189]). For example, given a context of utterance which is neutral regarding Bugandan and Adnagubian politics, E72 presupposes that there is a King of Adnagub (since E73 is coherent), but not that there is a King of Buganda (since E74 is incoherent).

E72  The King of Buganda is not balder than the King of Adnagub since Buganda has no King.

E73  There is a King of Adnagub. The King of Buganda is not balder than the King of Adnagub since Buganda has no King.

E74  * There is a King of Buganda. The King of Buganda is not balder than the King of Adnagub since Buganda has no King.

Coherence of a sentence, what van der Sandt expresses as "acceptability in a context", here comes down to the requirement that every clause is both consistent and informative. Van der Sandt's precise definition of acceptability, which is based upon a dynamic view of the interpretation of the logical connectives like that to be discussed in the following section, need not concern us here. So far as sentences containing logical connectives are concerned, a good approximation, which appears adequate for the examples van der Sandt discusses, is to define acceptability of a sentence $S$ in a context $\sigma$ as the requirement that for each clause $S'$ appearing in $S$ (other than within a presuppositional expression) $\sigma$ neither entails $S'$ nor entails the contrary of $S'$. If this requirement is not met, then $S$ will not be a maximally efficient (i.e. compact) way of communicating whatever information it conveys in that context. I simplify by taking a context to be a set of sentences, although van der Sandt allows for contexts to contain certain additional information.
Cancellation and Filtering

Definition D10 (Presuppositions in van der Sandt's Cancellation Account)

Given that all the potential presuppositions (or elementary presuppositions in van der Sandt's terminology) of S are collected in the set \( \pi \), the presuppositions of S in context \( \sigma \) are those propositions \( \phi \) such that:

1. \( \phi \in \pi \)
2. For any \( \psi \in \pi \), \( \sigma \cup \{\phi, \psi\} \not\models \bot \)
3. S is acceptable in the context \( \sigma \cup \{\phi\} \)

Although there are problems associate with this definition\(^5\), the intuition is clear, as the treatment of as treatment of E70 (repeated below) should illustrate:

E70' If Mary is sleeping then Fred is annoyed that she is sleeping.

Suppose that the context is empty. For E70, \( \pi \) is just the singleton set \{Mary is sleeping\}, the one potential presupposition being triggered by the factive 'annoyed'. We can test whether the potential presupposition is actually presupposed by adding it to the context and checking that all the subsentences in E70 not appearing in presuppositional expressions are neither entailed nor contradicted in the resulting context. Since the resulting context \{Mary is sleeping\} entails one of the subsentences, i.e. the antecedent of the conditional, we can conclude that the proposition that Mary is sleeping is not being presupposed, for if it were then E70 would be inefficient, and hence unacceptable.

\(^5\)The definition is essentially that given by van der Sandt as “D-7” [vdS88, p.203]. There appear to be two major errors. A first problem is that the second clause only checks for consistency of pairs of potential presuppositions. It is easy to manufacture an example where all pairs are consistent but the triples are not. Suppose the context \( \sigma \) contains the proposition that exactly two people whistled, and that \( S= '\text{Sherlock has discovered that Watson whistled, or he's discovered that Mycroft whistled, or he's discovered that Moriaty whistled}' \). Now we might take \( \pi \) to be the set \{Watson whistled', 'Mycroft whistled', Moriaty whistled\}. Any pair of elements of this set is consistent with \( \sigma \) although, assuming non-identity of Watson, Mycroft and Moriaty, the three elements together are inconsistent with \( \sigma \). The above definition would incorrectly predict that all elements of \( \pi \) become full presuppositions even in a context where their joint addition produces inconsistency.

A similar problem ensues from the third clause, which checks that addition of each presupposition to \( \sigma \) would not make \( S \) unacceptable, but does not ensure that if all the presuppositions are added to \( \sigma \) the resulting context accepts \( S \). Again we can manufacture a rather artificial example to illustrate the point. Suppose \( \sigma \) is empty, and \( S= 'If John is an only child then he doesn't regret that he has no brothers and he doesn't regret that he has no sisters.' \) It seems plausible that \( \pi \) should be the set \{John has no brothers, John has no sisters\}. Since these are consistent with each other, and since \( S \) is acceptable in either of the contexts produced by adding an element of \( \pi \) to \( \sigma \), van der Sandt predicts that both members of \( \pi \) become full presuppositions. This is inappropriate, since both elements of \( \pi \) taken together entail that John is an only child, so that if both are being assumed then the antecedent of the conditional is uninformative. In a context to which those presuppositions have been added, \( S \) will convey only the same information as the sentence 'John doesn't regret that he has no brothers and he doesn't regret that he has no sisters.'

Clearly the technical apparatus proposed by van der Sandt does not quite square up with what I take to be the intuition behind that apparatus, namely that in a context containing the presuppositions, \( S \) should be maximally efficient. See also Burton-Roberts review article, [Bu89e], for some quite different criticisms of van der Sandt's D-7.
4.5 Presupposition and Entailment

Uncertainty is sometimes expressed about whether potential presuppositions of a simple sentence should also be considered as part of the asserted content (i.e. amongst the entailments) of the sentence. Since in a traditional trivalent account of presupposition it is not even possible to define unentailed presuppositions, one naturally wonders whether the possibility of defining presuppositions which are not also assertions gives the cancellation account significant extra empirical coverage, or whether this possibility is just an artefact. The same issue arises with respect to two dimensional accounts such as Herzgerger's, or that of Karttunen and Peters, which also allow presuppositions not to be part of the asserted content.

Let me digress to point out that although the question has been posed both with respect to different presuppositional constructions, and with respect to various different models of presupposition, it is not always the same question. In particular, whereas Gazdar's model involves presuppositions being epistemic statements, with content like 'The speaker knows that there is a French King', this is not the case in most other models of presupposition, and the question of whether such an epistemic statement is entailed by 'The King of France is bald' is obviously different from the question of whether 'There is a King of France' is entailed. However, the epistemic aspect of Gazdar's potential presuppositions could fairly easily be removed. Suppose that the potential presuppositions \( \pi \) were just taken to be the standard expressions triggered by various lexical items etc., and not statements of the form \( K(\phi) \), and that the asserted content of an utterance was added to the previous context simpliciter, also without embedding under a K operator. Thus the update of a context \( \sigma \) would be given by \( (\sigma \cup \{\alpha\}) \cup !t \cup !\pi \). As things stand this would prevent conversational implicatures, which are inherently epistemic statements, from interacting properly with presupposition, but this is easily rectified. We simply replace the definition of consistency with: \( \text{cons}(X) \iff \{K(x) \mid x \in X\} \not\models \bot \). So this is just Gazdar's theory but with the \( K \)'s added at a later stage. It is not significant that the implicature expressions end up doubly embedded under K operators in the consistency.

\( ^6 \)Both Blok [Blo93] and Horton [Hort87], in developing theories of presupposition along Gazdarian lines, have suggested refinements of the underlying epistemic logic. Presumably, then, they would disagree with the impression I have given that the epistemic nature of Gazdar's presuppositions is inessential. But here I should qualify my intentions. I do think that most (or all) of the data dealt with in [Gaz79a] could be handled without making presuppositions into intrinsically epistemic statements, but I accept firstly that there may be philosophical motivation for presuppositions being epistemic, and secondly that there may be further applications of the theory for which the epistemic operators are essential. With regard to the first point, it should be noted that in this thesis presuppositions will be given an epistemic interpretation (roughly as what the speaker believes, or acts as if he believes, is mutually known), although this epistemicity will not be explicit in the formal system. With regard to the second, both Blok and Horton model the beliefs of multiple agents in a dialogue setting, whereas Gazdar concentrates on just the beliefs of just one agent (the speaker). When providing a full account of the evolution of multiple sets of beliefs, it is natural to utilise explicit belief operators.
test, because of the validity of \( K\neg K\phi \models \neg K\phi \) and \( KK\phi \models K\phi \) in the modal logic which Gazdar uses (Hintikka's epistemic logic).

To return from the digression, regarding Gazdar’s account it is very easy to answer the question of whether presuppositions should be part of the asserted content, at least for the majority of presuppositional expressions. Although presuppositions arising from triggers in embedded contexts produce defeasible presuppositional inferences, the corresponding inference connected with a trigger in a simple affirmative sentence is typically indefeasible. Gazdar [Gaz79a, pp.119-123] describes the inferences associated with factive verbs, definite descriptions, aspectual verbs, and clefts as being indefeasible in simple affirmative sentences. Since potential presuppositions are always defeasible in Gazdar’s model, and since the only inferences which are indefeasible in his model are those associated with the asserted content, Gazdar is forced to claim that the potential presuppositions of these constructions are also entailments.

Although Gazdar hedges with regard to the question of whether presuppositions should be part of the asserted content for other classes of presupposition trigger, he discusses at least one example where it is crucial to his analysis that the trigger does not entail the presupposition. The following, from [Gaz79a, p.150], is of a type discussed by Heinämäki [Hein72]:

\[
E75 \quad \text{Max died before he finished his autobiography.}
\]

Standard projection tests indicate that the complement of a before clause is presupposed. But in E75 the presupposition (that Max finished his autobiography) is apparently canceled, in spite of the fact that the ‘before’ clause occurs in a simple positive context. Gazdar’s successful treatment of this example implicitly assumes that the presupposition is not entailed. Van der Sandt makes it clear [vdS88, pp.196-198] that he regards the flexibility of a theory in which presuppositions do not have to be entailed as a major boon. Apart from ‘before’ clauses, van der Sandt also suggests that three other types might be non-entailed. Firstly, Fillmore’s [Fi71] verbs of judging, verbs like ‘accuse’ and ‘criticise’: E76 is claimed to presuppose but not entail E77. Secondly, van der Sandt mentions emotive factives, like ‘is glad’, ‘regrets’, a class of verbs which Gazdar [Gaz79a, p.122–123] argues do entail their presuppositions. Gazdar’s claim runs contra to earlier observations of Klein [Kl75], discussed by Gazdar, that utterances of sentences like E78 do not indicate the complement of ‘regret’ to be true. Thirdly is the particle ‘even’: E79 has been claimed (and this dates back as far as Horn’s [Ho69]) to be truth conditionally synonymous with E80.

\[
E76 \quad \text{John criticised Harry for writing the letter.}
\]

\[
E77 \quad \text{Harry wrote the letter.}
\]
E78 Falsely believing that he had inflicted a fatal wound, Oedipus regretted killing the stranger on the road to Thebes.

E79 Even John left.

E80 John left.

I do not accept that in any existing cancellation account a case has been made for allowing non-entailed presuppositions. For making this move creates as many problems as it solves. Firstly, let us consider 'before'. According to cancellation accounts, presuppositions can be suspended if they contradict entailments or implicatures. Thus E81, where the presupposition that Max won is explicitly denied, and E82, where the first sentence carries an implicature that the speaker does not know whether Max won, should both be cases where the presupposition associated with the before clause is unproblematically canceled. Yet both examples are very odd.

E81 ?Max trained assiduously before he won the race, and/but never did win it.

E82 ?Perhaps Max won the race. He trained assiduously before he won the race.

In cases where the presupposition of a 'before' clause is canceled, there is always some causal connection between the sentence which the 'before' clause modifies and the main clause. Thus in understanding E75 we are able to make a causal connection between the sentence 'Max died' and the 'before' complement 'Max finished his autobiography'. Perhaps we could say that there is a hypothetical causal chain which runs from a time just in advance of Max's death to a time when Max's autobiography is complete, but that this chain is interrupted by Max's death.7 Regarding instances of before in positive contexts, it appears crucial to the cancellation that the interruption to the relevant causal chain is announced by the main clause. This is not be reflected in current cancellation accounts, all of which allow a wide range of extraneous conflicting information to cancel the presupposition.

An additional problem with postulating an unentailed presupposition occurs with embedding of the presuppositional construction in an intensional context: Sentence E83 suggests E84. If the 'before' complement is not entailed, none of the cancellation accounts predict this inference.8

7This suggestion that the presence of a causal chain is crucial is reminiscent of Moens and Steedman's analysis of the oddity of examples like When my car broke down, the sun set [MS88, p.23].

8On the other hand, assuming an entailed presupposition, whilst it would produce an inference from E83 to E84, would not predict defeasibility of such inferences. Thus 'Jane thinks Max died before he finished his autobiography' does not suggest that Jane thinks Max finished his autobiography. So I am not saying we should conclude outright that the complement of 'before' is part of the asserted content, but that no current theory gets the data right whether or not this conclusion is accepted. Another
E83 John thinks Max trained assiduously before he won the race.

E84 John thinks Max won the race.

Regarding verbs of judging, postulation of an unentailed presupposition is once again problematic. Firstly, consider embedding in an intensional context:

E85 Bill thinks John criticised Harry for writing the letter.

E86 Bill thinks Harry wrote the letter.

If E86 follows from E85, and I would say it follows at least as strongly as E76 does, then this will be unexplained under a cancellation account which does not assume entailment of the presupposition.

Secondly, observe that (on my intuitions at least) in each of the following two pairs of examples, the first does not suggest that Mary actually did whistle, but the second does:

E87 a. John didn't criticise Mary for whistling, and didn't even indicate that she had whistled.
   b. John didn't realise that Mary whistled, and didn't even indicate that she had whistled.

E88 a. If John indicated that Mary whistled, he criticised her for whistling.
   b. If John indicated that Mary whistled, he realised that she had whistled.

If I am right about this data, then that might push us in favour of postulating a presupposition for 'X criticised Y for doing Z' other than 'Y did Z'. I am not convinced that 'X indicated that Y did Z' is presupposed, but it does not seem implausible that it is along the right lines, and that there is a further non-presuppositional inference step involved in reaching 'Y did Z'.

open problem is the licensing of negative polarity items in 'before' (but not 'after') clauses, as in 'Max died before finishing anything'. Explanations have been offered for this (see e.g. [SWZ94]), no unitary explanation has been offered both for the negative polarity item and for the cancellation data. I am (unjustifiably) optimistic that one day, when we really find out what 'before' means, a clear link between the two phenomena will emerge.

9Note that 'X indicated that Y did Z' is certainly entailed — witness the oddity of 'John criticised Mary for whistling, but he never indicated that she had whistled'. It is not clear whether the sort of analysis I am proposing would apply to other verbs of judging. I have considered just one other — 'dislike'. For instance, it might be said that 'John disliked Mary for whistling' presupposed but did not entail 'Mary whistled'. Here I would propose the alternative presupposition 'John believed Mary whistled'. This is backed up by the fact that neither of the two cases 'John didn't dislike Mary for whistling, since he didn't believe she did whistle' and 'If John believed Mary whistled, then he disliked her for whistling' seem to suggest that Mary did whistle. Once again, we can show using parallel examples that it is not simply
The argument from embedding in an intensional context can again be applied in the case of emotive factives. If the emotive 'regret' presupposes its complement but does not entail it, then cancellation accounts will fail to predict that E90 follows from E89, although they will typically predict that E91 follows:

E89 John thinks Mary regretted leaving.
E90 John thinks Mary left.
E91 Mary left.

Moving to the case of 'even', here, I think, the argument that there is a non-entailed presupposition simply falls flat on its face. It may be attractive to analyse E79 as being truth-conditionally equivalent to E80, but I know of no good reason to doubt that the presupposition is entailed. Horn's original suggestion of truth conditional synonymity was not couched within a formal theory where it was possible to make a distinction between entailed and non-entailed presuppositions, and the evidence he presented in [Ho69] does not bear on the issue. It is natural to take the presupposition of E79 to be E92. But the negation of E92 is inconsistent with a simple positive assertion of E79, as witnessed by the oddity of E93. To account for this in a cancellation theory, the natural move, maybe the only move, would be simply to assume that the presupposition was also an entailment. 10

E92 People other than John left.
E93 ?Even John left, but nobody else did.

4.6 Combining Cancellation and Filtration

The cancellation and filtering theories are largely complementary in terms of which data they get right. For instance, Karttunen's theory fails (without the introduction implicatures associated with 'John believed...' that are doing the canceling. Both 'If Bill believed Mary whistled, then John disliked her for whistling' and 'John didn't dislike Mary for whistling, since Bill didn't believe she did whistle' do seem to suggest (to me at least) that Mary whistled.

The inference from 'John believed Mary whistled' to 'Mary whistled' of course would still need explaining, presumably as some sort of conversational implicature. If such an explanation could be found, then it might also apply in the case of emotive factives. For instance, we might say that 'John regretted that Mary whistled' presupposed not 'Mary whistled', but 'John believed Mary whistled'. Note also that Karttunen and Peters, in [KP77] argue that verbs of judging are not associated with a presupposition, but that the purported presupposition in fact results from a particularised conversational implicature. However, they admit that they are unable to explain why this implicature should arise when the verb of judging is embedded under negation.

10Embedding 'even' in an intensional context does not appear to produce clear results. This may be because even is an anaphoric trigger, of the type discussed later in §6.8.3.
of a second plug negation, as well as other plug connectives) on presupposition denial cases, whilst Gazdar's theory fails on a class of examples like the following (as discussed by Heim [Hei83a] and Soames [So82]):

E94 If none of Mary's friends come to the party, she'll be surprised that her best friends aren't there.

E95 If John is munching his way through a packet of biscuits, then Bill will be glad that John is eating something.

These are cases where the antecedent of a conditional strictly entails a presupposition of the consequent, so that Karttunen, correctly I think, predicts cancellation. Thus I take it that a hearer would not infer from E94 that Mary's best friends will not be at the party, and would not infer from E95 that John is eating, although these presuppositions are triggered in the respective consequents of the conditionals. In each case, all of the cancellation accounts discussed predict projection of the presupposition. Earlier I discussed a case E70 where a presupposition triggered in the consequent of a conditional was canceled by a clausal implicature generated in the antecedent. But in E94 and E95 the relevant implicature is not strong enough. For instance, in E95 a clausal implicature is generated that the speaker does not know whether John is munching his way through a packet of biscuits, but this implicature does not imply that the speaker does not know whether John is eating at all, and the presupposition remains unchecked. To put this in terms of van der Sandt's cancellation model, addition to the context of the elementary presupposition that John is eating something does not make the conditional inefficient (for believing that John is eating something does not determine whether or not he is munching his way through a packet of biscuits, and does not determine Bill's mental state), and so this presupposition is added.

\[11\] In E94, focal stress on 'surprised', especially combined with destressing of 'that her friends aren't there', does seem to encourage the presuppositional inference. Similar comments apply in the case of E95. A possible line of explanation for this type of phenomenon is developed in my [Bea94b].

\[12\] Landman [La81] proposed a solution to this problem, which essentially consisted of strengthening conversational implicatures so as to force cancellation of the problematic presuppositions. This may solve the immediate problem with presuppositional predictions, but in the process creates quite undesirable effects from the implicatures themselves. See Soames' discussion in [So82]. It should be noted that the problems faced by cancellation models with this type of example do not result from the 'if-then' construction specifically, since variants on the examples which do not use this construction are still problematic. In each of the following triples, the cancellation models predict a contrast between the (a) and (b) sentences. Specifically they predict that the presupposition glossed in (c) is canceled in the (a) case by a clausal implicature, but projected in the (b) case. I am unable to detect any such contrast.

- (a) Either Mary's best friends will come to the party, or she'll be surprised that her best friends aren't there.
- (b) Either some of Mary's friends will come to the party, or she'll be surprised that her best friends aren't there.
- (c) Mary's best friends won't be at the party
To move to a different type of example which is discussed by van der Sandt in [vdS88], consider the following:

E96 John claimed that the President of France is coming to dinner: the idiot doesn't realise that France is a monarchy!

Karttunen identifies 'claim' as a plug, so that on his account the presupposition that France has a President vanishes, whilst the presupposition that France is a monarchy is projected. However, on Gazdar's or Mercer's cancellation accounts, the potential presuppositions that there is a President of France and that France is a monarchy conflict, so these accounts incorrectly predict that a hearer of this example would not infer that the speaker takes France to be a monarchy.

There are, then, many cases where Karttunen correctly predicts that some presupposition is filtered, but where various of the cancellation accounts incorrectly predict projection. On the other hand, there are numerous examples where the cancellation accounts correctly predict that some presupposition is canceled, but where Karttunen's filtering model incorrectly predicts projection. Witness, for instance, the following two examples repeated from above. In the first, the elementary presupposition that there is a King of France is canceled, and in the second the elementary presupposition that there is a King of Buganda is canceled. In both cases this is predicted by the cancellation models, but not by Karttunen's filtering model.

E23' If the King of France is bald, then I'm a Dutchman: there is no King of France!

E46' Either the King of Buganda is now opening parliament, or else the person who told me Buganda is a monarchy was wrong.

Having observed the complementarity in coverage between Gazdar's theory and a later version of Karttunen's theory, Soames [So82] proposed a synthesis of the two accounts. However, the later version of Karttunen's theory (to which we will turn shortly) is not a filtering theory in the sense defined above. The presuppositions that a complex sentence is predicted to have are not a subset of the potential presuppositions of its parts. This complicated the Soames attempt to unify the insights of the two account in a single theory. To give an idea of the difficulties faced, ask yourself this question: when looking for a synthesis between two accounts, where the first account makes all presuppositions members of the set of potential presuppositions, and the second account does

* (a) Jane and Mary think Bill ate something, and they think he knows that they know that he ate something.
(b) Jane and Mary think Bill munched his way through a packet of biscuits, and they think he knows that they know that he ate something.
(c) John ate something.
not, should the resulting theory be expected to make all presuppositions members of the set of potential presuppositions? (Soames in fact answers in the negative.) A much simpler integrated theory, but one which still preserved Soames' central insight, could be formed by combining the Karttunen 1973 theory, as discussed above, with Gazdar's. The most obvious way to join the two theories so as to address both defeat of presuppositions by inconsistency and filtering of presuppositions which are locally entailed, would simply be to take the intersection of the set of presuppositions predicted by each of the two models. Such a joint Gazdar-Karttunen model would provide a formidable account of presupposition, combining relative simplicity with a clear improvement over any of the other models discussed in this section. But it would still face a number of problems common to all cancellation and filtration theories, and most of these problems remain unsolved in the combined account that Soames did in fact propose.

4.7 Quantification in Cancellation and Filtering Models

The cancellation and filtering models can be likened to the two dimensional theories discussed in the previous chapter in that meaning is divided into separate presuppositional and assertional components, although the method the models discussed in the current chapter use to calculate the presuppositional component is quite different from that in the two dimensional accounts. It was shown in the previous chapter that in a two dimensional theory such as that of Karttunen and Peters difficulties arise regarding the treatment of quantified presuppositions, i.e. cases where an elementary presupposition appears free in some variable, and that variable is bound by a quantifier. Those problems arise precisely because presuppositional and assertional components are separated, so that no variable binding can occur between them. Thus one should expect that similar problems will occur with cancellation and filtering theories. It is, indeed, notable that (to my knowledge) none of the cancellation and filtering models discussed in this chapter have been applied to the problem of quantified presuppositions.

One thing that marks the cancellation theories apart from Karttunen and Peters' model is that for most triggers, cancellationists assume that the presupposition is also part of the asserted content. This assumption alleviates somewhat the problem of quantified presuppositions. For instance, if 'curtsied' not only carried a presupposition that its argument is female, but also entailed it, then 'Somebody curtsied' would presumably entail that some female curtsied. But if examples like 'Somebody curtsied, and I was surprised to see that it was a man' lead us to postulate that the presupposition, by virtue of its cancellability in positive sentences, was not part of the asserted content, problems would ensue. For without the assumption of an asserted presupposition, 'Somebody curtsied' would, just as in the Karttunen and Peters model, presuppose that somebody
was female, and assert that somebody performed the act of curtseying, and not provide any implication that the performer of the curtseying act was female.\textsuperscript{13}

Even if presuppositions are assumed to be asserted, problems arise. For instance, it seems reasonable to maintain that the following sentence cannot be true unless there is some watch-owning woman who realised that if her watch was slightly wrong she’d be in danger of shooting the wrong man:

\begin{equation}
\text{E97} \quad \text{Exactly one woman realised that if her watch was slightly wrong, she'd be in danger of shooting the wrong man.}
\end{equation}

If 'her watch' is taken to have a presupposition of the form \textit{has-a-watch}(x), an open proposition, then it is completely unclear what should be done with this proposition in a cancellation theory. Saying that the open proposition is presupposed (with the effect of universal quantification) is inadequate, because the presupposition is clearly only relevant to some salient set of women. On the other hand, the mere fact that \textit{has-a-watch}(x) is asserted in its local context does not help, for the phrase 'her watch' occurs within a conditional. The assertion would be predicted to be something along the lines of 'exactly one woman came to believe that if she owned a watch and that watch was slightly wrong then she would be in danger of shooting some wrong man', but this would not enforce that it was a watch owning woman who had the relevant realisation.

\section*{4.8 Conditional Presuppositions}

Consider the following stock examples (as in [Kar73, p.184], the general type and the second example being attributed by him to McCawley):

\begin{itemize}
\item \textbf{E98} If LBJ appoints J. Edgar Hoover to the cabinet, he will regret having appointed a homosexual.
\item \textbf{E99} If Nixon Invites Angela Davis to the Whitehouse, then Nixon will regret having invited a black militant to his residence.
\end{itemize}

In the consequent of E98 the elementary presupposition that LBJ has appointed a homosexual is triggered, and in the consequent of E99 the elementary presupposition

\begin{itemize}
\item\textsuperscript{13}This could be construed as yet another argument (c.f. §4.5) for not allowing non-asserted presuppositions in the cancellation models, for allowing any non-asserted presupposition would present major problems as soon as quantification into presuppositions was allowed. Take the case of ‘before’. If the presupposition (that what is described in the complement of ‘before’ actually took place) were not entailed, then it is hard to see how any sensible interpretation could be given for a sentence like ‘Everybody knocked before entering’. An assertion that everybody knocked and presupposition that everybody entered, even if such a presupposition could be derived, would still fail to enforce the proper temporal relationship between each knocking event and the following entering event.
\end{itemize}
that Nixon will have invited a black militant to his residence is triggered. In neither case would one expect a hearer to conclude that the elementary presupposition holds, so it is reasonable to claim, as Gazdar does with respect to a slight variant, that the presupposition is simply canceled. Gazdar [Gaz79a, pp.151–152] suggests that his treatment of this type of example represents one of the strengths of his theory. I suggest the contrary, firstly because I believe examples like these do carry non-trivial presuppositions (albeit not the elementary presuppositions triggered in the consequents), and secondly because I do not accept the assumptions Gazdar makes in order to demonstrate that his system yields cancellation. Gazdar’s argument as he applies it to a slight variant on E99 appears to run as follows:

1. A clausal implicature is triggered of the form ‘The speaker does not believe that Nixon invites Angela Davis.’

2. The context contains the fact that Angela Davis is a black militant.

3. The phrase ‘a black militant’ can be anaphoric on the NP ‘Angela Davis’, so that the potential presupposition can be given as ‘The speaker believes Nixon invites Angela Davis.’

4. The clausal implicature conflicts with the potential presupposition, and correctly prevents projection.

My first objection stems from the fact that when somebody who (like me) has no knowledge of Angela Davis’ skin colour or political tendencies is confronted with E99, they will tend to infer that she is a black militant. Thus the assumption that the context contains the fact that Davis is a black militant seems unjustified, and irrelevant to peoples’ understanding of the example. Likewise, if somebody who has no presumptions about Hoover’s sexual preferences is confronted with E98, I would suggest that they would tend to infer that Hoover is a homosexual. That such inferences occur can be seen as offering support for the claim that there are non trivial presuppositions. In the Strong Kleene and Karttunen and Peters accounts discussed earlier, in the hybrid model which Soames actually did propose in [So82], and in some of the dynamic semantic accounts to which I will shortly turn, conditionalised presuppositions are predicted for such examples. These presuppositions can be glossed as follows:

E100 If LBJ appoints Hoover he will have appointed a homosexual.

E101 If Nixon invites Davis, he will have invited a black militant.

As stated, these glosses do not strictly entail that Hoover is a homosexual or that Davis is a black militant, although the glosses are highly suggestive of these conclusions.
Some strengthening is needed, and an appropriate mechanism will be introduced later in the thesis, in Chapter 10. With or without explicit formalisation of a strengthening mechanism, prediction of a conditionalised presupposition seems to me closer to the mark than prediction of no presupposition.\textsuperscript{14}

My second objection to Gazdar's analysis concerns his assumption that the indefinite NP in the consequent is anaphoric, which I will argue against. Before that, I wish to point out that even if anaphoricity were present in the original examples, it could not be used to explain away very similar variants. Consider the following:

\begin{enumerate}
  \item If LBJ appoints J. Edgar Hoover to the cabinet, he will regret that his cabinet is no longer entirely made up of heterosexuals.
  \item If Nixon invites Angela Davis to the Whitehouse, then Nixon will regret that his house is no longer devoid of black militants.
\end{enumerate}

I take it that hearers of E102 would not uniformly infer that LBJ's cabinet is no longer entirely made up of heterosexuals, so that Gazdar might wish to say that the potential presupposition is (or at least can be) canceled. Similarly, he would presumably want to say that in E103 the potential presupposition that Nixon's house is no longer devoid of black militants is canceled. But in this variant there is no expression which can act anaphorically on the NP 'Angela Davis', and the argumentation offered for the original examples would fail. Now we move on to consider whether there was any anaphoric link in the original examples. Consider the following:

\begin{enumerate}
  \item If LBJ appoints J. Edgar Hoover to the cabinet, he will realise that he has appointed a homosexual.
  \item If Nixon invites Angela Davis to the Whitehouse, then Nixon will realise that he has invited a black militant to his residence.
  \item If Nixon invites Angela Davis to the Whitehouse, then Nixon will realise that he has invited the black militant to his residence.
\end{enumerate}

Examples E104 and E105 are both examples where assuming the indefinite NP in the consequent was coreferential with the proper name in the antecedent would be disastrous. We would not want to say that E105 says only that if Nixon invites Davis then he will realise that he has done so. This is exactly the (peculiar) reading we would get in a truly anaphoric case such as E106, which has an interpretation such that even if

\textsuperscript{14}Note that the conditionalised presupposition survives under embedding. For instance, 'Perhaps if Nixon invites Davis he will regret having invited a black militant' seems to imply 'If Nixon invites Davis, he will have invited a black militant.'
Cancellation and Filtering

Nixon invites Davis, he does not necessarily realise that she is a black militant, although the speaker does realise this. Conclusion: the indefinite 'a black militant' in E105 is not anaphoric, at least not in the sense that in the LF of E105 'a black militant' acts as if it were merely a repeat of the NP 'Angela Davis', which is the type of assumption that would enable Gazdar's argument to go through. A similar conclusion can be drawn regarding E99.

So, if we take it that 'a black militant' in E99 is non-anaphoric, or we consider examples like E103 or E105, what results does the cancellation model predict? This can be answered very easily. Whether or not the fact that Davis is a black militant is taken to be in the context, the clausal implicature that 'The speaker does not believe that Nixon invites Angela Davis' will not conflict with the potential presupposition 'The speaker believes that Nixon invites a black militant'. In fact, on any of the cancellation accounts, E99 should be expected to presuppose, incorrectly, that Nixon has invited a black militant (except in very special contexts, such as those already containing the negation of this proposition, or containing the assumption that Nixon invited at most one person).

Both Karttunen's revised 1973 account and the joint Gazdar-Karttunen model proposed above correctly predict filtering provided the proposition that Davis is a black militant is assumed to be in the initial context. But these models still fail to account for the fact that even hearers who are not aware Davis' skin colour and militancy will tend to infer it. A better model would be one which predicted a conditionalised presupposition as discussed above.

If a sentence has the form 'If A then B \(\land\) C' (meaning that B carries presupposition C), then the relevant conditionalised presupposition is 'If A then C'. As it happens, the relevant 'If A then C' conditional is part of the asserted content for the cancellationist. This is because factives are assumed to carry their potential presuppositions as entailments. For example, the assertion of the McCawley sentence E99 may be paraphrased 'If Nixon invites Davis, he'll have invited a black activist and regret it', which classically entails 'If Nixon invites Davis, he'll have invited a black activist.' Thus if one sets up the initial context so as to force cancellation, then although cancellation models do not yield conditionalised presuppositions for the McCawley sentences, they do still predict that the speaker is committed to the relevant 'If A then C' proposition. An example of such a context is that set up by the modal statement 'Perhaps Nixon will invite a black militant to the Whitehouse', where an implicature is triggered that the speaker does not know whether Nixon will in fact invite a black militant.

It might then seem that there is hope for the cancellationist, if only the conditions for cancellation could be strengthened in these cases so that the elementary presupposition itself is cancelled as Gazdar claimed. Then what others might claim to be the
conditional presupposition of a McCawley sentence would be identified as part of the asserted content by the cancellationist. But this appearance of hope is illusory, for it is easy to find cases for which other theories would still predict a rather plausible conditional presupposition, but where the cancellation model will not predict this conditional to be either a presupposition or an assertion:

E107 If LBJ appoints J. Edgar Hoover to the cabinet and regrets having appointed a homosexual, he will try to fire him.

E108 If Nixon invites Angela Davis to the Whitehouse and regrets having invited a black militant to his residence, then he will organise a cover-up.

Strong Kleene, the theory of Karttunen and Peters, that of Soames, and the dynamic theories in the next chapter all predict the same conditional presuppositions for E107 and E108 as for E98 and E99 respectively, and this prediction seems appropriate. But for the latter pair of examples there is no prospect of a cancellation theory reproducing the relevant conditional presupposition as an assertion. The cancellationist's assertion for E108 could be glossed as 'If Nixon invites Davis and has invited a black militant and regrets it, he will organise a cover-up.' Clearly this does not entail 'If Nixon invites Davis then he will have invited a black militant', which the other models mentioned would, correctly I think, predict as a presupposition. To summarise, not only is it the case that, as was shown in the previous chapter, there are no solid arguments against conditionalised presuppositions, it is also the case that cancellation and filtering theories are demonstrably inadequate in their coverage of a range of cases where other models do predict a conditionalised presupposition, and where this presupposition seems appropriate.
Chapter 5

Dynamic Semantics

All of the major contemporary theories of presupposition projection are in one way or another dynamic theories, making crucial use of the way in which the epistemic state of an agent changes as the interpretation process proceeds. We have already seen that the cancellation theories of Gazdar, van der Sandt and Mercer, although based on a classical static semantics, involve pragmatic mechanisms controlling the evolution of a set of accepted propositions. Another theory which combines a static (although non-classical) semantics with a dynamic pragmatics is that of Seuren [Seu85]. In all these theories we may say that the static interpretation of a sentence acts as a middleman between the syntax of language and pragmatic processes controlling the changing state of the language user. In this chapter we will be concerned with theories which try to cut out this middleman, so that language is interpreted directly into a domain of state-changing operations. The term dynamic semantics will be used to describe systems in which strings of sentential category are mapped onto a certain class of operations, and these operations act on the state of information of some agent to produce a new state.

In general, the successful performance of an operation may require that certain preconditions are met. Open heart surgery requires a steady hand and a fair amount of equipment, and the operation of buttering toast also requires both a steady hand and a certain minimal set of ingredients. The central idea of the dynamic semantic approach to presupposition is that the operation of modifying an information state may require certain of the ingredients to be already present. For instance 'Oh no! I've dropped the knife.' may be understood as an operation to update a state which in some way determines a salient knife (the crucial ingredient) with the information that the object in question has been lost. This will lead to a formal model of presupposition which is intuitive in the sense that it accords closely with the everyday usage of the term as a proposition taken to be accepted in advance. The outline of the model runs as follows:

- An information state is comparable to a partial model, with respect to which some
and will then show how this reformulated model was further adapted by Heim to produce a dynamic formulation of the shifting token theory (as discussed in the last chapter). I will begin the discussion of dynamic semantic accounts with Karttunen's 1974

neutrosophic determinations of propositional truth-values.

neutrosophic determinations of propositional truth-values. Although Karttunen and Peters do suppose that there are further pragmatically determined truth-values in their theory, Aloni and Peters offer no further discussion of this matter. I would use the term semantically for the determination of the truth-value of a proposition on the basis of the act of speaking a proposition is pragmatistic. On the other hand, with regard to calculation of the potential truth-values of a proposition, there is an interesting divergence between the two authors. Peters defines his truth-values without reference to the syntactic structure of the proposition, whereas Karttunen defines his truth-values with reference to the syntactic structure of the proposition. In this sense, Peters' truth-values are defined by a pragmatic part which describes the process of speaking a proposition, while Karttunen's truth-values are defined by a semantic part which describes the process of interpreting a proposition. In this chapter, I will follow Peters' model of the interpretation of a proposition. I will describe how the potential truth-values of a proposition are determined by the syntactic structure of the proposition and the semantic structure of the proposition.

To illustrate the use of the term dynamic semantics is not meant to imply that the

expressions are all admitted in their local interpretative spaces, but that all steps are demanded by the syntax. The syntax is said to be undecidable in that stage, where the semantic values of the function should determine, what input states are used locally in the interpretative process. Basic propositional logic are interpreted by the

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a first integrated dynamic model of presupposition and anaphora. In Heim's model of presupposition information states are given an abstract formulation in terms of possible worlds. Information states can be thought of as mental representations, and much recent work on the dynamics of the interpretation process has concentrated on providing accounts of structured representations and how they are manipulated. Heim's model of interpretation was originally stated in this way, and Kamp's DRT [Kam81, KRe93], is another good example of this type of approach. Previously to the work of Heim and Kamp, linguistic context had typically been modelled as a set of parameters such as a speaker parameter, a hearer parameter, and/or a single world of evaluation. In a few accounts, principally those discussed in the last chapter, linguistic context had been given more structure, being defined as a set of propositions, and had been allowed to evolve in an interesting way in the course of a conversation. Both Heim's account and DRT, provide sophisticated notions of linguistic context, as information states of agents, and such notions of context facilitate a type of treatment of presuppositions that might be termed anaphoric. In such a treatment, presupposition triggers may stand in a relation to previous linguistic material which is analogous to (or, according to van der Sandt, identical with) the relationship between an anaphoric pronoun and its antecedent. The last part of this chapter will be taken up with describing phenomena which motivate such an anaphoric account of presupposition, and showing how it may be realised in the frameworks set up by Heim and Kamp.

5.1 From Projection to Satisfaction

The second of Karttunen's two 1973 definitions of presupposition, as discussed in the previous chapter, involved a special contextual parameter for "a set of assumed facts", utterance presuppositions being calculated relative to such a set. However, it is not clear in this theory how the set of assumed facts and the set of (utterance) presuppositions are to be understood, and what, from a philosophical perspective, is meant to be the relation between them. In [Kar74] Karttunen brilliantly resolved these difficulties, essentially by turning the projection problem, as then conceived, on its head. Instead of considering directly how the presuppositions of the parts of a sentence determine the presuppositions of the whole, he suggests we should first consider how the global context of utterance of a complex sentence determines the local linguistic context in which the parts of the sentence are interpreted, and derive from this a way of calculating which global contexts of utterance lead to local satisfaction of the presuppositions. He gives a formal definition of when a context satisfies-the-presuppositions-of — or admits — a formula. A simple sentence S will be admitted in a context $\sigma$ (here written $\sigma \triangleright S$) if and only if the primitive presuppositions of S are satisfied in $\sigma$. Here the logical form of a
sentence S is written in italics, S, a context is understood as a set of such logical forms, and the natural notion of contextual satisfaction is just classical entailment. When a complex sentence is evaluated in some context, however, presuppositions belonging to the parts of the sentence need not necessarily be satisfied in that context. For example, if a sentence S of the form 'S_1 and S_2' occurs in a context \( \sigma \), the conditions for S to be admitted in \( \sigma \) are that S_1 is admitted in \( \sigma \) and S_2 is admitted in a new context produced by adding the logical form of S_1 to \( \sigma \). Note that essentially the same idea was independently developed by Stalnaker, who comments [St73, p.455] in the case of conjunction: "If one asserts a proposition using a conjunctive sentence ... the presuppositions will change in the middle of the assertion. The first conjunct will be added to the initial presuppositions before the second conjunct is asserted." In reading this quote it is perhaps illuminating to substitute information state for presuppositions, since Stalnaker's notion of presupposition is intended to capture something like the set of propositions assumed by the speaker to be in the common ground, and not any specific set of propositions attached to a sentence. Definition D11, where the \( \pi \)-function is, as above, assumed to map a simple sentence to its presuppositions, collects Karttunen's admittance conditions for simple sentences and for the logical connectives:

**Definition D11 (Admittance in Karttunen 1974)**

\[
\begin{align*}
(8) & \quad \sigma \triangleright S \iff \sigma \models \pi(S) \quad \text{for any simple sentence } S \\
(9) & \quad \sigma \triangleright \neg S \iff \sigma \triangleright S \\
(10) & \quad \sigma \triangleright S_1 \text{ and } S_2 \iff \sigma \triangleright S_1 \text{ and } \sigma \cup \{S_1\} \triangleright S_2 \\
(11) & \quad \sigma \triangleright \text{if } S_1 \text{ then } S_2 \iff \sigma \triangleright S_1 \text{ and } \sigma \cup \{S_1\} \triangleright S_2 \\
(12) & \quad \sigma \triangleright S_1 \text{ or } S_2 \iff \sigma \triangleright S \text{ and } \sigma \cup \{
eg S_1\} \triangleright S_2
\end{align*}
\]

Presupposition may be defined as follows:

**Definition D12 (Dynamic Presupposition)**

\[
\phi \gg \psi \iff \forall \sigma \quad \sigma \triangleright \phi \rightarrow \sigma \models \psi
\]

The empirical motivation Karttunen presents for this theory is much the same as for his earlier theory. Let us consider an example:

**E109** If \([A \text{ Jane noticed that }] [B \text{ Fido was hungry}]), and \([C \text{ she fed him}], \text{ then } [D \text{ he'll be glad that she fed him}].\)

1. The elementary presuppositions are given by \( \pi(B) = \pi(C) = \emptyset, \pi(A) = B, \text{ and } \pi(D) = C.\)
2. We want to know for which contexts $\sigma$ it is the case that $\sigma \vDash 'If A and C, then D'$. From definition D11, this will hold just in case: (i) $\sigma \vDash A$, (ii) $\sigma \cup \{A\} \vDash C$, and (iii) $\sigma \cup \{A, C\} \vDash D$.

3. From the first clause of D11, and given that $\pi(A) = B$, we see that (i) only holds if $\sigma \models B$ does. Since $\pi(C) = \emptyset$, (ii) imposes no further constraint. And since $\pi(D) = C$, (iii) will hold just in case $\sigma \cup \{A, C\} \models C$, which is trivial, so (iii) also imposes no constraint.

4. So the example is admitted in any context which entails B. In other words E109 presupposes that Fido was hungry (and all entailments of this sentence).

This is similar to the result that would have obtained in the Karttunen '73 theories discussed earlier: the elementary presupposition in the antecedent of the conditional is projected, but the elementary presupposition in the consequent is effectively canceled. The “more or less” caveat is needed because whereas Karttunen as of 1974 would predict not only that B is presupposed, but also that all its entailments are presupposed, this is not the case with the [Kar73] models. This difference is revealing, for it shows that [Kar74] is not a filtering model at all, for the presuppositions of a sentence are not in general a subset of the elementary presuppositions of its parts.1 Furthermore, the difference is not just that entailments of presuppositions are predicted to be presupposed. More interestingly, we will see that there is a whole class of cases where D11 predicts a non-trivial presupposition which is not a member of the elementary presupposition set at all, when the earlier Karttunen model would predict no presupposition at all.2 Here is a summary of the presupposition projection properties arising from definitions D11 and D12:

**Fact F2**

*If A presupposes B then:*

1. ‘Not A’, ‘A and C’, ‘If A then C’, ‘A or C’ all presuppose B.

2. ‘C and A’ and ‘If C then A’ both presuppose ‘If C then A’.

1Under the condition that $\pi$ always mapped simple sentences onto logically closed sets of elementary presuppositions, the Karttunen '74 theory would of course also be a filtering theory. For that matter, so would the multivalent and partial accounts of Chapter 3. However, without this restriction, the Karttunen '74 model, and most of the accounts of Chapter 3 could not be described as filtering theories. One difference is that in filtering/cancellation accounts, tautologies will not normally be presupposed, whereas in all the accounts described formally in Chapter 3 and in Karttunen's '74 model, all tautologies are presupposed by every sentence. In Karttunen's '73 filtering account, a tautology will only be presupposed if it is an elementary presupposition of the leftmost subsentence.

2A similar point concerning the difference between Karttunen's '73 and '74 models is made by Geurts in [Geu95].
3. 'C or A' presupposes 'Unless C then A' (i.e. 'If not C then A').

It can be seen that when a presupposition trigger is found on the right-hand side of a connective, a conditional presupposition results, although this conditional will not in general be one of the elementary presuppositions itself. Let us consider a concrete case where the predictions of the earlier and later Karttunen models vary (sticking to the earlier labelings for the subsentences):

E110 If $[B \text{ Fido was hungry}], [D \text{ he'll be glad that } [C \text{ Jane fed him}]]$

Let us make the assumption that $B \nrightarrow C$. The first version of the '73 theory (that specified in D6) will simply predict projection of C, so that the sentence presupposes that Jane fed Fido. On the other hand, it can be seen from F2 that the '74 theory will predict a presupposition 'If B then C', i.e. that if Fido was hungry then Jane fed him. The second version of the '73 theory D7 presents an interesting halfway house: if the set of assumed facts associated with an utterance of the sentence entails 'If B then C' then there will be no presupposition, but otherwise C will be presupposed.\(^3\)

I have already discussed, in §3.3.3 and §4.8, structurally similar examples to E110 (conditionals with presupposition triggers in the consequent), and have argued in favour of 'If B then C'-type conditionalised presuppositions, which also arise in the Strong Kleene and Karttunen and Peters' systems. Of course, it is scarcely surprising that the Karttunen '74 and Karttunen and Peters' systems manifest the same behaviour, since the latter was developed from the former. Despite the Karttunen and Peters system being historically the more recent of the two, it is in a certain sense less radical than the Karttunen '74 theory, in that, from a technical perspective, the Karttunen and Peters' system is based on a multivalent but static semantics, whereas the Karttunen '74 system utilises a dynamic conception of the interpretation process. In Chapter 11 I will discuss the insights (originally due to Peters, [Pe77]) which allowed a connection to be drawn between dynamic and multivalent systems.

\(^3\) Having observed that there are differences between the '73 and '74 theories, one might wonder what the relation between the models is. It is straightforward to formally define the Karttunen '74 notions of admittance and presupposition in terms of the '73 definition of utterance presupposition. I will write $X 73$-presupposes $Y$ if, on the Karttunen '73 model as defined in D7 in Chapter 4, $Y$ is a presupposition of $X$ relative to a set of assumed facts $F$. Then we have:

1. $\sigma$ admits $X$ iff for any proposition $Y$ such that $X 73$-presupposes $\sigma$, $Y$, $\sigma$ entails $Y$.
2. $X 74$-presupposes $Y$ iff for any $\sigma$, if there is no $Z$ such that $X 74$-presupposes $\sigma$, $Z$, then $\sigma$ entails $Y$. 
5.2 Context Change Potential

Although Karttunen’s 1974 model resolved the tension created by the simultaneous presence in his earlier work of distinct notions of assumption and utterance presupposition, it left unresolved one crucial issue: what is supposed to be the relationship between the definition of admittance for an expression and the semantics of that expression? Judging from the developments in Karttunen and Peters’ later joint work, one might conclude that admittance conditions and semantics are separate and unrelated parts of a grammar, but some authors see this as a weakness of the theory. Gazdar [Gaz79b, pp. 58–59], who does not distinguish between Karttunen’s ’73 and ’74 accounts, caricatures Karttunen’s justification for why presuppositions sometimes disappear as “Because those presuppositions have been filtered out by my filter conditions.” Gazdar suggests that an explanatory adequate model should not only stipulate filtering conditions, but provide independent motivation for why those conditions are as they are. Although it is difficult to give any definitive characterisation of exactly when a theory of presupposition is explanatory adequate — and Gazdar provides no such characterisation — it is at least clear that it would be desirable to justify a particular choice of filtering or admittance conditions. Heim [Hei83a] attempts to provide such a justification, and at the same time to clarify the relationship between admittance conditions and semantics. In particular, Heim provides a method of stating semantics, based on the approach developed in her earlier work [Hei82], in such a way that admittance conditions can be read off from the semantic definitions without having to be stipulated separately. Crucially, Heim’s semantics involves a significant deviation from the classical Tarskian approach, in that rather than viewing meaning as a static relation holding between language and truth in the world, she takes the meaning of an expression to be a method of updating the information state of communicating agents. As will be seen in Chapter 7, Heim’s claim of providing explanatory adequacy has proved difficult to establish, and a goal of this thesis is to bolster her position on the matter.

Following Stalnaker [St79], Heim initially takes an information state, or context, to be a set of possible worlds, representing the set of alternative worlds compatible with an agent’s knowledge. For a simple sentence, the admittance condition must be stipulated by the grammar, as in other approaches. Thus a simple sentence with a factive will be admitted in a context if and only if all the worlds in the context are worlds where the propositional complement of the factive is true. The Context Change Potential (CCP) of the sentence is a procedure for updating a context to provide a new context. In the case of a simple sentence S, standard semantics must be taken to provide a set of worlds where S is true, call these the S-worlds. The CCP is an operation of intersection between the set of worlds in the old context, call it σ, and the S-worlds, the result being denoted σ+S. But this CCP should only be applied to a context in case the sentence is admitted.
in that context, in which case the CCP will provide an update:

**Definition D13 (Admittance as Definedness of Update)**

\[\sigma + S \text{ is defined } \iff \sigma \triangleright S\]

In the case of simple sentences, the admittance conditions must be defined by fiat, such that the sentence is only admitted in its local context if its presuppositions are satisfied in that context. Here the notion of satisfaction is different from that of Karttunen, since the notion of context is different. A sentence S is satisfied in a Stalnakerian context just in case the context contains only S-worlds. The important innovation over Karttunen’s approach is that for complex sentences the admittance conditions are not given directly, but are derived from the definitions of CCPs for complex sentences. She gives the following definitions of the CCPs for negative sentences and conditionals, where the slash denotes set subtraction:

**Definition D14 (Heimian Negation and Conditionals)**

\[
\begin{align*}
\sigma + \text{Not } S & = \sigma \setminus (\sigma + S) \\
\sigma + \text{If A then B} & = \sigma \setminus (\sigma + A \setminus (\sigma + A + B))
\end{align*}
\]

Thus to update a context with the negation of a sentence, find the result of updating with the sentence, and subtract the resulting set of worlds from the original context. To update with a conditional, find the set of worlds where the antecedent is true, take away those worlds where both the antecedent and consequent are true, and subtract the result from the original context. Admittance conditions are now definedness conditions on updating, so \(\sigma\) admits ‘Not S’ just in case \(\sigma \setminus (\sigma + S)\) is defined, which will be the case whenever \(\sigma + S\) is defined. Thus the admittance conditions for S are identical to those for ‘Not S’, just as in Karttunen’s system above. Similarly, a context admits ‘If A then B’ just in case \(\sigma + A\) and \(\sigma + A + B\) are defined, so that \(\sigma\) must admit A, and \(\sigma\) incremented with A must admit B. Again, this yields the same admittance conditions as Karttunen gave.

These CCPs for complex sentences are supposed to serve two purposes. Firstly, and as we just saw, they provide a means for predicting whether a complex sentence is admitted in a context in terms of admittance of the parts of the sentence in local contexts. Secondly, they provide an account of the new information conveyed by complex sentences, encoding the normal truth conditional meaning in such a way that truth conditions can always be read off from a CCP. She says “I believe, without offering

\[^4\text{Heim [Hei83a, p.118] provides the following definition for truth of a sentence in terms of its CCP:}\]
justification here, that ... a compositional assignment of CCPs to the sentences of a
language can fully replace a compositional assignment of truth conditions of the sort
normally envisaged by semanticists, without any loss of empirical coverage.” As I indi-
cated in the introductory chapter, a principal goal of this thesis is to show in detail how
such a compositional assignment of CCPs to the sentences of a language can be achieved,
and to demonstrate not only that it can replace the classical paradigm of compositional
assignment of truth conditions, but also that it can improve on the empirical coverage
available in that paradigm.

5.3 Quantification in the CCP model

One of Heim’s main claims in [Hei83a] is that the CCP model provides a way to deal
with the presuppositions of open sentences — sentences which at LF might be expected
to contain a free variable bound only externally to the sentence. Heim suggests that
contexts should register not only factual information, information determining which
world we are in, but also information about the values of variables. To achieve this she
introduces the notion of a sequence, a mapping from indices (natural numbers) to the
domain of individuals, and redefines contexts as sets of sequence-world pairs. Thus the
CCP of a sentence ‘Shei is happy’, if ‘shei’ is treated as a variable free in the sentence,
becomes an intersective operation on this new type of context, mapping a set of world-
sequence pairs onto that subset containing only pairs (g, w) where the index i is mapped
by the sequence g onto an individual that is happy in w. Heim then gives (essentially)
the following CCP for universal sentences:

Definition D15 (Heimian Universal) If g_a is an i-variant of g, differing only by
mapping i to some a, an element of the domain of individuals D (it being assumed that
D is constant across worlds), then:

\[ \sigma + \text{Every}_i A, B = \{ (g, w) \in \sigma | \forall a \in D, (g_a^i, w) \in \sigma + A \rightarrow (g_a^i, w) \in \sigma + A + B \} \]

Heim also places an additional requirement that i is a “new variable” in \( \sigma \), which
she takes to mean that for any sequence-world pair \( \langle g, w \rangle \) in \( \sigma \), and for any individual
a, there is another sequence-world pair \( \langle g_a^i, w \rangle \) in \( \sigma \). In other words, as far as \( \sigma \) is
concerned i could have any value at all in any world in the context.\(^5\) So, the definition

\(^5\) Note that this requirement of i being unconstrained corresponds to what is enforced in semantics
for programming languages by the operation of random assignment. In the recent spate of work on
dynamic semantics initiated by [GS91a], the requirement is normally made explicit in the semantics of
the quantifier.
Dynamic Semantics

says that the update of a context $\sigma$ with a sentence $\text{Every}_i A, A$ is the set of sequence-world pairs $(g, w)$ in $\sigma$ such that all the $i$-variants of $g$ that survive update with the restrictor also survive sequential update with the restrictor and then the scope. To what admittance conditions does this definition lead? The conditions are parallel with those for conditionals: the restrictor $(A)$ must be admitted in the input context, and the scope $(B)$ must be admitted in the context formed by updating the input context with the scope. Suppose $A$ contains a presupposition that is itself free in the quantified variable, as in the following example from [Hei83a]:

E111 Everyone who serves his king will be rewarded

On the relevant reading, the phrase 'his king' is understood to mean the king of the individual $x_i$'s country, and the presupposition is thus that $x_i$ has a king. The restrictor will be admitted in an input context $\sigma$ only if the proposition that $x_i$ has a king is satisfied in that context. But for that to be the case, it must be that every value onto which index $i$ is mapped in $\sigma$, by assumption every individual in the domain, must be established to have a king in $\sigma$. Heim predicts (contra, for example, Karttunen and Peters [KP79], and contra the empirical survey in [Bea94a]) that in general bound presuppositions in a quantificational sentence become universal presuppositions of the whole sentence, and, in particular, that 5.3 presupposes that everybody has a king.\(^6\)

The problem seems more serious in connection with the treatment of indefinites. Witness the presuppositions assigned to the following example, again from [Hei83a]:

E112 A fat man was pushing his bicycle.

Heim, in common with Kamp [Kam81], assumes that indefinites carry no quantificational force of their own, but merely mark that their index represents a new variable. Taking $i$ as the indefinite's index, updating a context $\sigma$ with E112 consists in firstly adding that $x_i$ is a horizontally challenged adult male to form an intermediary context $\sigma'$, and then updating with 'x$_i$ was pushing x$_i$'s bike.' But this latter sentence is assumed only to be admitted in contexts which satisfy 'x$_i$ has a bike', so that in order for update to continue, every sequence-world pair in $\sigma'$ must map $x_i$ onto a bike owner. Since in $\sigma'$ there are valuations mapping $x_i$ onto each individual in each world such that the individual is a fat man in that world, the whole sentence is predicted to presuppose that every fat man owns a bicycle. This, of course, conflicts with intuition.\(^7\) In Chapter 9 a solution to the problem of universal presuppositions from existential sentences,

\(^6\)As Heim points out, a mechanism of local accommodation (to which we will turn shortly) could make these predictions no more than a default.

\(^7\)Heim does suggest a possible line of solution, utilising the mechanism of local accommodation to be discussed in the following chapter. To jump ahead, the solution involves locally accommodating the presupposition within the scope of the existential. I refer the reader to Soames discussion [So89, pp.559-600] where it is shown that Heim's solution is not without problems of its own.
and a more generally adequate treatment of presuppositions of open sentences, will be presented.\(^8\)

### 5.4 Projection from Propositional Complements

D11 omits Karttunen’s 1974 account of how presuppositions triggered within propositional complements are projected. Karttunen divides lexical items taking a propositional complement into three classes: verbs of saying (e.g. *say, announce*), verbs of propositional attitude (e.g. *believe, want*), and others. The ‘others’ class includes various presupposition triggers, such as factive verbs, as well as an assortment of predicates which Karttunen takes to be *holes*, such as modals and internal negation. On Karttunen’s account, the simplest cases are the first and the third: presuppositions triggered within the complement of a verb of saying do not impose any constraint on the context of utterance, whilst for members of the third class all presuppositions triggered within the complement must be satisfied. Thus ‘John says that the king of France is bald’ should be acceptable in any context, and ‘John knows that the king of France is bald’ should only be acceptable in contexts where there is a (unique) king of France. For a sentence with propositional attitude verb as matrix, Karttunen argues that it is the beliefs of the subject of the sentence which are crucial: for a context \(\sigma\) to admit the sentence, the beliefs of the subject in that context must satisfy all the presuppositions of the propositional complement. So ‘John hopes that the king of France is bald’ should only be admitted in contexts where it is satisfied that John believes there to be a king of France. In favour of this analysis is the fact that for instance the discourse ‘Although France is not a monarchy, John believes that there is a reigning French king. He hopes that the King of France is bald’, although contrived, is felicitous.

Assuming that neither verbs of saying nor verbs of propositional attitude induce any new presuppositions, and omitting members of the *other* class apart from factives, the following are essentially Karttunen’s acceptability conditions:

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\(^8\)The particular problem of universal presuppositions from existential sentences does not occur in some other recent dynamic systems, such as those of Chierchia [Ch:MS], van Eijck [Ei93, Ei94, Ei95] and Krahmer [Krah93, Krah94, Krah:MS], or in my own earlier [Bea92]. I regret that I have been unable to include a full discussion of these proposals.
Definition D16 (Karttunen '74 Attitudes)

\[ \sigma \triangleright \text{saying-verb}(x, S) \]
\[ \sigma \triangleright \text{attitude-verb}(x, S) \iff \{ \phi \mid \sigma \models \text{believes}(x, \phi) \} \triangleright S \]
\[ \sigma \triangleright \text{factive-verb}(x, S) \iff \sigma \models S \]

For dynamic semantics of attitude verbs embodying such admittance conditions, the reader might consult Heim's [Hei92]. Note, however, that the semantics presented there involves essentially a stipulation of Karttuneseque admittance conditions within the definitions of the context change potentials associated with attitude verbs. It is not the case that Heim claims motivation of these admittance conditions independently of presuppositional phenomena, through any 'deep' understanding of the concepts associated with such verbs. Zeevat [Ze92], however, does give a dynamic semantics for 'believe' in which Karttunen type admittance conditions arise quite naturally.

5.5 Anaphoricity

Over the last decade a number of authors, most notably van der Sandt [vdS89, vdS92] and Kripke [Krip:MS] (which unfortunately remains unpublished), have argued that there is a tight connection between presupposition and anaphora. Van der Sandt has pointed out that for every example of what might be called discrepant anaphora, by which I mean those cases where the anaphoric link is not naturally treated using standard binary quantifiers to interpret determiners and bound variables for pronouns, parallel cases of discrepant presupposition can be found. In the following four triples, the (a) examples exemplify discourse anaphora, donkey anaphora, bathroom sentences and modal subordination, respectively. In each case, a corresponding example is given, as (b), in which a presupposition is triggered (by the adverb 'still') in the same structural position as the anaphor occurred, but in which this presupposition is satisfied.\(^9\) The

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\(^9\) Although I have defined formal notions of presupposition satisfaction, I have not said what it means as a description of a text to say that in the text a certain (elementary) presupposition is satisfied. Indeed, such terminology is commonplace in recent presupposition literature, but I do not know of any pre-theoretic analysis of satisfaction. Perhaps a direct test for satisfaction could be developed. To start the ball rolling, I propose the following method of determining whether an elementary presupposition P in a text segment T uttered in a context C is satisfied (where the presence of an elementary presupposition must be determined by standard embedding tests applied to the clause containing the putative elementary presupposition):

If the dialogue consisting of
A: I don't know whether P
B: I see. Well, T
is felicitous in context C, then the elementary presupposition P is satisfied in the text T in this context.

For example, set T = 'If Mary is vigilant, then she knows that someone ate a biscuit', and P = 'A biscuit
third member, (c), completes the circle, showing that the argument of the presupposition trigger can itself be pronominalised with no change of meaning.

E113 a. A farmer owns a donkey. He beats it.
   b. Wanda used to beat Pedro. She still beats him.
   c. Wanda used to beat Pedro. She still does.

E114 a. If a farmer owns a donkey then he beats it. \[Geach\]
   b. If Wanda used to beat Pedro then she still beats him.
   c. If Wanda used to beat Pedro then she still does.

E115 a. Either there is no bathroom in this house or it’s in a funny place. \[Partee\]
   b. Either Wanda never beat Pedro, or she still beats him.
   c. Either Wanda never beat Pedro, or she still does.

E116 a. A wolf might come to the door. It might eat you.
   b. Perhaps Wanda used to beat Pedro, and perhaps she still beats him.
   c. Perhaps Wanda used to beat Pedro, and perhaps she still does.

The parallel is compelling, and furthermore similar examples are easily constructed involving all standard presupposition types. But evidence for the anaphoricity of presuppositions goes beyond cases, like those above, where the presupposition is satisfied because it is in some sense anaphoric on a textual antecedent. The reverse of the coin is that, for at least some types of presupposition trigger, if a textual antecedent is not present the presupposition cannot be satisfied. Kripke observes that a common analysis of ‘too’ would make the presupposition of sentence E117, below, the proposition that somebody other than Sam is having supper in New York tonight. However, this proposition seems uncontroversial, so the standard account provides no explanation of why the sentence, uttered in isolation, is infelicitous.

was eaten’. I find it hard to imagine a context in which the following dialogue would be felicitous:
A: I don’t know whether a biscuit was eaten.
B: I see. Well, if Mary is vigilant, then she
knows that someone ate a biscuit.
On the other hand set \(T = \text{If John ate a biscuit, then Mary knows that someone did}'\), and \(P = \text{‘A biscuit was eaten’}\). The dialogue
A: I don’t know whether a biscuit was eaten.
B: I see. Well, if John ate a biscuit, then Mary
knows that someone did.
is, if still rather strained, more acceptable than the previous one, especially if B’s reply is followed by ‘Perhaps she can help you.’ Similarly, applying the test to the (c) example in E113 we obtain a felicitous text, and so conclude that the presupposition is satisfied:
A: I don’t know whether Wanda beats Pedro.
B: I see. Well, Wanda used to beat Pedro. She
still does.
I leave it to the reader to apply the test to the remaining (b) and (c) examples.
Tonight Sam is having supper in New York, too. [Krip:MS]

Notably, E117 is felicitous when it follows a sentence saying of somebody other than Sam that he is having dinner in New York tonight, e.g. 'Saul is having dinner in New York tonight...'. It might be argued that E117 places a requirement on its local context that there is a salient having-supper-in-NY-tonight event. Although one could imagine introducing event discourse markers, and some ontology of events, into the framework we have sketched so far, less effort will be required if we restrict ourselves to an alternative suggestion in [Hei90]. This is the hypothesis that E117 is felicitous in contexts where there is a discourse entity of which it is locally satisfied that the entity is having supper in New York tonight. Adapting from Heim somewhat, we might give the following sketch of an admittance condition for a sentence of the form 'S too', where the word 'too' is assumed to be co-indexed with some focussed NP:

Definition D17 (Heimian 'too')

\[ \sigma \triangleright S \text{ too}_i \iff \sigma \triangleright S, \text{ and there is some index } j \text{ such that } S[i/j] \text{ is satisfied in } \sigma \]

(\text{where } S[i/j] \text{ represents the sentence } S \text{ with all instances of NPs indexed } i \text{ replaced by } x_j\)

E117 would be indexed 'Tonight Sam is having supper in New York, too', and would only be admitted in contexts where for some \( j \), 'Tonight \( x_j \) is having supper in New York' was satisfied.\(^{12}\) We would thus expect E117 only to be admitted in a restricted range of contexts, but 'If Saul is having supper in New York tonight, then Sam is having supper in New York, too.' to carry no presupposition at all.\(^{13}\)

\(^{10}\)To back up the suggestion that the presence of a discourse marker is essential to the felicity of 'too', observe that of the following two discourses (adapted from a well known pronominalisation example due to Partee) A is odd, but B is felicitous.

A: I have ten marbles and you have one. Only nine of mine are transparent. Your marble is opaque too.
B: I have ten marbles and you have one. One of mine is not transparent. Your marble is opaque too.

\(^{11}\)Kripke does not limit his consideration to cases where an NP is in focus, and, of course, a fuller analysis than that given here would allow non-NPs to be focussed constituents as well.

\(^{12}\)In order for definition D17 fully to meet Kripke's objections, an additional constraint on Heimian contexts would be required, roughly that they contain only information introduced in the immediately previous discourse. Otherwise an instance of 'too' might be predicted to be satisfied by material that was not introduced in the preceding text.

\(^{13}\)Kripke makes the provocative claim that the presupposition of a discourse like 'If Herb comes to the party the boss will come too' is that Herb and the boss are distinct individuals. This is interesting, and perhaps it is right in the pragmatic sense of presupposition, in as much as it would be usual for the speaker to be assuming distinctness. But I do not think that this is a presupposition which is conventionally associated with 'too', and I am not sure it is helpful to call it a presupposition at all. Consider firstly the following dialogue segment:

A: If Clark is at the party then is Lois in Washington?
B: No. If Clark is at the party then Lois is in New York too.
For which presupposition triggers is an anaphoric analysis appropriate? Van der Sandt gives a straightforward answer: all presupposition triggers are anaphors. Perhaps it can be imagined how analyses like that for 'too' above could be given for other presupposition types. For instance, to make factives anaphoric, one might introduce discourse markers for propositions and facts, a development which would anyway be essential to treat propositional anaphora within texts (c.f. [As93]). One could then make acceptability of a factive verb with propositional complement $\phi$ conditional on the presence of a factual discourse marker (perhaps a discourse marker identifying a proposition satisfied in the local context) with interpretation related to $\phi$ in some yet to be specified manner. The addition of discourse markers for uttered propositions would yield a fine grained notion of information. An information state would record in much greater detail exactly what statements had been used to update it than is found in the dynamic systems discussed above. For instance, Stalnaker's notion of an information state as a set of worlds can only distinguish between asserted statements up to classical equivalence, and Heimian contexts go only a little further. Van der Sandt's approach to providing an anaphoric account of presupposition does not, however, involve refining Stalnaker's sets of worlds or Heim's contexts. Instead van der Sandt utilises a rather different sort of dynamic system, Kamp's DRT [Kam81, KRe93], with which I will assume the reader's familiarity.

Van der Sandt is not the only one to have provided an account of presupposition in DRT, but his is the most developed account, and others, such as Kamp and Ross-deutscher's [KRo94, Ros94] are closely related. Accordingly, when discussing the relevance of the dynamics of DRT interpretation to presupposition, I will concentrate on

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In the B sentence, the antecedent of the conditional acts as an anaphoric antecedent for the presupposition in the consequent, and we arrive at a presupposition to the effect that if Clark is at the party then Clark is in New York. And indeed, there does seem to be an assumption associated with the sentence that Clark, and hence the party, is in New York. This presupposition can be removed by adding extra information to the antecedent, as in 'If the party is in New York and Clark is at the party, then Lois is in New York too.', but it cannot be canceled simply by adding contradictory information. The following dialogue segment is infelicitous if it occurs discourse initially (when there is no other possible antecedent for the 'too'):

A: If Clark is at the party then is Lois in Washington?
B: ? No. If Clark is at the party then Lois is in New York too, although the party is in Seattle.

However, the claimed distinctness presupposition behaves differently, and can be canceled simply by denying its truth later. The following discourse is felicitous:

A: I never see Clark Kent and Superman together, so if Clark Kent is at the party, then Superman isn't.
B: If Clark is at the party, then Superman is definitely there too, since Clark is Superman!

I would favour a Gricean explanation of the distinctness implication, whereby each clause of a sentence or discourse is normally required to be informative. A sentence 'X Ys too' will only be informative if in its local context X is not established to Y. But if the presupposition that some salient entity Ys is satisfied by X itself, then clearly 'X Ys too' does not add any new information to that context. Note that on this basis van der Sandt's DRT-based theory, which incorporates such an informativeness constraint as a condition on DRS well-formedness, could account for Kripke's distinctness effect without any need to specify distinctness in the lexical entry for 'too'.
van der Sandt's account. Note that in this chapter I will only be discussing the part of van der Sandt's account which takes advantage of the inherent dynamism of standard DRT, and it is only in the next chapter that I will discuss the considerable further developments that van der Sandt has made in the form of a theory of accommodation.

Discourse Representation Structures provide a very fine grained notion of information state, one which is ideal for an anaphoric account of presupposition, since so much of the original surface structure of utterances is recorded. But crucially, although van der Sandt's model operates under the motto presupposition is anaphora, it does not treat presuppositions as anaphors in the strict sense of requiring a textual antecedent. Rather, van der Sandt claims that a presupposition trigger is anaphoric at the level of discourse representation. The heart of the theory involves a structural relation between the position at which a presupposition trigger is represented in a DRS, and the point at which its antecedent is represented. The antecedent must be represented somewhere along the anaphoric accessibility path from the representation of the trigger, this condition being exactly the same requirement as is placed on anaphoric pronouns and their antecedents in standard DRT. The treatment of E118 should illustrate.

E118 Fred is escaping, but Mary doesn't realise that somebody is escaping.

Initially a DRS like the following, in which the presence of a presupposition is indicated using a double thickness box, is constructed:

\[
\begin{array}{c|c|c|c|c}
| \text{f m} | & \text{escaping}(f) | & \text{x} | & \text{escaping}(x) | & \text{realises}(m, y) | \text{escaping}(y) \\
\hline
\end{array}
\]

The global DRS is accessible from within the negation. The marker \(x\) can be resolved with the marker \(f\), and in this case both the universe of the presupposition (now \(f\)) is
accessible in the global universe, and the condition in the presupposition is accessible as a global condition. Thus the presupposition has an antecedent. The double-lined presupposition box, which plays no further role in DRS construction, and does not enter into the model theoretic interpretation of the completed DRS structure, is simply removed, to yield the final logical form:

\[
\begin{array}{c|c|c}
\text{f m} & \text{escaping(f)} \\
\hline
\rightarrow & \text{y} \\

\end{array}
\]

\[
\begin{array}{c|c|c}
\rightarrow & \text{realises(m, escaping(y))} \\
\end{array}
\]

Note that it would make little difference to the treatment of E118 if the word 'somebody' had been replaced by 'he'. Van der Sandt thus provides an interesting twist to the DRT treatment of noun phrase semantics, since in his extended DRT an indefinite can (when embedded in a presuppositional environment) behave to some extent anaphorically.

This model of presupposition resolution, as will be seen shortly, is just one part of van der Sandt's theory of presupposition. Let us explore the relation between van der Sandt's resolution model and the other dynamic theories considered in this section, a job done much more thoroughly by Zeevat [Ze92]. The dynamics of van der Sandt's model is not stated in terms of update functions as in Heim's work. Although some effort has been devoted to providing a more declarative statement of the model (see [vdS92, SG91]), it remains explicitly procedural. For instance, it is important that the anaphors and presuppositions of a sentence are dealt with only after processing of previous discourse is complete. The dynamics can be said to reside in at least three aspects of the theory: the (extended) DRS construction algorithm, the standardly dynamic DRT semantics of implication and quantifiers, and the statement of anaphoric accessibility conditions.

The notion of accessibility is implicitly directional, in that it is invariably defined using an anti-symmetric relation, and reflects Karttunen's conditions on context incrementation. We might restate accessibility conditions in a way that brings this out. Say that a DRS $\alpha$ is a pair $(\alpha_0, \alpha_1)$, with $\alpha_0$ a set of discourse markers and $\alpha_1$ a set of conditions. Define $\text{var}(\alpha)$ as the set of markers mentioned in the conditions $\alpha_1$, and take the
context $\sigma$ of any sub-DRS to be a set of discourse markers: this should be thought of as the set of markers external to a DRS which are accessible from within it. The markers of a DRS $\alpha$ in a context $\sigma$ are completely accessible, written $\sigma \succ \alpha$, if $\text{var}(\alpha) \in a_0 \cup \sigma$. Then the following two rules state whether the variables in the sub-DRSs of negations and implications are accessible:

$$\sigma \succ \alpha \to \beta \text{ iff } \sigma \succ \alpha \text{ and } \sigma \cup a_0 \succ \beta$$

$$\sigma \succ -\alpha \text{ iff } \sigma \succ \alpha$$

These rules, which must be extended to allow for van der Sandt's notion of accessibility of DRS conditions as well as DRS markers, are obviously close to Karttunen's admissibility conditions, as given above (definition D11). Differences arise with conjunction and disjunction, however. Regarding disjunction, it is fair to say that Karttunen's, Heim's and van der Sandt's theories all have problems. The problems with Karttunen and Heim's account are analogous to those facing multivalent accounts of presupposition — see the discussion in Chapter 3. The difficulties with disjunction in van der Sandt's model will be discussed in the following chapter, after the main component of van der Sandt's theory, the accommodation mechanism, has been introduced. The absence of any conjunction operation between DRSs in standard DRT makes comparison on this count difficult, but at least in the case of sentence sequencing, the fact that sentences are processed in a definite order will have the effect that the context of one sentence includes information from previous sentences, which is just what is given in Karttunen's admittance rule for conjunction (again in D11). In other cases there will be a difference in predictions. Van der Sandt's model, unlike the Karttunen or Heim theories, does not seem to predict any difference in acceptability between the following two examples:

E119 If John has children and John's children are at home, he's elsewhere.

E120 If John's children are at home and John has children, he's elsewhere.

To deal with this in van der Sandt's theory, one would presumably have to replace the set of conditions in a DRS with a sequence of conditions, and make one condition accessible from another within the same DRS only if the first preceded the second in the sequence. To make such an adjustment, of course, would increase even further the similarity between van der Sandt's model and the other dynamic accounts which have been discussed.

Anaphoricity is generally understood as a structural relation, whether the structures involved are texts, syntactic trees, or DRSs. But it must be pointed out that whilst such structures place some constraints on which items can stand in the relation, it would be wrong to suppose that this was the end of the story. The following examples all concern
counterfactual conditionals, although I think the points I will make could be addressed to any intensional predicate which creates a local context that might be inconsistent with the global context:

E121 Mary owns a donkey. If she had been a farmer, she would have beaten it.

E122? Mary owns a donkey. If she had not owned any animals, she would have beaten it.

E123? Mary owns a donkey. If she had owned a mule instead, John would have owned a donkey too.

The first of these, E121, shows that in principle a pronoun in the consequent of a counterfactual conditional can stand in an anaphoric relation to an object introduced outside of the conditional. In DRT terms, one would have to say that the global DRS is accessible from the consequent DRS of a counterfactual conditional just as it is from the consequent box of a non-counterfactual conditional. But E122, which I take to be infelicitous, shows that one cannot arbitrarily resolve pronouns in the consequent of a counterfactual to relevant objects in the global box. There seems to be some extra non-structural condition: perhaps, given an appropriate theory of the semantics of counterfactual conditionals, one could say that not only must the antecedent to a pronoun be on the accessibility path, it must also correspond to an object which exists (in an intuitive sense which I will not attempt to clarify) in the local DRS. But in stating such a constraint, we would be complicating our notion of anaphoricity, placing semantic preconditions on when an anaphoric link could hold. In other words, we would be providing pronouns, the paragons of anaphoricity, with something like semantic presuppositions. 14 Similarly, in E123 it seems that regarding the structural relationship between ‘Mary owns a donkey’ and ‘owned a donkey too’, anaphora should be licensed. Van der Sandt’s model, as it now stands, would certainly predict simple resolution of the presupposition. But this is clearly wrong. E123 is infelicitous, and this shows us that conceiving of the anaphoricity of ‘too’ purely structurally, whilst a good approximation in many cases, does not work in general. It is at least arguable that the Heim-style ‘too’ given above, which involves semantic constraints on the local context, should fare better in such cases, but such a claim remains vacuous in the absence of a CCP semantics for counterfactual conditionals. Heim actually discusses such a semantics in [Hei92], but I will not attempt to combine it with the above analysis of ‘too’ here.

14 Gender and number requirements can also be seen as semantic presuppositions, but there is at least the possibility of defining these requirements as grammatical constraints which are determined syntactically.
Chapter 6

Accommodation

"...ordinary conversation does not always proceed in the ideal orderly fashion described earlier. People do make leaps and short cuts by using sentences whose presuppositions are not satisfied in the conversational context... But... I think we can maintain that a sentence is always taken to be an increment to a context that satisfies its presuppositions. If the current conversational context does not suffice, the listener is entitled and expected to extend it as required. He must determine for himself what context he is supposed to be in on the basis of what is said and, if he is willing to go along with it, make the same tacit extension that his interlocutor appears to have made." [Kar74, p. 191]

The process Karttunen here describes, whereby a "tacit extension" is made to the discourse context to allow for update with otherwise unfulfilled presuppositions, is what Lewis later called accommodation [Le79]. Theories which utilise a mechanism of accommodation, are not classical static theories of meaning, but rather theories about the dynamics of the interpretation process. Yet theories of accommodation could reasonably be said to involve a dynamic pragmatics, in that accommodation is not usually thought of in compositional terms, but as an extra process operating in addition to the normal composition of meanings.

In this chapter I will describe the contributions of Heim and van der Sandt to the theory of accommodation, and will detail van der Sandt's recent theory of presupposition and accommodation in DRT, this being by far the most comprehensive and fully specified

1 Stalnaker [St72, p. 398] expresses similar sentiments to those in the above Karttunen quotation, commenting that presuppositions "need not be true", and that in some cases a "Minor revision might bring our debate in line with new presuppositions." Interestingly, in the same paragraph Stalnaker talks of certain things being "accommodated" in the light of new presuppositions, although what he is describing here is not how we change our assumptions (the Lewisian notion of "accommodation"), but how after we have changed our assumptions we may reinterpret earlier observations.
current theory of presuppositional accommodation. A large part of the chapter will be
taken up with discussion of van der Sandt's model, with a number of sections devoted
to its empirical inadequacies, but also some discussion of the strengths it has above
competing theories of presupposition. In the penultimate section of this chapter I will
attempt some synthesis of different approaches to presupposition, showing that a wide
range of superficially quite different theories of presupposition can be recast in terms
of accommodation. Finally, I will briefly comment on the relevance of Langendoen and
Savin's notion of projection to an accommodational theory of presupposition.

6.1 Heim and van der Sandt

Two questions are central to understanding the characteristics an accommodation-based
theory of presupposition might have:

1. Given that the interpretation of a discourse involves not one linguistic context, but
   a series of contexts corresponding to different parts of the interpretation process
   and different parts of the discourse's meaning, in which context should accommo-
dation occur?

2. Given some decision as to the context in which accommodation occurs, exactly
   how should a hearer determine what the new context is supposed to be?

Heim, in [Hei83a], was the first author to recognise the significance of the first ques-
tion, noting that quite different effects could result according to which point in the
interpretation of a sentence accommodation occurs. In the Heim/Karttunen account
one can distinguish two types of context. There is the global context which represents
the information agents have after complete interpretation of some sequence of sentences
of text, but there are also local contexts, the contexts against which sub-parts of a
sentence are evaluated.

Under definition D14 above, updating a context \( \sigma \) with a conditional 'If A then B' will
involve local contexts \( \sigma + A \) and \( \sigma + A + B \) (to be read left-associatively) which are involved
during the calculation of the update. Suppose that B contains some presupposition which
is unsatisfied in the context \( \sigma + A \), so that \( \sigma \) does not admit the conditional. In that case
accommodation must occur, adjusting one of the contexts involved in the calculation so
that A is admitted in its local context of evaluation. This might take the form of adding
some sentence P directly to the local context in which B is to be evaluated, so that the
final result of updating with the context would not be \( \sigma \setminus (\sigma + A \setminus (\sigma + A + B)) \), but
\( \sigma \setminus (\sigma + A \setminus (\sigma + A + P + B)) \): this would be called local accommodation. On the other
hand, an agent might backtrack right back to the initial context, add a sentence Q to the
global context, and then start the update again. This is termed global accommodation,
and the result of updating would be $\sigma + Q \setminus ((\sigma + Q + A \setminus (\sigma + Q + A + B))$. There is at least one other possibility. The agent might just backtrack as far as the evaluation of the antecedent, and add some extra information, say a proposition $R$, into the context in which the antecedent is evaluated, producing a result like $\sigma \setminus (\sigma + R + A \setminus (\sigma + R + A + B))$. Since this last option involves accommodation into a context intermediate between the global context and the context in which the problematic presuppositional construction is actually evaluated, it can be termed *intermediate accommodation*. Clearly the Heimian view on accommodation is highly procedural, and the exact options which are available for accommodation will be dependent on the details of how updating actually occurs, such processing details not being fully specified by the CCP alone.

The Heimian answer to question (1), then, is that accommodation might take place at any time during the interpretation process such as to ensure later local satisfaction of presuppositions. Put another way, accommodation might potentially take place in any of the discourse contexts used in the calculation of a sentence's CCP. Unfortunately, Heim has given no indication of how question (2) should be answered.² The first theory of accommodation which provides a fully explicit answer to both questions is that of van der Sandt [vdS92], and having described one part of that theory in the previous section, I will now present the theory in full. As mentioned, in van der Sandt’s theory Heimian contexts are replaced by explicit discourse representations. Consequently, whereas for Heim accommodation must consist in augmenting a set of world-sequence pairs, van der Sandtian accommodation is simply addition of discourse referents and conditions to a DRS. This difference could be minimised if the CCP model were presented in terms of Heimian filecards (c.f. [Hei82, Hei83b]), so that accommodation would consist of either creating new filecards, or adding conditions to existing ones. Regarding question (1), van der Sandt’s theory shares the flexibility of Heim’s. If a presupposition lacks an antecedent in a DRS, van der Sandt allows accommodation to take place in any discourse context that is accessible from the site of the trigger. Thus once again we can talk of *local accommodation*, meaning accommodation in the DRS where the trigger is represented, *global accommodation* meaning addition of material in the global DRS, and

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²Witness the following quote from [Hei83a]: "Suppose [a sentence] S is uttered in a context $\sigma$ which doesn’t admit it.... simply amend the context $\sigma$ to a richer context $\sigma'$, one which admits S and is otherwise like $\sigma$, and then proceed to compute $\sigma'$ [updated with] S instead of $\sigma$ [updated with ] S." Here she does not specify the relation between $\sigma$ and $\sigma'$, except to say that $\sigma'$ is richer than $\sigma$, and strong enough to admit S. Her later comparison with Gazdar’s theory, a comparison to which we will turn shortly, does seem to suggest that she considers accommodation to consist in adding exactly the proposition that Gazdar would have labeled the potential presupposition, but, as Heim (p.c.) has pointed out, she nowhere says this explicitly. It seems I was mistaken in assuming, in an earlier version of this work [Bea93b], that Heim was committed to a structural account of accommodation, a term which will be explained shortly. Zeevat [Ze92] has also assumed that Heimian accommodation consists in adding the proposition signalled as presupposed by the trigger. On the other hand, Geurts [Geu95] supposes that the most natural explicitation of Heim’s theory would involve accommodation of the logically weakest proposition needed to guarantee local satisfaction.
intermediate accommodation meaning addition of material in any DRS intermediate on the accessibility path between the global DRS and the site of the trigger.

Van der Sandt's answer to question (2), the question of what is accommodated, is as simple as it could be: if a trigger has an antecedentless presupposition, then accommodation essentially consists of transferring the discourse markers and conditions of the presupposition from the trigger site to the accommodation site. An example will demonstrate the power of the accommodation mechanism. At the same time, the example should illustrate an analogy that might be drawn between van der Sandt’s theory and a transformational account of syntax, with van der Sandt’s equivalent of move-α being an operation on DRSs.

E124 If Mary chose the Chateau Neuf, then she realises it’s a good wine.

Assuming, just so that we can concentrate on the treatment of the factive 'realises', that 'Mary' and 'the Chateau Neuf' and 'it' are simply represented as discourse markers, we derive the following DRS:

To produce a DRS in which there is no antecedentless presupposition, a transformation must take place whereby α, the presupposition \([\text{good-wine}(c)]\), is moved to one of the three sites accessible from the site of the trigger, producing the following three representations:

Global Accommodation (Gloss: 'CN is good, and if Mary orders it then she realises it's good.')

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3When giving DRSs in the running text, I use a linear notation, whereby \([a,b][p(a,b),q(a)]\) represents a DRS which introduces markers a and b, and has conditions p(a,b) and q(a).
Intermediate Accommodation (Gloss: ‘If CN is good and Mary orders it, then she realises it’s good.’)

Local Accommodation (Gloss: ‘If Mary orders CN then it’s good and she realises it’s good.’)
Given all these forms of accommodation, and, in van der Sandt's theory, additional options when resolution is possible, how are we to decide which treatment is preferred? Heim offered only one heuristic: "I suggest that the global option is strongly preferred, but the local option is also available in certain circumstances that make it unavoidable." [Hei83a, p.120] Van der Sandt provides much more detail. He offers a number of constraints that any solution must obey, and also suggests a group of preferences between alternative solutions that satisfy those constraints, including a preference for global over local accommodation. 4 The following versions of the preferences and constraints are at some points revised, but I think capture van der Sandt's intentions 5:

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4 In earlier versions of van der Sandt's theory the preferences between solutions were stated less explicitly, as side effects of a general algorithm for treating presuppositions. This algorithm, which he termed the "anaphoric loop" consisted of the following steps: on encountering a presupposition, firstly check each DRS along the accessibility path from the trigger, moving successively outwards, and attempting to resolve the presupposition, and if after reaching the top box no resolution site has been found, check each box in the reverse direction (i.e. from the top box to the trigger site) attempting to accommodate. Thus resolution is attempted first, and only if that fails is accommodation attempted.

5 In particular, the presentation of constraints here differs considerably from, for instance, the presentation in [vdS92]. Firstly van der Sandt gives two consistency constraints, but these should both be subsumed under the one constraint given here. Secondly, van der Sandt's formulations of informativity and consistency constraints seem to involve a notion of local entailment of sub-DRSs, although I am not aware of such a notion ever having been formalised. Thus his equivalent of my local informativity (given as (iii)a on p.167) is "Resolving a DRS K₀ to produce a new DRS K₁ does not give rise to a structure in which . . . some subordinate DRS is entailed by the DRSs which are superordinate to it". Whilst he does not formalise what it is for a DRS to be entailed by the DRSs which are superordinate to it, the formalisation of local informativity given here, in terms of the standard notion of DRS embedding and a simple syntactic operation on DRSs, hopefully ties up that loose end, and is in the spirit of the definitions used in van der Sandt's formalisation of the notion of acceptability in his earlier non-DRT work.
Definition D18 (Absolute Constraints on van der Sandtian Solutions)

1. Trapping. If a presupposition containing a discourse marker d is triggered in an environment where d is bound, the presupposition will be resolved or accommodated at a site from where the relevant binding occurrence of d is accessible.

2. Global Informativity. If some DRS K is incremented with information from a new sentence, such that after solution of all presuppositions the new DRS is K', then $K \neq K'$

3. Local Informativity. No sub-DRS is redundant. Formally, if K is the complete DRS structure and K' is an arbitrarily deeply embedded sub-DRS, K' is redundant if and only if $\forall M, f (M, f \models K \rightarrow M, f \models K[K'/\top])$. Here $K[K'/\top]$ is a DRS like K except for having the instance of K' replaced by an instance of an empty DRS, and $\models$ denotes the DRT notion of embedding.

4. Consistency. No sub-DRS is inconsistent. Formally, if K is the complete DRS structure and K' is an arbitrarily deeply embedded sub-DRS, K' is locally inconsistent if and only if $\forall M, f (M, f \models K \rightarrow M, f \models K[K'/\bot])$. Here $K[K'/\bot]$ is a DRS like K except for having the instance of K' replaced by an instance of an inconsistent DRS.

Definition D19 (Preferences Between van der Sandtian Solutions)

1. Resolution is preferred to accommodation.

2. One resolution is preferred to another if the first is more local (i.e. closer to the site of the trigger).

3. One accommodation is preferred to another if the first is more global (i.e. further from the site of the trigger).

I will illustrate these constraints with some examples. Firstly, trapping:

E125 Nobody regrets leaving school.

Initially the following DRS might be constructed:
The presupposition cannot be accommodated globally because the discourse marker $x$ would become unbound. The next most preferred accommodation site is in the antecedent box. This produces the final structure, the meaning of which can be glossed as ‘Nobody who leaves school regrets having left school’:

Next, application of the informativity constraint. This is exemplified by E126:

E126 If Jane is married then her husband is on holiday.

Global accommodation of the presupposition that Jane has a husband (triggered by ‘her husband’) would produce the following DRS:
But, on the assumption that models are constrained by meaning postulates in such a way that if somebody has a husband then they are married, this DRS breaks the informativity constraint: replacing the DRS in the antecedent of the conditional, \( [\text{married}(j)] \), by the empty DRS \( [] \) would not alter the range of models in which the global DRS could be embedded. Thus, once again, intermediate accommodation is preferred, producing a structure glossable as 'If Jane is married to x, then x is on holiday':

The next two examples, which I will not discuss in detail, illustrate the consistency and global informativity constraints, respectively:

E127 Either Jane is a spinster, or else her husband is on holiday.

E128 Jim is Fred’s friend, and Fred is married. He is married too.

The reader should verify that for E127, the consistency constraint prevents global accommodation of the presupposition that Jane is married, forcing local accommodation, and that for E128 the global informativity constraint prevents resolution of the variable associated with 'he' to the discourse marker for Fred.⁶

⁶Note that in van der Sandt’s system pronouns are treated in the same way as standard presupposition triggers, except that the presupposed DRS associated with a pronoun (something like \([x]\)) is assumed to contain insufficient conditions to support accommodation.
Like the combined Gazdar-Karttunen theory described earlier, or Soames’ similar synthesis of Gazdar’s and Karttunen’s work, van der Sandt’s DRT-based model of presupposition gets right the cases which Gazdar’s theory handles well (i.e. where presuppositions are either explicitly denied, or appear to be out-competed by implicatures) and the cases which Karttunen’s theories handle well (typically where a presupposition is entailed in its local context). However, *none* of the cancellation accounts discussed, *none* of the various theories proposed singly or in joint work by Karttunen, and *neither* the above combined Gazdar-Karttunen theory *nor* Soames own combined model provides an adequate account *either* of presupposed open propositions and their interaction with quantifiers, *or* of Kripkean cases of anaphoric presupposition. Van der Sandt’s model treats both of these phenomena. It is on this basis that I would claim that the most successful fully formalised⁷ model of presupposition to date is van der Sandt’s, whose theory, with a judicious mixture of resolution and accommodation, successfully handles a wide range of problems from the literature and more besides⁸

However, there remain considerable problems for van der Sandt’s theory. Some of these difficulties seem to me to be of such a general nature as to be relevant to any theory of accommodation, but firstly I will discuss a few problems which seem particular to van der Sandt’s formalisation.

### 6.2 The Cancellationist Heritage

Many cases where the various Karttunen-derived models would predict no substantive presupposition, because the presuppositional requirements of the trigger are entailed in their local context, are, as has been discussed, problematic for cancellation theories. Van der Sandt’s DRT based model does not filter out entailed presuppositions, but presuppositions which have an anaphoric antecedent. Van der Sandt is able to show that in the standard cases where local entailment plays a role in the theories of Karttunen,

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⁷What it is for a model to be *fully formalised* is a matter of judgement. None the less, it is clear that van der Sandt’s model goes further than most of its competitors. For instance, perhaps Seuren’s model will in principle yield comparable coverage, but at least one central component of the theory, i.e. *backward suppletion*, Seuren’s equivalent of accommodation, remains unformalised to my knowledge (but see the developments in Chapter 11 of this thesis). Heim, though presenting an account with many superficial similarities to van der Sandt, has likewise not offered a detailed formal model of accommodation. One could transport a van der Sandtian view of accommodation into Heim’s model (as indeed Zeevat has done [Ze92]) or into Seuren’s, but then one produces, not surprisingly, a model with very similar descriptive coverage to van der Sandt’s account. Or take the accounts of Burton-Roberts and Link. Both have offered promising starting points, but push much of the work over to an as yet unformalised pragmatic component. Another justification for calling van der Sandt’s account “fully formalised” is that it has reached a stage where it can be implemented in an NLP system — see van der Sandt and Geurts’ [SG91] and Bos’ [Bo94].

⁸For the “more besides” see especially §6.7 below on anaphora from accommodated presuppositions. Also see Saebo’s [Sa94] development of van der Sandt’s model which involves applying the model to data not usually thought of as presuppositional.
Karttunen and Peters, and Heim, his model predicts that there is a suitable anaphoric antecedent (or else one can be unproblematically constructed), and thus that no presuppositions are globally accommodated. However, the cancellationist history of van der Sandt's model shows itself in the treatment of the following examples:

E129 Either John didn’t solve the problem or else Mary realises that the problem’s been solved.

E130 Unless John didn’t solve the problem, Mary realises that the problem’s been solved.

E131 Either Mary’s autobiography hasn’t appeared yet, or else John must be very proud that Mary has had a book published.

E132 Unless Mary’s autobiography hasn’t appeared yet, John must be very proud that Mary has had a book published.

I do not think that E129 and E130 presuppose that the problem has been solved, or that E131 and E132 presuppose that Mary has had a book published.9 In a Karttunen derived satisfaction model one might explain that the local context in which the second disjunct of, for instance, E131 is evaluated is one in which the negation of the first disjunct has been added. Since the negation of ‘Mary’s autobiography hasn’t appeared yet’ presumably entails (given appropriate meaning postulates on autobiographies, books, etc.) that Mary has had a book published, no substantive presupposition is predicted. However, given the DRT notion of accessibility, the negation of the first disjunct is not anaphorically accessible from within the second disjunct, so anaphoric resolution is ruled out. Van der Sandt will then predict that the preferred reading is one involving global accommodation of the presupposition, yielding a DRS having interpretation corresponding to ‘Mary has had a book published and either her autobiography hasn’t appeared yet or else John must be very proud that Mary has had a book published.’ The same comments apply mutandis mutandi to E129. Further, if unless is analysed in DRT simply as setting up a two box structure, with one box for the unless clause, and one for the matrix, then we will similarly not expect the negation of the unless clause to be anaphorically accessible from within the matrix, and van der Sandt’s model will make the same erroneous prediction of preferred global accommodation.

9However, E129 and E130 are certainly compatible with the problem having been solved, and with suitable stress perhaps this inference will be made. But I do not think the inference can arise from the presupposition of the factive realises alone. Similar comments apply to E129 and E130. See [Bea94b] for discussion of such issues. It is not essential to my arguments that there is no presuppositional reading, but only that the non-presuppositional reading is available, which I take to be uncontroversial.
6.3 Accommodation of Unbound Presuppositions

A relatively minor problem with van der Sandt's model, but still one that requires attention, involves the fact that unbound presuppositions can become accommodated into quantificational contexts. Consider the following two peculiar examples:

E133? I don't know whether the Pope has measles. But every Catholic realises that the Pope has measles.

E134? I don't know whether the Pope has measles. But every protestant who realises that the Pope has measles is converting.

I will represent the main condition introduced by the first sentence in these examples as \( \neg \text{KW}(\text{has-measles}(p)) \). After processing 'the pope', the DRS for the first example might be:

\[
\begin{array}{c|c}
\text{pope}(p) & \neg \text{KW}(\text{has-measles}(p)) \\
\hline
\text{catholic}(x) & \Rightarrow \\
\end{array}
\]

Global accommodation of the condition has-measles(p) would render the condition \( \neg \text{KW}(\text{has-measles}(p)) \) uninformative, and hence be blocked. The next option is accommodation in the restrictor, to produce:
This DRS is then predicted to be the final meaning of the sentence. Its truth conditions may be glossed as follows: 'I don’t know whether the Pope has measles, but if he does then every Catholic realises it.’ Now, whilst this is a perfectly good meaning for a discourse to have, it is not the meaning of the first example above, and there is no explanation of why the sentence is odd. This problem could perhaps be solved by adding an extra constraint on accommodation: call it releasing, since it is approximately the reverse of trapping. Whereas trapping prevents bound presuppositions leaving the quantificational context of the binder, releasing would prevent presuppositions not free in a certain variable from being accommodated in a quantification over that variable.

6.4 The Partial Match

Another problem with van der Sandt’s theory, but again one which I suppose will be solved as the theory matures, is that anaphoric resolution is underconstrained. If what is known about a discourse referent does not conflict with known information about a presupposed referent, then resolution can occur, identifying the two referents, and adding information about the previously introduced referent. This is what van der Sandt has called a partial match. Furthermore, if such a resolution is available, it is preferred to any accommodation involving the addition of completely new referents. But this results in rather bizarre predictions. In the following, each of the (a) sentences is predicted to have the same meaning as the (b) sentences, at least on the preferred interpretation:

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10It might at first seem that the misanalysis is connected with the fact that I have represented the universal quantification using a conditional DRS, as in early DRT. But this is not relevant. Using a more recent version of DRT would still yield an interpretation glossed as above.

11For example, in E136(a), the presupposition that there is a purple farmer-eating donkey on the loose is triggered in the scope of a universal. In the restrictor DRS of the quantifier there is a marker for a donkey, and since being a donkey is compatible with being a purple farmer-eating donkey on the loose, anaphoric resolution will take place as the preferred option, adding to the restrictor the extra material necessary so that the presupposition has an adequate antecedent.
E135 a. Every farmer who owns a donkey realises that a farmer has been injured.
b. Every injured farmer who owns a donkey realises that a farmer has been injured.

E136 a. Every farmer who owns a donkey realises that a purple farmer-eating donkey is on the loose.
b. Every farmer who owns a purple farmer-eating donkey which is on the loose realises that a purple farmer-eating donkey is on the loose.

E137 a. If a farmer owns a donkey, he realises that a farmer has been injured.
b. If an injured farmer owns a donkey, he realises that a farmer has been injured.

E138 a. If a farmer owns a donkey, he realises that a purple farmer-eating donkey is on the loose.
b. If a farmer owns a purple farmer-eating donkey which is on the loose, he realises that a purple farmer-eating donkey is on the loose.

Let us reconsider the McCawley conditional E99, and its variant E108:

E99' If Nixon invites Angela Davis to the Whitehouse, then Nixon will regret having invited a black militant to his residence.

E108' If Nixon invites Angela Davis to the Whitehouse and regrets having invited a black militant to his residence, then he will organise a cover-up.

For both of these, the preferred reading on van der Sandt’s theory will presumably involve a partial match of the black militant with one of the earlier discourse entities. Nixon may be pragmatically ruled out as the antecedent on a number of grounds, such as common knowledge that he is not a black militant and is not the sort of person to invite himself to the Whitehouse. In that case, the model will predict just the anaphoric connection for which Gazdar argued. But the predictions will be none the less bizarre, producing DRSs that may be squibbed as follows:

E99' If Nixon invites Davis and Davis is a black militant, then Nixon will regret having invited a black militant.

E108' If Nixon invites Davis, Davis is a black militant, and Nixon regrets having invited a black militant, then he will organise a cover-up.

Under this claimed interpretation, the speaker is not committed as to whether Davis is a black militant, but if it turned out that she was not, both E99 and E108 would be validated (regardless of Nixon’s mental state). This cannot be right. What is worse, we will see shortly that even if the partial match can be prevented in these cases, the remaining interpretations predicted by van der Sandt’s model are also inappropriate.
6.5 Intermediate Accommodation

The following examples from [Bea94a] are clearly infelicitous. A small group of correspondents were asked to rate eighteen discourses on a five point scale from 'weird' upwards. All informants judged both of these examples weird (with various extra comments, like "I think I'm missing something."): 

E139 How many team members and cheerleaders will drive to the match?

* Few of the 15 team members and none of the 5 cheerleaders can drive, but every team member will come to the match in her car. So expect about 4 cars.

E140 How many of your employees with company cars had problems with their car radiators last year?

Although few of the sales staff had any problems with their cars last year, all of the management discovered that their car radiators had sprung a leak. * However, most of the management didn't have a single problem with their car radiator the whole year: they are generally quite conscientious about car maintenance.

What is wrong with these discourses? It seems that the only reading available for the bold sentences is the one corresponding to local accommodation. Thus in the first case, every team member must have a car, and in the second case every member of the management must have had problems with their car radiators (the presupposition being triggered by the factive verb 'discover'). In each case, this contradicts information elsewhere in the discourse. Van der Sandt's prediction that a presupposition in the scope of a quantifier can trigger domain restriction is falsified by the oddity of the above examples, since on the domain restriction (or rather, intermediate accommodation) reading there would be no contradiction. Furthermore, it is not open for the defender of van der Sandt's model to claim that some extraneous fact about the discourse would mean that intermediate accommodation would result in an incoherent discourse, as is shown by the following two variant examples:

E141 How many team members and cheerleaders will drive to the match?

Few of the 15 team members and none of the 5 cheerleaders can drive, but every team member who owns a car will come to the match in her car. So expect about 4 cars.

E142 How many of your employees had problems with their car radiators last year?

Although few of the sales staff had any problems with their cars last year, all of the management whose car radiators sprang a leak discovered that their
car radiators had sprung a leak. However, most of the management didn’t have a single problem with their car radiator the whole year: they are generally quite conscientious about car maintenance.

In these examples, the crucial sentences from the earlier examples are replaced by new sentences which have exactly the meanings that the earlier sentences would have been predicted to have on van der Sandt’s intermediate accommodation reading. Thus ‘every team member will come to the match in her car’ is replaced by ‘every team member who owns a car will come to the match in her car.’ Both discourses are clearly felicitous (although only the first was included in the survey). For variety, I also tried versions with an extraposed relative clause as a domain restrictor, like the following:

E143 How many team members and cheerleaders will drive to the match?

Few of the 15 team members and none of the 5 cheerleaders can drive, but every team member will come to the match in her car, if she owns one. So expect about 4 cars.

Although informants uniformly rated this example better than E139, some still thought it was “a bit odd”, which was only one point above “weird” on the scale that the informants were given. I had more success on the few informants I tested verbally: stress and rhythm appear to be critical. It might be that this extraposed relative construction is just more typical of spoken than written discourse, the sort of after-thought that is quite common in everyday speech, but not normal in the world of white-out fluid and delete keys. But the point remains that the sentence ‘Every team member will come to the match in her car’ does not have van der Sandt’s intermediate accommodation reading in the context set up in E139, and that the felicity of sentences with precisely that meaning shows there to be no independent reason for the reading to be blocked.

Similar comments apply to intermediate accommodation into the antecedent of a conditional. Consider the following two examples:

E144 If the problem was easy, I know that somebody solved it.

E145 What do I know about the problem?

Well, if it was difficult, it’s probably still unsolved.

?If it was difficult, then it was Morton who solved it.

Thats all I know.

Regarding E144, an implicature is triggered that the speaker does not know whether the problem was solved, and this prevents global accommodation. Van der Sandt’s theory would then allow for an intermediate accommodation reading whereby the sentence
Every woman who buys a car will sell her Cadillac.

Reading

The quantificational analogy of ET 144, which also lacks an intermediate accommodation, is clearly a problematic operation, and any reading is

ET 144 If a woman buys a car, she will sell her Cadillac.

ET 148 If a woman buys a car, she will sell her Cadillac.

Somewhat related: Consider the following part of examples:

This is hardly surprising, for in DRT quantifiers and conditionals are regarded as being sentences as were reached earlier with respect to quantificational examples. Indeed, we have treated precisely the same conclusion with respect to conditional accommodation of a woman who comes a Cadillac for some x introduced by the NP.  

Thus...
better off without it. The same applies to Heim’s theory.\textsuperscript{12}

6.6 Structural Accommodation and Conditional Presuppositions

I have taken as a defining characteristic of a theory of presupposition that a function from simple sentences to their elementary presuppositions is given. But this has left open how presuppositional inferences connected with complex sentences are explained. In canceling and filtering theories, complex sentences have presuppositions which are

\textsuperscript{12}I cannot deny that there are cases where domain restriction readings like those discussed above occur. My father recently made the following comment to me: ‘Everybody takes their pram into the supermarket.’

What did he mean? For a start, it seems to be a statement about pram-owners, so that domain restriction has occurred, even though, as I recall, nothing in the previous discourse had explicitly made the set of pram-owning people salient. But specifying the meaning precisely is difficult. Presumably he meant, roughly, that on occasions when people with a pram faced a choice as to whether to take their pram into the supermarket or not, they took it in. (We were standing outside a supermarket at the time, but it is not entirely clear whether ‘the supermarket’ referred just to this supermarket, or to supermarkets more generally.) Or, to put it another way, he meant that everybody, when finding themselves in a certain type of circumstance follows a certain course of action: the type of circumstance is ‘possession of a pram that might be taken into the supermarket’, and the course of action is ‘taking the pram into the supermarket’. So the sentence might be understood as containing an implicit generic, and given an initial LF something like: every(x,person(x),gen(c, c is a circumstance where x considers taking x’s pram into the supermarket, c is a circumstance where x takes x’s pram into the supermarket)). Under this interpretation, the trigger ‘their pram’ is understood as occurring in the restrictor of an implicit generic. Accommodation that x has a pram might occur not in the restrictor of the initial universal, but in the restrictor of the generic. But this would then not be a case of intermediate accommodation, but of local accommodation in the restrictor of an implicit operator.

Whether or not such an analysis seems far-fetched, it should be borne in mind that some explanation must be found for the fact (and I take it to be so) that the claimed intermediate accommodation readings have only been found to occur in sentences that have a distinctly generic flavour. For instance, examples like ‘Every German woman drives her car to work.’ are sometimes given by van der Sandt. The intermediate accommodation reading appears to me to get distinctly less clear when the genericity is removed. For example, consider ‘Between 9:00 and 9:30 yesterday, every German woman drove her car to work.’ With this variant, I find a clear implication that every German woman has a car, although this implication would not be there on the intermediate accommodation reading. This is not to say that the sentence could not be used in a context where German car owners, or car owning women, were already salient, so that domain restriction would be licensed by the previous context independently of the presence of the presupposition trigger.

Perhaps some will contend that this latter example (‘Between 9:00 and 9:30 yesterday, every German woman drove her car to work’) also has an intermediate accommodation reading. Then I would have to accept the need for intermediate accommodation. But I would suggest that it is constrained in such a way that it is only applied when the quantificational domain of a statement is in some sense unclear. For instance, in the discourse ‘Ten women work at the company. Every woman drives her car to work’, the domain is explicit, and intermediate accommodation (to produce ‘Every woman who works for the company and has a car’) seems to be difficult or impossible. We could postulate that quantificational statements are always anaphoric on some set which is assumed to be salient, but that when this set has not been introduced explicitly, the hearer must globally accommodate a referent for the set. Then the intermediate accommodation readings would be explained without recourse to intermediate accommodation, but only in terms of global accommodation. Further, such readings would be blocked whenever the domain of a quantificational statement was clearly linked to an explicit antecedent.
simply a subset of the elementary presuppositions, whilst in multivalent and dynamic semantic theories complex sentences have presuppositions which need not come from this set. We may say that in the cancellation and filtering theories the possible presuppositions of complex sentences are structurally predictable from the set of elementary presuppositions, whilst in a theory employing multivalent or dynamic semantics this need not be so. In an accommodation theory, an explanation of presupposition related inferences may be given without any direct definition of what the presuppositions of complex sentences are. This is the case in van der Sandt’s recent theory, where accommodation cuts and pastes the elementary presuppositions into the logical form itself, until a logical form is produced containing no further presuppositions. Although we cannot say that the possible presuppositions of complex sentences are structurally predictable, since complex sentences are given no presuppositions, we may still say that van der Sandt has employed a *structural* notion of accommodation: what is accommodated is strictly drawn from amongst the elementary presuppositions.  

The problem that I see for a purely *structural* account of accommodation is as follows: it is not possible to predict on structural grounds alone exactly what should be accommodated. In general, the exact accommodated material can only be calculated with reference to the way in which world knowledge and plausibility criteria interact with the meaning of a given sentence. Consider the following Charles-and-Di example:

\[
E_{150} \text{ If Charles turns up, then everybody will be amazed that both Charles and Diana are here.}
\]

Here the relevant presupposition trigger is the factive *amazed*, and the elementary presupposition is that both Charles and Diana are here. However, in a situation where it is known that Charles and Diana generally try to avoid each other, there is a clear tendency to come to the conclusion that Diana is already here but that Charles is not (although he may turn up). Furthermore, accommodating non-globally does not help. Accommodating that both Charles and Diana are here in the antecedent of the conditional would yield a meaning like ‘If Charles turns up and both Charles and Diana are here, then everyone will be amazed that both Charles and Diana are here’. This was the most charitable version of accommodation into the antecedent which I could manage, and yet it is clear that it does not yield a sentence corresponding to the meaning of E150. Accommodating into the consequent yields a somewhat more plausible meaning,

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13 It could be argued that, however, that partial match cases involve a sort of non-structural accommodation, since identity statements are added to the DRS which are not part of the elementary presuppositions themselves.

14 If the word ‘both’ in the example is stressed, then this conclusion seems more likely regardless of our knowledge about Charles and Diana. However, I have nothing to say about the role of intonation, and my remarks concern an utterance with no sharp focal stress.
something like: 'If Charles turns up then both Charles and Diana are here and everyone will be amazed (by that).’ I think there are occasions of use of E150 where this meaning would be reasonable. However, the implication that Diana is here would remain unexplained. I would not wish to claim that no structural account of accommodation could lead to the correct result, but certainly no existing such account does, and van der Sandt's theory would require apparently ad hoc modifications.

The most glaring weak point of a structural account of accommodation concerns the fact that there is no way for it to produce conditional presuppositions, which I have argued are appropriate in many cases.\(^{15}\) Let us consider one such case:

**E151** If Spaceman Spiff lands on Planet X, he will be bothered by the fact that his weight is higher than it would be on Earth.

The ‘fact that’ construction in this case triggers the presupposition that Spiff’s weight is higher than it would be on Earth, and this is further re-enforced by the presence of the factive ‘bothered by’. Structural accounts of accommodation suggest that this proposition should be globally accommodated. However, this result is simply wrong: it is not normal to conclude from E151 that Spiff’s weight is higher than it would be on Earth. Indeed, it seems natural for this sentence to be uttered under conditions where Spiff is hanging about in space, and completely weightless. It is difficult to argue against a conditional presupposition, that if Spiff lands on X his weight will be higher than on Earth. Can non-global accommodation save the structural account? Accommodation into the antecedent produces something like ‘If Spaceman Spiff’s weight is higher than it would be on Earth and he lands on Planet X, he will be bothered by the fact that his weight is higher than it would be on Earth.’ I do not think this is a possible meaning of E151.

Accommodation into the consequent appears to improve on this, yielding (after charitable adjustment of tense) ‘If Spaceman Spiff lands on Planet X, his weight will be higher than it would be on Earth and he will be bothered by the fact that his weight is higher than it would be on Earth.’ This provides a reasonable meaning for E151, and suggests that if only some way could be found of removing the two incorrect readings, the structural account might be saved. Now recall the way in which cancellation models were shown to sometimes produce an analogue of a conditional presupposition for the McCawley type examples, but could not in general produce such readings (in §4.8, p.80). Much the same holds regarding a structural account of accommodation: it can sometimes produce

\(^{15}\)It is ironic, and worrying, that the occurrence of such readings in the CCP account without an accommodation mechanism continues to be taken as one of the most serious objections to Karttunen-derived models (see e.g. Geurts [Geu95]), and the non-occurrence of such readings in other accounts continues to be taken by Karttun-ists (such as myself) to be a serious failing of those theories. It is obviously too soon to say whether the arguments in this thesis will advance the debate any further, or merely leave the combatants as deeply entrenched as ever.
conditional readings, but it is easy to find cases where it cannot. Slight variations on E151 produce examples where the structural account produces multiple incorrect (or, at the very least, non-preferred) readings, and completely fails to yield the preferred reading:

E152 It is unlikely that if Spaceman Spiff lands on Planet X, he will be bothered by the fact that his weight is higher than it would be on Earth.

E153 If Spaceman Spiff lands on Planet X and is bothered by the fact that his weight is higher than it would be on Earth, he won't stay long.

The preferred readings of these sentences still involve the same conditional implication that if he lands on Planet X, Spiff's weight will be higher than it is on Earth. It is clear (given the treatment of and) that the Karttunen, Karttunen and Peters and CCP models will make this prediction for E153, and quite natural assumptions about the semantics of the 'it is unlikely' construction would lead to these theories making the same predictions for E152. But in these cases the structural accommodation account no longer yields the right reading after accommodation into the consequent of the conditional. Such accommodation is not even available as an option for E153, and in the case of E152 would yield 'It is unlikely that if Spaceman Spiff lands on Planet X, his weight will be higher than it is on Earth and he will be bothered by it', which does not imply that if he lands on Planet X, Spiff's weight will be higher than it is on Earth. On the contrary, one might expect from this gloss that if Spiff lands on Planet X his weight probably will not be higher than it is on Earth, which is clearly inappropriate.

Example E151 has essentially the same form as the McCawley example E99, a form which I earlier glossed as 'If A then B or'. This being so, it should be clear to the reader how my remarks could be transferred from the Spiff examples to the Nixon/Davis examples, which means that even if van der Sandt's incorrect partial match reading for the Nixon/Davis cases could be avoided, accommodation could still not produce the right results.

It might seem to the reader that the problems I am citing with regard to structural notions of presupposition are rather academic in nature, involving sentences far from the run of everyday conversation. But perhaps the greatest problem with structural accommodation concerns a phenomena which is quite obviously commonplace. This is the phenomenon of bridging whereby a new discourse entity is linked indirectly (i.e. not by identity) to an old one. Consider the following:

E154 Jane sat in the car. She adjusted the rearview mirror.

E155 If I go to a wedding then the rabbi will get drunk.
E156 An old woman hit me. The knuckle-duster cut deep.

E157 Whenever you remove the head stratifier from a hyperspace drive unit, remember to hold your nose.

In these examples, it seems that the rearview mirror is connected to the car, the rabbi is connected to the wedding, the knuckle-duster (U.S.: 'brass knuckles') is connected to the hitting event, or the old woman or both, and the head stratifier is connected to the hyperspace drive unit. But what enables us to make these connections? Perhaps it could be claimed that 'rearview mirror' is an intrinsically relational noun with an argument place for a car, and that the NP 'the rearview mirror' thus contains the content necessary to make the connection to the car Jane sat in. However, I am not sure that we would wish to analyse 'rabbi' as a relational noun with an argument place for religious ceremonies, or 'knuckle-duster' as having an argument place for an old woman. Furthermore, in the last example, it seems that all that is required to understand that hyperspace drive units have head stratifiers is competence in English and a little common sense, and not competence in hyperspace drive maintenance or a working knowledge of the concepts involved. In all these cases, accommodation might allow us to fill in the missing links. For instance, we might accommodate that a certain knuckle duster is being worn by a certain old woman, or we might accommodate more general rules, say that hyperspace drive units have (/can have) head stratifiers. But we cannot expect the accommodated material to be provided by our knowledge of grammar alone.16

6.7 Anaphora from Accommodated Material

Now, to provide at least a modicum of balance, I will turn to one aspect of van der Sandt's model which I consider a particular strength. The following two counter-examples to DRT constraints on accessibility of anaphoric antecedents date back to over a decade before DRT was introduced, from Karttunen's influential work on discourse reference [Kar76] (which was only published some years after its first presentation):

E158 Bill didn't realise that he had a dime. It was in his pocket.

16How crucial to van der Sandt's model is it that accommodation is a structural operation? I think the correct answer, and perhaps the answer van der Sandt would give, is that such structurality is not essential, it is merely a simplifying assumption. There has been some work which, though building on van der Sandt's ideas, drops the assumption that the accommodated material is wholly derived from the trigger. Lorenz [Lo92] has developed an account of presuppositions in the temporal domain which uses default world knowledge to generate the accommodated material, and Bos, Buitelaar and Mineur [BBM95] have developed an account of bridging descriptions. I find myself very much in sympathy with this direction of research: I think it shows that the gap between the van der Sandtian account of accommodation and the non-structural account which will be proposed in this thesis may itself, in due course, be bridged.
E159 John knew that Mary had a car, but he had never seen it.

In the first example, not only is 'a dime' embedded within an intensional context, but that context is itself embedded under a negation. In standard DRT, either of these embeddings would normally be sufficient to guarantee anaphoric inaccessibility. Since 'a car' in the second example is embedded within an intensional context, standard DRT incorrectly predicts it to be inaccessible. However, van der Sandt predicts that in both these cases global accommodation occurs. For instance the final DRS for E158 would be something like the following:

\[
\begin{array}{c|c|c}
 b & x & z \\
\hline
 b = \text{bill} \\
\text{dime}(x) \\
\text{had}(b,x) \\
\hline
 - \text{realise}(b, \\
 \text{dime}(y) \\
 \text{had}(b,y) \\
\hline
 \text{pocket-of}(b,z) \\
\text{in}(x,z) \\
\end{array}
\]

Here global accommodation of a DRS of the form \([x][\text{dime}(x),\text{had}(b,x)]\) creates an anaphorically accessible dime to which the pronoun in the second sentence can refer.

Such patterns of anaphoric reference can be demonstrated with a wide range of presuppositional constructions embedded in environments that would otherwise block anaphoric reference. Perhaps most significant of these presuppositional constructions are definite descriptions. For instance, in the following discourse, van der Sandt's theory predicts that the presupposition associated with 'the tallest mountain in the world' is globally accommodated, and hence correctly licenses subsequent anaphoric reference:

E160 John believes that he can see the tallest mountain in the world. But in fact it is completely obscured by mist.

What is most notable about this last case is that it shows that given a theory like van der Sandt's, a rather ad hoc stipulation in standard DRT, the promotion of proper
names and definites, can be dispensed with. This stipulation, that referents introduced by proper names and definite descriptions are automatically promoted to a position in the global DRS regardless of how deeply embedded they arose, was originally motivated only by the need to account for the special anaphoric accessibility of names and definites. But in van der Sandt’s account, the separately motivated theory of presupposition takes care of promotion (under the name of accommodation), and it is only necessary to make the relatively uncontroversial assumption that both definites and names are presuppositional.

6.8 Alternative Accommodation Strategies

Cancellation and filtering might be said to bring the woodsman’s axe and hacksaw into the linguistic operation theatre, allowing grisly brute force amputations of unwanted chunks of meaning. By comparison, accommodation is the modern surgeon’s toolkit, which in the right hands can enable precision repair of linguistic contexts or major transplants of lexical material across logical forms. As I will attempt to show, accommodation provides one of the great unifying themes of modern presupposition theory, since many theories of presupposition which were not originally proposed as accommodation theories can be thought of in terms of accommodation.

6.8.1 From Amputation to Accommodation

In a sense cancellation is the inverse of global accommodation. Heim [Hei83a], after suggesting her enhancement of the CCP model with an account of accommodation, makes the following observation:

Note that by stipulating a *ceteris paribus* preference for global over local accommodation, we recapture the effect of [Gazdar’s] assumption that presupposition cancellation occurs only under the threat of inconsistency.

I find this stunning. With one short remark buried in a terse paper Heim offers a simple synthesis between the two antitheses of 1970’s presupposition theory, namely the Karttunen 1974 derived model which her paper uses as its base, and Gazdar’s cancellation account. Perhaps implicit in Heim’s remark is the idea that global accommodation of an elementary presupposition may be identified with what was termed *projection* in earlier models. In this case whenever accommodation is not global, we have the effect of cancellation. Looked at this way, a preference for global over local accommodation becomes a preference for projection over cancellation, and given an appropriate stipulation of the circumstances in which this preference can be overridden (e.g. in order to avoid inconsistency), the effects of a cancellation theory can be mimicked. In a stroke
this shows a way to eliminate the bulk of existing counter-examples to the CCP model, in particular examples where a presupposition associated with an embedded trigger is eliminated by explicit denial. Further, and in common with van der Sandt's cancellation account, Heim's remark introduces a way of thinking about Gazdar's theory that preserves his insight that default reasoning is involved in the processing of presuppositions, whilst restoring the intuition that, in some sense, presuppositions are to do with what come first, with definedness conditions on the input rather than preferences on the output. Note that in [vdS88] van der Sandt is explicit in identifying his cancellation analysis as involving an accommodation-like mechanism, although this was not the case in his theory's first incarnation [vdS82]. Also note that for Heim's analogy between cancellation and accommodation theories to really drive home it is important that in the cancellation account it is assumed that presuppositions are also part of the asserted content, which, as elaborated above, is a reasonable assumption. Entailment of presuppositions is what produces the effect of local accommodation in cases where the presupposition is globally canceled.

6.8.2 The Transformation from Russell to van der Sandt

Now let us consider a very different type of theory, the neo-Russellian account discussed earlier. Recall that the essential idea of this theory was that alternative presuppositional readings are obtained only as a result of variations in logical scope, or, put another way, as a result of variations in logical form. Strangely, these scopal variations are mirrored by the alternative accommodation readings in van der Sandt's theory, save that Russell's logical forms happened to be expressed in FOPL, whereas van der Sandt's are expressed in the language of DRT. Russell gave few hints as to how his logical forms should be derived, and I see no obvious reason why a Russellian theory of scopal variation should not be developed where scope bearing operators are initially interpreted in situ to produce a first logical form, and are then moved about to produce the final logical form in a manner reminiscent of the semantic move-\(\alpha\) operations of van der Sandt's theory. Thus we see that the transformation from Russell to van der Sandt is surprisingly small. For instance, the neo-Russellian and van der Sandt accounts allow essentially the same two readings for sentences like the following:

E39' Pooh doesn't realise that Eeyore is sad.

The neo-Russellian narrow scope negation reading is given by the formula \([x - \text{sad}(e)](\neg\text{realises}(p, x))\). Corresponding to this is the following van der Sandtian global accommodation reading:
On the other hand the neo-Russellian wide-scope negation reading, \( \neg( \text{sad}(e)((\text{realises}(p, x))) \), is analogous to van der Sandt's local accommodation reading:

But this is not to deny that van der Sandt's theory incorporates important innovations. Firstly, lack of anaphoricity was mentioned earlier as being problematic in a purely Russellian account (§2.5, p.30), but van der Sandt's account includes not only an accommodation component, but also an anaphoric resolution component completely alien to the Russellian picture of definites. Secondly, van der Sandt not only allows for presuppositional elements to take different scopes, he also provides an account of which scopes are to be preferred, and this is again something absent from the Russellian account. Thirdly, and specifically as a result of being situated in DRT, van der Sandt's model allows for extra possibilities which would not be available to Russell. For instance, a presupposition \( \alpha \) triggered in the consequent of a conditional may, in van der Sandt's theory, eventually make its way to the antecedent of the conditional. Such a transformation would make no sense on the Russellian picture, since an element in the antecedent of a conditional could classically not bind material in the consequent. But are these extra scopes in van der Sandt's theory a good or a bad thing? Above
I suggested that intermediate accommodation into the antecedent of a conditional or restrictor produced problematic results, so that van der Sandt’s theory might actually improve its descriptive coverage by sticking to purely Russellian scopes.

6.8.3 Accommodation as a Journey through Mental Space

Fauconnier [Fa85] presents a representationalist theory in which meanings are rendered in a structured collection of interconnected mental spaces. Mental spaces are akin to Kamp’s DRS boxes (or, perhaps even more aptly, Seuren’s discourse domains). A few remarks should clarify the similarity with DRT:

1. Like DRS boxes, mental spaces can be seen as partial models in which a set of discourse entities bear certain properties and relations to each other, but in which the extensions of many other properties and relations are left undecided.

2. Like DRS boxes, mental spaces are arranged hierarchically, with some boxes being seen as subordinate to others. Properties of objects in subordinate daughter spaces may be inherited from their parent spaces. However, the links between entities in different spaces are not sustained by variable binding, but by a Lewisian counterpart relation. The inter-space links between entities are analogous to the connections between discourse markers in later versions of DRT [KRe93] where objects in intensional contexts are linked to objects outside by anchoring functions, these determining which objects are counterparts of which others.

3. Unlike Kamp, Fauconnier does not follow the Montagovian method of fragments. He does not provide a fully formalised method of constructing mental spaces for all the strings produced by a generative grammar.

4. Unlike in DRT, no semantic interpretation or Tarski truth definition is given for mental spaces, and no notion of logical consequence between mental spaces is defined.

5. Fauconnier considers a wide range of syntactic constructions and complications of the interpretation process which have so far eluded DRT. For instance, he discusses the special semantic environment set up by descriptions of pictures or the painting of pictures (sentences like ‘In this picture, Kamp and Fauconnier have been painted as the same person’) and cases of perspective shift, more commonly dealt with in literary theory than in generative linguistics.

In order to see what Fauconnier’s theory of presupposition [Fa85, pp.86–87] would look like in a van der Sandtian setting, let us assume that a space is just a DRT box (i.e. a set of discourse markers and a set of conditions), and assume a DRT-like notion of
accommodation. Let us say that a proposition is \textit{supported} in a space if it is a consequence of the conditions in that space, and that a proposition is \textit{accessible} from a space if it is a consequence of propositions in accessible (i.e. superordinate) spaces, and let us assume a standard logical definition of \textit{consistency} of a space, meaning consistency of the set of conditions in that space.$^{17}$ In certain cases (generally non-intensional contexts) Fauconnier also employs a notion of \textit{compatibility}, meaning consistency of the set of conditions either in the space or accessible from it. Fauconnier's theory of presupposition can be described as a theory of presupposition flotation, whereby locally triggered presuppositions float up through as many spaces as they can without creating inconsistency.$^{18}$ I would characterise the theory as follows:

1. Presuppositions must be supported in the local space of the trigger.
2. If a presupposition is accessible, then nothing further need be done.
3. Otherwise, the presupposition is accommodated into successively more global spaces along the accessibility path, until reaching the highest space where accommodation does not create inconsistency at the accommodation site, or incompatibility of any (non-intensional) subordinate space.$^{19}$

It is readily seen that, at least in the van der Sandtian form that I have presented it, Fauconnier's model will make predictions comparable to some of the other models that have been discussed in this chapter. The first clause means that in a sense Fauconnier always locally accommodates, whatever else he does. This produces the effect that in a cancellation account would be derived by assuming presuppositions to be part of the asserted content.$^{20}$ The second clause provides for something like van der Sandt's anaphoric resolution of presuppositions. In most cases this will presumably yield filtering

$^{17}$The relation \textit{supports} corresponds to Fauconnier's \textit{satisfaction}, but I refrain from using this term here since I have tended to use it elsewhere with a slightly different meaning. Fauconnier's satisfaction of a proposition means that the propositions in the local context entail the proposition, whereas my use means roughly that all the accessible conditions taken together entail the proposition. For exegetical purposes, I have also been rather cavalier with Fauconnier's notion of \textit{accessibility} of a proposition. I have assumed that propositions in all superordinate spaces are accessible, but Fauconnier is interested in a wide variety of intensional contexts such that (consequences of) propositions holding in parent spaces cannot in general be expected to hold locally. The approximation will suffice at least for treatment of logical connectives (provided counterfactuality is not involved).

$^{18}$The flotation metaphor is used by Fauconnier himself. Coincidentally, the same metaphor is chosen by Geurts [Geu95] when discussing van der Sandt's accommodation theory.

$^{19}$I take the \textit{incompatibility} requirement from Fauconnier's discussion of conflicting presuppositions in disjunctions [Fa85, p.92].

$^{20}$In a section entitled "Presupposition Transfer" [Fa85, pp.105-108], Fauconnier also discusses cases where a presupposition need not be supported in the local space of its trigger. For example, he discusses the sentence 'Hey, in this painting Gudule is beautiful again.' He allows that the sentence may be interpreted in a context where Gudule in reality was once beautiful, but is no longer, without committing the speaker to a proposition like 'In the painting Gudule was once beautiful.'
of entailed presuppositions as in Karttunen’s ’73 model. The third clause meanwhile will prevent global accommodation in case that would produce inconsistency, thus giving the effect of a cancellation theory in cases of presupposition denial.

There is one important respect in which the version of Fauconnier’s theory above makes different predictions from van der Sandt’s. Under Fauconnier’s accommodation strategy as a presupposition floats upwards, it leaves a shadow behind (i.e. a copy of the presupposition) in every space through which it passes. But van der Sandt’s strategy depicts presuppositions as bubbling up without leaving any trace of their journey. In fact Zeevat has compared an accommodation strategy just like Fauconnier’s to van der Sandt’s, although Zeevat attributes what I call Fauconnier’s strategy to Heim. Distinguishing the two strategies Zeevat says [Ze92, p.396]: “The one remaining difference [i.e. between his version of van der Sandt’s theory and his version of Heim’s theory] is the question whether we should add the presupposition everywhere between the position of the trigger and the highest position where it can be accommodated, or whether we can be satisfied with adding it just once at that position.” So which is the right strategy? Zeevat comes to an interesting conclusion: both are right, but for different classes of presupposition trigger. The two classes Zeevat delimits are what he calls anaphoric and lexical presuppositions. The anaphoric (or resolution) triggers are those “whose primary function is — like anaphora — to collect entities from the environment in order to say new things about them.”[Ze92, p.397] This class, which presumably at least includes definite noun phrases, and discourse particles like too and again, is the one for which Zeevat supposes the van der Sandtian strategy to be appropriate. The following data back up his point:

E161 a. Bill called Mary a Republican. And it is clear from Mary’s diary that John insulted her too.

   b. It is clear from Mary’s diary that Bill insulted her.

E162 a. Bill called Mary a Republican. And it is clear from Mary’s diary that Bill thinks that John insulted her too.

   b. It is clear from Mary’s diary that Bill insulted her.
   c. It is clear from Mary’s diary that Bill thinks he insulted her.

In Zeevat’s terms, the too in E161(a) and E162(a) is used because the speaker is collecting up a property which he takes to already be realised in the context, the property of insulting Mary, and saying something new about the extension of that property. I would say that on hearing either E161(a) or E162(a) a hearer would normally conclude that the speaker thinks that Bill insulted Mary, presumably in the act of calling her a Republican. So it would seem that ‘Bill insulted Mary’ — or the proposition that the
event of Bill calling Mary a Republican is identical to an event of Bill insulting Mary— is globally accommodated. But (and I hope readers can convince themselves of this) I do not think that on the basis of (a) a hearer would conclude that the speaker believes (b). This is just what would be predicted on van der Sandt’s strategy, since the local context to the trigger, the mental space set aside for what is clear in Mary’s diary, would not need to contain the presupposition. Similarly, I do not think a hearer of (a) would normally infer that the speaker believes either of (b) or (c), although these propositions are certainly compatible with what the speaker has said. Thus the presupposition arguably skips over both the space assigned to what Bill thinks in Mary’s diary, and the space assigned to what is clear in Mary’s diary, just as van der Sandt predicts. On the other hand, on Fauconnier’s strategy both (b) and (c) would be inferred.

The lexical triggers are those where the presupposition is a condition on the application of a concept, so that the presupposition must hold in any context where the trigger is applied if the application of the concept is to be meaningful. Factive verbs are presumably in this class. From the definition of lexical triggers, we can see that the presupposition should be expected to hold not only at the highest accommodation site, but also locally. Zeevat goes further in requiring lexical presuppositions to hold Fauconnier fashion in all the intermediary contexts, but the following examples perhaps provide some support for this analysis:

E163 a. Bill called Mary a Republican. And it is clear from Mary’s diary that she realised that he had insulted her.
   b. It is clear from Mary’s diary that Bill insulted her.

E164 a. Bill called Mary a Republican. And it is clear from Mary’s diary that Bill thinks she realised that he had insulted her.
   b. It is clear from Mary’s diary that Bill insulted her.
   c. It is clear from Mary’s diary that Bill thinks he insulted her.

That E163(b) follows from E163(a) seems indisputable. E164(a) is obviously a more complicated case, and requires considerably more effort to comprehend. But my feeling is that both E164(b) and E164(c) do follow from it, in accordance with Zeevat’s prediction that the Fauconnier (or Heim) algorithm is appropriate in this case.21

21Cases like E164 constitute counterexamples not only to van der Sandt’s theory, but to any theory where accommodation occurs at only one site. As discussed above, all the cancellation and filtering theories can be thought of as falling into this class. The problem will typically occur whenever a lexical presupposition is embedded under an operator which is itself embedded in an intensional context. For instance, ‘Fred thinks Mary doesn’t know that she won’ involves the lexical presupposition trigger ‘know’
Whilst I have adduced some support for Zeevat's distinction and his choice of accommodation strategies, the data is not clear cut, and there is clearly room for more research on the issue. My aim was not in fact to argue for the distinction per se, but to demonstrate that a rather promising vista is opened up when we start comparing different theories in terms of the alternative accommodation strategies they manifest. In turn this suggests that we should strive for a better technical understanding of what accommodation is, and that is one of the main goals of this thesis.

6.9 Accommodation and Projection

What is the relationship between accommodation and projection? As mentioned in §sec:fata, one could view projection as global accommodation. However, in a theory like Heim's or van der Sandt's, presuppositions play an important role in determining meaning even if there is no global accommodation. Contemporary presupposition theorists are concerned not merely with whether a presupposition is projected, but also with what happens to the presupposition if it is not projected. Furthermore, it need not be assumed that what is accommodated corresponds directly to an elementary presupposition, and neither Heim's account, nor that to be developed in the second part of this thesis, require such a correspondence. In van der Sandt's account, resolution of presuppositions with a partially matching antecedent is accompanied by some accommodation of information, but the conditions added, which include equality conditions to bind the presupposition to its antecedent, need not be strictly part of the elementary presupposition. If it is not elementary presuppositions which are accommodated, but something else, then it is hard to see how we could determine when it is appropriate to say that an elementary presupposition has been projected. Perhaps it should be recognised that Langendoen and Savin's the projection problem, as usually conceived, has outlived its usefulness?

embedded under a negation operator itself embedded under 'thinks'. The example suggests not only that Mary won, which is predicted by cancellation theories, but also that Fred thinks she won, which is not predicted by these accounts.

For this reason, I will follow the bulk of the presupposition literature in not distinguishing between Zeevat's two classes of presupposition in the main body of this thesis. But I do believe that the theory to be developed would be compatible with making such a distinction, or indeed further distinctions amongst presuppositional constructions. Goldberg et al [GKS:MS] motivate a division between what they term external and internal presuppositions, the idea being that external presuppositions hold in the model, but internal presuppositions need only be satisfied in the discourse context. At least at a schematic level, it seems natural to equate their term external with Zeevat's lexical, and their internal with Zeevat's resolution, although I will not pursue this line any further here.

Other theories of presupposition that compare with van der Sandt's in much the way as Fauconnier's are those of Dinsmore [Di81b, Di92], and Schiebe [Schi79]. Like the theories of van der Sandt and Fauconnier, these accounts are explicitly procedural, and explicitly representational. Note that although Schiebe talks of worlds of evaluation, one of his uses of the term world is akin to Fauconnier's term mental space.
Part II

The Assertion: What Comes First in Dynamic Semantics
Chapter 7

Two Birds and One Stone

7.1 Aim

Two apparently disparate aspects of natural language meaning, presupposition and epistemic modality, can be tackled using a single, suitably dynamic, theory of information. The current chapter has two main objectives. Firstly it should serve as an introduction to the treatment of presupposition within a dynamic framework, and thus to the more ambitious developments in the remainder of the thesis. Secondly the chapter is intended to show that the underlying assumptions behind the Context Change model of presupposition, which was developed in the seventies and early eighties and is thus one of the first instances of a dynamic theory of natural language meaning, can be independently motivated through a consideration of epistemic modality.

In the remainder of the thesis I will focus on two types of presuppositional construction, definites, which I take to include not only definite descriptions, but also pronouns and proper names, and factives. I will take as paradigmatic examples like the following, containing the attitude verb 'realise' which is factive and is said to presuppose the truth of its propositional complement:

E165 Anna realises that Bertha is hiding.

Whilst I will occasionally mention presuppositional constructions other than definites and factives\(^1\), the formal theory as such will be limited to just these two. I will not take

\(^1\) Karttunen [Kar71] distinguished between two classes of factive verbs, full factives and semifactives. He suggested that whilst both assertions and denials of sentences with matrix factives require the truth of the factive complement, the two subclasses differ in their behaviour when embedded in hypothetical contexts such as conditionals. Supposedly, whereas a full factive which is embedded in a hypothetical context such as the antecedent of a conditional still requires the truth of the factive complement, this is not the case for a semifactive. However, Stalnaker [St74, pp.207-210] argued convincingly that the source of the differences Karttunen had observed does not reflect an important taxonomic division, and instead suggested a pragmatic explanation for the original data. Stalnaker's point of view seems to have won the
the raw data to consist of sentences and their presuppositions, and then present a theory which predicts for each sentence its presuppositions. In a sense I will not specifically be addressing the Projection Problem for Presuppositions. Instead, I will consider that old and familiar projection problem of which presupposition is but a tiny part, namely the problem of predicting the meanings of complex sentences in terms of the meanings of their parts, or the problem of compositionality as it is best known.

I will attempt to follow a long standing tradition of philosophers and semanticists. The data will consist of implications between sentences of natural language, and to account for the data I will define logics which yield these implications as entailments between formulae, combined with a general way of translating from natural language into the logic. An obvious advantage of using implications as the raw data is that it makes perfect sense to talk of a complex discourse having a particular sentence as an implication, whereas, as I have said, there is no test for presuppositions which applies to multiple sentence texts. In summary, the projection problem for presuppositions forms just one part of a much larger projection problem, and not a very clearly defined part. I will address the larger, better defined problem, but in the process the sub-problem, whatever the details of its definition, should be covered.

So, what are the special demands that presuppositional constructions place on a compositional theory of meaning? I will now present some simple, illustrative data, more complicated cases being considered later in the thesis. Firstly, utterances of either E165 or its negation E166 would tend to imply that Bertha was hiding:

E166 Anna does not realise that Bertha is hiding.

It is this characteristic behaviour of presuppositions under negation which most clearly marks them out as different from ordinary entailments. For example an utterance of E167 would also lead to the conclusion that Bertha was hiding, but this would not be the case with an utterance of E168. So E167 is said to entail but not presuppose that Bertha is hiding.

day, for later work rarely mentions the distinction. For instance a cursory inspection of the authoritative collection in [O&D79] revealed only one mention of semifactives, that being in a passage in Gazdar's article [Gaz79b, pp.86-87] where he agrees with Stalnaker that Karttunen's distinction is "otiose", and provides a formalisation of Stalnaker's argument (the same passage is found in [Gaz79a, pp.153-154] modulo minor, mostly typographical alterations and a unit decrement of example numbering). Accordingly, I will not distinguish between subclasses of factive verbs. But the reader should be aware that the major part of this thesis will not be concerned with explaining the differences that Karttunen observed, and the compositional grammar to be developed will not predict any such differences. Only in Chapter 11 will it be shown formally how the approach developed in the thesis could be adapted so as to account fully for the defeasibility of presuppositional inferences, this defeasibility lying at the heart of the Stalnaker/Gazdar account of Karttunen's observations. Note further that the observational differences between classes of factive verbs pointed out by Karttunen concerned first-person uses, and to my knowledge have not been observed in third-person uses. Since the examples I will consider are uniformly third person, there is reason to think that Karttunen's observations do not bear on the data to be discussed.
E167 Bertha is hiding in the attic.

E168 Bertha is not hiding in the attic.

An utterance of E169, in which a presuppositional construction is embedded in the antecedent of a conditional, would implicate that Bertha was hiding. Similar predictions seem warranted for embedding under the modality 'might', as seen in E170, and for iterative embedding of presuppositional constructions as in E171.

E169 If Anna realises that Bertha is hiding, then she will find her.

E170 Anna might realise that Bertha is hiding.

E171 Bertha regrets that Anna realises that she is hiding.

However, complex sentences do not uniformly preserve the presuppositions of their parts. For instance, an utterance of E172 would not implicate that Bertha was in the attic, in spite of the occurrence of 'Bertha is in the attic' as the complement of a factive verb. Similarly, neither E173 nor E174 seems to implicate that Bertha is hiding at all, although they both involve a factive verb with complement 'Bertha is hiding'.

E172 If Bertha is not in the kitchen, then Anna realises that Bertha is in the attic.

E173 If Bertha is hiding, then Anna realises that Bertha is hiding.

E174 Perhaps Bertha is hiding and Anna realises that Bertha is hiding.

In the remainder of the chapter I will weave backwards and forwards between the topics of presupposition and epistemic modality. Beginning with presupposition, in §7.2 I focus on a problematic methodological issue concerning the Context Change model. In §7.3 I jump to the semantics of epistemic modality, presenting some data and suggesting informally how a dynamic semantics could improve over more traditional static accounts of modality. A formal presentation of a dynamic system appropriate to the treatment of the data in §7.3 is given in §7.4. In §7.5, it is shown that a minor extension to this system provides us with an account of presupposition, and we see how the account fares with the data presented above.

Note that the verb 'realise' which is being used in some of these examples is one Karttunen classed as semifactive. It has been pointed out to me by Gerald Gazdar that differences between factives and semifactives can surface under the types of embedding being considered here, for instance in conditionals. Thus to be sure of the generality of the inference patterns found in these examples one should consider variants with full factives. The reader should verify that substituting 'be glad' (which Karttunen classed as a full factive) for 'realise' does not affect the relevant presuppositionally derived inferences (or, in some cases, absence of them) in any of the examples considered.
7.2 Descriptive versus Explanatory Adequacy

One objection to Karttunen's account was that the CCPs of complex sentences are defined arbitrarily and with no independent motivation. In early formulations the meanings of lexical items included separate specifications of truth conditions and heritage conditions, where by "heritage conditions, I mean the rules which say how presuppositions will be projected."

Consider the definition for the conditional discussed in §5.1, in which the truth conditions are as for material implication and the heritage conditions are given by: "Context X satisfies-the-presuppositions-of 'If A then B' just in case (i) X satisfies-the-presuppositions-of A, and (ii) X [augmented with] A satisfies-the-presuppositions-of B." Since the truth conditions are specified independently of the heritage conditions, it would be possible to imagine a child mistakenly learning the correct truth conditions of the conditional but the wrong heritage conditions. This does not seem plausible, and we would clearly prefer a theory in which the heritage conditions were not specified separately, but somehow derived from the truth conditions and other general principles.

Heim attempted to rescue Karttunen's approach by showing how truth conditions could generally be derived from appropriate specifications of Context Change Potential. On this basis she claimed that the CCP model had at least the explanatory adequacy of its competitors, such as Gazdar's theory. However Mats Rooth (as cited in [Hei90]) and Scott Soames have noted that whilst the correct truth conditions derive from the CCPs for connectives that Heim specifies, this would also be the case for a number of other specifications of the CCP, and some of these alternative specifications would give incorrect heritage conditions. So we could still imagine a child learning CCPs for some connectives that gave the correct truth conditions but incorrect heritage conditions. On this basis, Heim (e.g. in [Hei90]) has been forced to retract her claim.

I believe that the CCP model can be saved from the quagmire of non-explanatory ignominy. But of course it is impossible to justify this without some definition of what it is for a theory to be an explanation for some phenomenon, and previous authors on presupposition have tended to take this notion for granted without actually saying what it is. Neither will I attempt a definition here. But I will assume that at least one factor determining explanatoriness is independent motivation: if phenomenon X and phenomenon Y are detected through independent observations, then a model motivated by phenomenon X alone can be seen as having some explanatory power with regard to phenomenon Y if it predicts Y without substantive addition. Those who are not prepared to accept that this is what is intended by explanatory might simply substitute the phrase independently motivated whenever they encounter explanatory in the remainder of this.

---

3 This is similar to the argument in [Hei83a].
thesis.

The key observation is that the recent work of Frank Veltman [Ve91] on the semantics of epistemic modal operators relies on a strikingly similar underlying notion of context change to that utilised by Heim. And indeed this is hardly surprising given that both have taken inspiration from the same philosophical well-springs, for instance from the work of Robert Stalnaker and David Lewis.

I will borrow from Veltman’s work to show how the context sensitivity of words like ‘might’ and ‘must’ motivates a dynamic semantics. None of the alternative CCPs for connectives that have been suggested by Rooth and Soames would be compatible with this semantics, and it is hard to imagine how a relevantly different dynamic semantics could still get the facts right about the meanings of the epistemic modalities.

I will then show how a simple extension to the logic developed in §7.3 — in fact the addition of a single unary operator — produces a system with all the presupposition inheritance properties we would expect of a CCP model. In the process, the connection between presupposition and the epistemic modalities, and also the logic of presupposition itself, will become transparent.

7.3 Hide and Seek with Epistemic Modalities

Imagine the following situation, which is very like an example considered by Veltman in [Ve86]. The difference is that he had misplaced his marbles, whereas I have lost a number of women.

- Anna is seeking Bertha, Clothilde and Daisy, and for our benefit she is recording her thoughts on a small portable cassette recorder.

- Anna has searched almost everywhere, and she knows that the only remaining hiding places are the cupboard (which is not in the attic) and the attic (which is not in the cupboard.)

- Only one person fits in the cupboard.

- Anna, having heard some noises, knows that somebody is in the cupboard.

Let us consider what we would expect to find on Anna’s tape, restricting our attention to discourses involving interesting mixtures of connectives and epistemic modalities, as they might occur when Anna tells us what she has found out. Firstly, look at the following two examples involving conjunctions, which I take to include sentence sequencing as well as the particle ‘and’, the word ‘perhaps’, which I take to mark epistemic possibility, and ‘must’ which seems to act as a sort of epistemic necessity operator:
E175 Perhaps it is Bertha in the cupboard and . . . it is Clothilde in the cupboard. Got you!
So Bertha must be in the attic.

E176? It is Clothilde in the cupboard and . . . perhaps it is Bertha in the cupboard. So Bertha
must be in the attic.

It is quite plausible that we might find E175, but by contrast it is hard to imagine an
occurrence of the discourse in E176. The reason for this is clear. As Anna learns about
where everybody is hiding, she gradually eliminates possibilities. So what is possible
at one point may not be possible after the addition of new information. However, the
reverse does not hold. So long as Anna has not been in anyway deluded, and provided
she is suitably cautious in her reasoning process and does not make any unwarranted
eliminations of possibilities, then the addition of new information can never increase the
number of open possibilities.

The only significant difference between examples E175 and E176 seems to be in the
ordering of conjuncts, and this will motivate the definition of an asymmetric conjunction.
We will also need to define epistemic operators that are sensitive to the local context of
interpretation. Thus the meaning of a sentence ‘A and B’, or a sequence of sentences ‘A.
B.’, will be expressed as an update with A followed by an update with B. An occurrence
of ‘perhaps A’ at a particular point in a discourse will mean that at that point in the
discourse the possibility of A remains, and we will take ‘must A’ to mean that at the
phrase’s point of occurrence, the possibility of A being false has been excluded.

Next, consider a case involving a conditional:

E177? Daisy might be in the cupboard. So if Daisy is not in the cupboard, then she might
be in the cupboard.

We could not account for this data by interpreting the conditional as material im-
planation, and taking ‘perhaps’ and ‘must’ to be standard, say S5, modal possibility and
necessity operators using the same modal accessibility relation. Under such static as-
sumptions the consequents of conditionals would be evaluated with respect to the same
context (i.e. the same possible world) as the conditionals as a whole. The intuitively
invalid argument in E177 would be valid in the standard picture, since if it was pos-
sible that Daisy was in the cupboard, then any conditional with an expression of this
possibility in the consequent would be true.\footnote{I mention S5 merely as an example. In fact, the data given here are not predicted by any of the
logics in the standard modal hierarchy, such as K, T or S4. The difference between examples E175 and
E176 is one of ordering. Depending on translation, this may be ordering of conjuncts or ordering of
premises in an argument. Either way, the logics K–S5 do not predict a difference, since in these logics
permutation of conjunctions and permutation of premises both preserve argument validity or invalidity.}
Clothilde is in the attic. Now although Bertha might be hiding in the cupboard, and might be hiding in the attic, I conclude that if Daisy is not hiding in the cupboard then Bertha must be hiding there, and if Daisy is hiding in the cupboard then Bertha must be hiding in the attic.

By contrast E178 is a valid argument, but would appear invalid on the standard picture. Standardly, E178 would come to imply that:

i. There are accessible worlds where Bertha is in the cupboard,
ii. There are accessible worlds where Bertha is in the attic,
iii. Daisy is in the attic implies that Bertha is in the cupboard in all accessible worlds,
iv. Daisy is in the cupboard implies that Bertha is in the attic in all accessible worlds.

Suppose that Daisy was in the cupboard. Then (iv) could only be true if Bertha was in the cupboard in all accessible worlds, which contradicts (ii). On the other hand, suppose that Daisy was not in the cupboard. Then (iii) could only be true if Bertha was in the attic in all accessible worlds, which contradicts (i). The relevant cases being thus exhausted, the discourse as a whole would appear logically inconsistent. However, this is at odds with our intuitions, for it seems that E178, although somewhat convoluted, is a perfectly reasonable thing for Anna to say.

Such examples support an analysis of the conditional as an assertion of the consequent under the assumption of the antecedent: this will be stated more formally below. Thus, concerning the conditional in E178, the assumption that Daisy was in the cupboard, would exclude the possibility that she was not there, and the conditional would not be supported by the given assumptions. In E178 whilst Anna is still open to the possibility that Bertha is in the cupboard, under the additional assumption that Daisy is in the cupboard she is prepared to assert that Bertha is not in the cupboard. Similarly,

As far as E177 is concerned, standard translation into modal logic would produce a valid argument. This is easily seen: translate E177 as having premise φ and conclusion ψ → φ. This form is classically valid, and remains valid in the all modal logics which are generated from K by addition of axioms. In these logics it is irrelevant to the validity of this inference that the φ formula is modalised (‘she might be in the cupboard’). Note also that Hintikka’s proposal [Hi62] to translate all sentences as intrinsically epistemic by embedding them under a belief operator has no direct bearing on the issue since it does not introduce any order sensitivity into the logic (two arguments which, like E175 and E176, differ only by permutation are not distinguished) and does not affect the validity of the argument in E177 (since in Hintikka’s S4 Bφ |= B(ψ → φ) regardless of whether φ is itself a modal statement). Note that Hintikka’s motivation for assuming embedding under an extra belief operator arose from Moore’s paradox, which concerns the oddity of utterances of discourses like ‘The cat is on the mat. I don’t believe that the cat is on the mat.’ Clearly the oddity of such a discourse must be linked to the oddity of ‘The cat is on the mat. It might not be on the mat.’, which is analogous to the first sentence of E176 (in the special context assumed). However, Moore’s paradox does not involve making distinctions between permuted variants like the first sentences of E175 and E176, and thus the literature on Moore’s paradox impinges only indirectly on the data analysed here. For recent discussion of Moore’s paradox, see e.g. Blok’s dissertation [Blo93, p.126–128].
whilst she is open to the possibility that Bertha is in the attic, under the assumption that Daisy is in the attic it would have to be the case — since she would have no other open possibilities — that Bertha is in the cupboard.

I will not discuss the meaning of negation in detail. Suffice it to say that we must define a negation consistent with our picture of an agent gradually eliminating possibilities. It must predict that in a case like E179, when we hear Anna telling us the negation of 'Daisy is in the cupboard', we need no longer consider alternatives where Daisy is in the cupboard. And, considering examples like E180 — I leave the reader to invent some more, or search for counter-examples, according to taste — it should predict that epistemic possibility and necessity behave as logical duals.

E179? Daisy is not in the cupboard. Perhaps Daisy is in the cupboard.

E180 Daisy might not be in the cupboard. So it's not the case that Daisy must be in the cupboard.

7.4 Update Logic (UL)

To meet the requirements of the data presented above, I will define a logic along the lines of one presented by Veltman [Ve86]. The significant difference will be syntactic, in that I will allow arbitrarily deep embeddings of the epistemic modalities whereas Veltman prefers to keep his modalities near the surface.

For the moment I will restrict myself to a propositional language defined over some set of atomic formulae, such as the proposition that Bertha is in the cupboard and the proposition that she is in the attic. I will let $\mathcal{P}$ stand for the set of atomic propositions.

Unlike in the Tarskian scheme, where semantics concerns itself with determining the truth or falsity of propositions, the main concern of Update Logic is the potential of a proposition to change an agent's information state. An information state will be identified with the range of open possibilities an agent has with regard to her knowledge of reality. Each open possibility, or possible world, will provide a complete picture of reality. To this end a proposition will be identified with a set of possible worlds, intuitively the set of worlds in which the proposition is true, and an information state will be a set of possible worlds.

**Definition D20 (Models for UL)** A model of UL is a pair $(W, F)$ where $W$ is a set of possible worlds and $F$ is an interpretation function mapping propositional constants to sets of worlds.
Definition D21 (Information in UL) An information state (context) in UL is a subset of $W$. Thus the minimal information state is $W$ itself, which will also be written $\top$, and the maximal information state is the empty set of worlds, also written $\bot$. Nonmaximal information states will be called consistent.

Definition D22 (Syntax of UL) The sentences of an Update Logic restricted to the propositions in $P$ are formed in the usual way from the atomic formulae in $P$, the unary operators $\text{NOT}$, $\text{MIGHT}$, $\text{MUST}$ and the binary connectives $\text{AND}$ and $\text{IMPLIES}$. We will use $p, q$ as metavariables over atomic formulae, and $\phi, \psi$ as metavariables over arbitrary sentences of UL.

The above definitions seem to assume worlds as ontological primitives. However, Veltman's system has a more syntactic flavour, in that worlds are not basic but identified with sets of atomic formulae. To see how a set of atomic formulae can be equated with a possible world think of the atomic formulae in the set as those which are true in that world, and those not in the set as false in that world. Later, when we work through some examples, it will be useful to view worlds in this syntactic light, and the following definition gives a method of constructing the appropriate models:

Definition D23 (Term Models for UL) A term model for UL over the atomic formulae $P$ is a pair $(W, F)$ where $W$ is the powerset of $P$ and $F$ is a function such that if $p \in P$ and $w \in W$ then $w \in F(p)$ if $p \in w$.

Now we are in a position to define the semantics of UL. The meaning of an expression $\phi$ of UL, written $[\phi]$ will be defined as a relation, written in infix notation, between two information states, intuitively an input and an output state. In general we should think of this relation as holding between a given pair of states just in case when we are in the first state the new information could leave us in the second state:

Definition D24 (Update) If $\sigma[\phi] \tau$ then $\tau$ is said to be an update of $\sigma$ with $\phi$.

In fact, in the following definition all UL formulae will denote relations which are total functions on the domain of information states, and I have diverged superficially from Veltman by specifying the semantics relationally rather than functionally: this will become important later. Another notational point: iteration of update relations will be allowed, such that $\sigma[\phi] \tau[\psi] \nu$ will mean that both $\sigma[\phi] \tau$ and $\tau[\psi] \nu$ hold.\footnote{Note that although this is a common convention of iteration, for instance being used in statements like "$x \geq y \geq z$", it is not the only convention used in logic. For example, "$\phi \lor \psi \lor \chi$", although ambiguous, would never be used to mean "$(\phi \lor \psi) \land (\psi \lor \chi)$".}
Definition D25 (Semantics of UL) For all models $\mathcal{M}$ and information states $\sigma, \tau$, the relation $\llbracket \cdot \rrbracket^\mathcal{M}_{UL}$ (sub- and super-scripts omitted where unambiguous) is given recursively by:

1. $\sigma[p_{\text{atomic}}] \tau$ if $\tau = \{w \in \sigma \mid w \in F(p)\}$
2. $\sigma[\phi \text{ AND } \psi] \tau$ if $\exists \upsilon \sigma[\phi] \upsilon[\psi] \tau$
3. $\sigma[\neg \phi] \tau$ if $\exists \upsilon \sigma[\phi] \upsilon \land \tau = \sigma \setminus \upsilon$
4. $\sigma[\phi \text{ IMPLIES } \psi] \tau$ if $\sigma[\neg (\phi \text{ AND } \neg \psi)] \tau$
5. $\sigma[MIGHT \phi] \tau$ if $\exists \upsilon \sigma[\phi] \upsilon \land$
   
   $((\upsilon \neq \bot \rightarrow \tau = \sigma) \land$
   $((\upsilon = \bot \rightarrow \tau = \bot)$
6. $\sigma[MUST \phi] \tau$ if $\exists \upsilon \sigma[\phi] \upsilon \land$
   
   $((\upsilon = \sigma \rightarrow \tau = \sigma) \land$
   $((\upsilon \neq \sigma \rightarrow \tau = \bot)$

Let us consider the clauses of definition D25 individually:

(1) **Atomic Formulae** The base case of the recursion says that to update an information state with an atomic formula, you must remove all those worlds in that state which are incompatible with the new proposition, and what remains is the outgoing state.

(2) **Conjunction** The meaning of a conjunction is defined as a relational composition between the meanings of the conjuncts. This definition corresponds to the informal analysis above suggesting that to update with a conjunction, you should update with the first conjunct, and then with the second.

(3) **Negation** This is defined in terms of a set complement operation. We find those worlds in the input state which are compatible with the negated proposition, and the output state is what remains after removing these worlds from the input.

(4) **Implication** Implication is defined using a standard equivalence, and it is the fact that a dynamic conjunction is used within that equivalence that gives the implication its dynamic flavour. In particular, the consequent is only evaluated in the context set up by a previous assertion of the antecedent.

(5) **Epistemic Possibility** There are two cases to be considered in the definition of the MIGHT-operator, which corresponds to Veltman's "might" operator. Either the propositional complement of the MIGHT corresponds to one of the open possibilities in the incoming information state (which is established by attempting to update
with the argument and checking that the result is not the absurd state) in which case the outgoing state is identified with the incoming one, or else the complement is already falsified by the incoming state, in which case the result is absurdity.

(6) Epistemic Necessity Again there are two relevant cases. Either adding the complementary proposition would not remove any worlds from the incoming state, in which case the complementary proposition "must" hold in the input state and the outgoing state is again identified with the incoming one, or else the complement would remove some worlds. In this case the complement is not yet established, it is clearly false that the complement "must" be true in the incoming state, and the final result is absurdity.

There are several notions of entailment that can be appropriate to a dynamic logic like UL, and for discussion the reader is referred to [Ve91]. The definition below says that a sequence of UL premises entails a conclusion just in case the relational composition of the meanings of the premises has in its range only fixed points of the conclusion. In other words, once we have updated any information state with all the premises, updating with the conclusion would add no new information.

**Definition D26 (Entailment in UL)**

\[ \phi_1, \ldots, \phi_n \models \psi \iff \forall \sigma_0, \ldots, \sigma_n \]
\[ \sigma_0[\phi_1][\phi_2] \ldots [\phi_n] \sigma_n \rightarrow \sigma_n[\psi] \sigma_n \]

The following clause gives a derivative notion of entailment against a particular background of assumptions:

**Definition D27 (Contextual Entailment in UL)** If \( \sigma \) is an information state, then:

\[ \phi_1, \ldots, \phi_n \models_\sigma \psi \iff \forall \sigma_1, \ldots, \sigma_n \]
\[ \sigma[\phi_1][\phi_2] \ldots [\phi_n] \sigma_n \rightarrow \sigma_n[\psi] \sigma_n \]

**Examples**

We will now consider some simple-minded translations of examples E175 - E180 above. We will confine ourselves to an update language restricted to the six atomic formulae \( bc, cc, dc, ba, ca \) and \( da \), which concern who is hiding where. For instance \( bc \) is the proposition that Bertha is hiding in the Cupboard, and \( da \) is the proposition that Daisy is hiding in the Attic. In the translations below I have ignored the presuppositional component of the it-clefts in some of the examples, and have also ignored the propositional content of 'Got you!'. Further, I have treated discourses of the form 'A. So B.' and 'A.
I conclude that B. as consisting of two parts, an assertion of the content of A, and a meta-level assertion that A entails B in the context σ of the given assumptions. In the following discussion, the context σ will correspond to the hide-and-seek situation described at the beginning of §7.3.

Firstly the cases motivating the asymmetric definition of conjunction:

E175 a. Perhaps it is Bertha in the cupboard and ... it is Clothilde in the cupboard. Got you! So Bertha must be in the attic.

b. $\text{MIGHT}bc \text{ AND } \text{cc, } (\text{MIGHT}bc \text{ AND } \text{cc} \models_{\sigma} \text{MUST}ba)$

E176 a. ?It is Clothilde in the cupboard and ... perhaps it is Bertha in the cupboard. So Bertha must be in the attic.

b. $\text{cc AND MIGHT}bc, \ (\text{cc AND MIGHT}bc \models_{\sigma} \text{MUST}ba)$

In explaining the contrast between these two examples (and also for the discussion of the remaining examples), we will need to consider what would happen to our information state as we heard them playing on Anna’s cassette recorder. It will be helpful to construct the relevant information states using the term model over the above six atomic formulae. A world will be a subset of the six atomic propositions, and an information state will be a set of such worlds.

However, since we already know that one person cannot be in two places at once, and that each person is in at least one place, our initial information state need not contain surreal possible worlds like $\{ba, bc, ca, cc\}$, which would depict Bertha and Clothilde as being in both the cupboard and the attic, and Daisy as being nowhere. Furthermore we know that only one person fits in the cupboard, so we can eliminate possible worlds like $\{bc, cc, dc\}$, which would paint a picture of a very crowded cupboard indeed. And one more piece of information: somebody is in the cupboard. The only three possible worlds compatible with all this information are $w_1 = \{ba, ca, dc\}, w_2 = \{ba, cc, da\}$ and $w_3 = \{bc, ca, da\}$, and if we initially have just this information, our information state will be $\{w_1, w_2, w_3\}$.

Now consider the effect of updating this information with the formulae in E175b. The reader should verify that the only possible sequence of information states starting with $\{w_1, w_2, w_3\}$ is:

E175 c. $\{w_1, w_2, w_3\} [\text{MIGHT}bc] \{w_1, w_2, w_3\} [\text{cc}] \{w_2\} [\text{MUST}ba] \{w_2\}$

On the other hand, the only possible sequence of states resulting from an update with the formulae in E176b, and starting from the same initial state is:

E176 c. $\{w_1, w_2, w_3\} [\text{cc}] \{w_2\} [\text{MIGHT}bc], [\text{MUST}ba]$
Thus the oddity of E176 arises because updating a state which does not allow for the possibility of \( ba \) with the proposition \( \text{MIGHT} ba \) yields a contradictory information state.

Regarding the conditionals in E177 and E178, we see that the contextual entailment in the first is not valid, since in the context of \( \{w_1, w_2, w_3\} \), 
\[
[(\not dc) \implies \text{MIGHT} dc]
\]
is not a fixed point, but the contextual entailment in the second is valid since \( \{w_1, w_3\} \) is a fixed point of:
\[
\left[\left(\not dc\right) \implies \text{MUST} bc\right) \text{ AND } (dc \implies \text{MUST} ba)\]

E177 a.  
\(\text{MIGHT} dc \), \( (\text{MIGHT} dc \models \not dc \implies \text{MIGHT} dc) \)

b. \( \{w_1, w_2, w_3\} \left[\text{MIGHT} dc\right]\{w_1, w_2, w_3\} \)
\[
\left[\left(\not dc\right) \implies \text{MIGHT} dc\right)\{w_1\} \]

c. \( \{w_1, w_2, w_3\} \left[\text{ca}\right]\{w_1, w_2, w_3\} \left[\left(\text{MIGHT} bc\right) \text{ AND } \text{MIGHT} ba\right]\{w_1, w_3\} \)
\[
\ldots \left[\left(\not dc\right) \implies \text{MUST} bc\right) \text{ AND } (dc \implies \text{MUST} ba)\left]\{w_1, w_3\} \]

E178 a.  
\text{Clothilde is in the attic. Now although Bertha might be hiding in the cupboard, and might be hiding in the attic, I conclude that if Daisy is not hiding in the cupboard then Bertha must be hiding there, and if Daisy is hiding in the cupboard then Bertha must be hiding in the attic.}

b. \( \text{ca} \), \( (\text{ca AND MIGHT} bc \text{ AND MIGHT} ba \models \left. \left(\not dc\right) \implies \text{MUST} bc\right) \text{ AND } (dc \implies \text{MUST} ba)\) \)

c. \( \{w_1, w_2, w_3\} \left[\text{ca}\right]\{w_1, w_2, w_3\} \left[\left(\text{MIGHT} bc\right) \text{ AND } \text{MIGHT} ba\right]\{w_1, w_3\} \)
\[
\ldots \left[\left(\not dc\right) \implies \text{MUST} bc\right) \text{ AND } (dc \implies \text{MUST} ba)\left]\{w_1, w_3\} \]

We have already considered some examples involving negation, so it should by now be clear to the reader why E179 is anomalous. The statement that 'Daisy is not in the cupboard' removes any alternatives in which Daisy was in the cupboard, and the following assertion that there still remains the possibility of Daisy being in the cupboard leads to absurdity.

E179 a.  
\(\text{Daisy is not in the cupboard. Perhaps Daisy is in the cupboard.}\)

b. \( (\not dc) \text{ AND MIGHT} dc \)

c. \( \{w_1, w_2, w_3\} \left[\not dc\right]\{w_2, w_3\} \left[\text{MIGHT} dc\right] \perp \)

However, E180 adds something new to the discussion, since it involves a negation outscoping an epistemic modality:

E180 a.  
\(\text{Daisy might not be in the cupboard. So it's not the case that Daisy must be in the cupboard.}\)

b. \( \text{MIGHT} \not dc \), \( (\text{MIGHT} \not dc \models \not \text{MUST} dc) \)
That E180 is consistent follows from the fact that the first and second sentences of E180 translate into equivalent formulae of UL. This, of course, is just one example of a more general equivalence, namely that MIGHT and MUST are logical duals:

**Fact F3** For any formula ϕ and information states σ, τ:

\[ \sigma \models \text{MUST} \phi \iff \sigma \not\models \text{NOT} (\text{MIGHT} \neg \phi) \]

Proof: From the definitions of NOT and MIGHT it can be seen that:

\[ \sigma \models \text{MIGHT} \neg \phi \iff \exists \sigma[\phi] \forall \land \]

\[ (u \neq \sigma \rightarrow \tau = \sigma) \land \]

\[ (u = \sigma \rightarrow \tau = \bot) \]

Using the definition of NOT once more we obtain:

\[ \sigma \not\models \text{NOT} (\text{MIGHT} \neg \phi) \iff \exists \sigma[\phi] \forall \land \]

\[ (u \neq \sigma \rightarrow \tau = \bot) \land \]

\[ (u = \sigma \rightarrow \tau = \sigma) \]

But this is just the definition of MUST.

To understand how examples like E180 constrain the definition of negation, we need only consider alternative possible definitions which would be consistent with the classical picture of negation, but not preserve the logical duality of the dynamic modal operators. For instance, we could have defined negation by:

**Definition D28 (Pointwise Negation)**

\[ \sigma \not\models \phi \iff \tau = \{ w \in \sigma \mid \{ w \}[\phi] \bot \} \]

This negation is pointwise in that it looks at the individual worlds in the incoming state, and checks which ones are incompatible with the negated proposition. Using such a negation would not have affected examples E175 – E179, since it is easily verified that it is equivalent with the earlier negation provided the negated proposition contains no epistemic modalities. But the entailment in E180 would not have held, since the conclusion would no longer be a fixed point in the context set up by the premise, as is seen from the following sequence of updates:

\[ \{ w_1, w_2, w_3 \}[\text{MIGHT} \not dc] \{ w_1, w_2, w_3 \}[\text{MUST} dc] \{ w_2, w_3 \} \]. In fact we would have the unlikely equivalence: \( \not\models \phi \equiv \not\models \phi \). The original definition of negation, in
which the negated proposition is evaluated with respect to the entire incoming context rather than just its parts, is clearly preferable.

7.5 A Presupposition Operator

The dynamic account above leads to a straightforward characterisation of the CCP notion of presupposition. A context can only be updated with a sentence if the presuppositions of the sentence are already satisfied in the context. More formally:

**Definition D29 (Satisfaction)** A context $\sigma$ satisfies a formula $\phi$ iff $\models_\sigma \phi$ (or equivalently $\sigma[\phi]_\sigma$).

**Definition D30 (Admission)** A context $\sigma$ admits (can be updated with) a formula $\phi$ iff there is a context $\tau$ such that $\sigma[\phi]_\tau$.

**Definition D31 (Presupposition)** A formula $\phi$ presupposes a formula $\psi$ iff for all contexts $\sigma$, if $\sigma$ admits $\phi$ then $\sigma$ satisfies $\psi$.

In these terms, the formulae of UL carry no non-trivial presuppositions, since every context can be updated with any formula of UL. This is because the meanings of UL formulae define the equivalent of total functions on the domain of contexts. However, I will now extend UL with a single unary operator which allows us to restrict the incoming contexts for which an update is defined. In the resulting Partial Update Logic (PUL), some formulae will define the equivalent of partial functions on the domain of contexts.

**Definition D32 (Models for PUL)** As for UL.

**Definition D33 (Information in PUL)** As for UL.

**Definition D34 (Syntax of PUL)** As for UL but with an additional unary operator, $\partial$, “the presupposition operator”.

**Definition D35 (Semantics of PUL)** As for UL but with the following additional clause:

$$\sigma[\partial\phi]_\tau \iff \tau = \sigma \land \sigma[\phi]_\sigma$$

**Definition D36 (Entailment and Contextual entailment in PUL)** As for UL.

The presupposition operator $\partial$ is reminiscent of the modal operator $\text{MUST}$ defined previously. Given any formula $\phi$ which itself contains no presuppositions, both $\partial\phi$ and $\text{MUST}\phi$ have the same fixed points as $\phi$. That is, for all states $\sigma$, if $\sigma[\phi]_\sigma$ then $\sigma[\partial\phi]_\sigma$.
and $\sigma[MUST\phi]\sigma$. However, the two operators differ with respect to the non-fixed points of $\phi$ — that is the states $\sigma$ such that there is a state $\tau \neq \sigma$ for which $\sigma[\phi]\tau$. The presupposition operator will not define a transition for such points. That is, if $\sigma$ is a non-fixed point of $\phi$ then $\sigma$ does not admit $\partial \phi$. On the other hand, the necessity operator does define a transition for the non-fixed points: for any state $\sigma$ which is not a fixed point of $\phi$, we have that $\sigma[MUST\phi]$. So the $\partial$-operator is importantly different from the MUST-operator in that all contexts admit MUST, whereas no consistent context for which updating with MUST$\phi$ would yield an absurd state admits $\partial \phi$.

I will now show how the presupposition operator can be used to reproduce the CCP treatment of presupposition as it concerns the examples from §7.1. Consider E165a together with the suggested translation in E165b:

E165 a. Anna realises that Bertha is hiding.
   b. $\partial bih$ AND $cb_{a}.bih$
   c. $\partial bih$ AND $cb_{a}.bih \models bih$

I have assumed two atomic formulae in this translation, $bih$, the proposition that 'Bertha is Hiding', and $cb_{a}.bih$, the proposition that 'Anna' has 'Come-to-Believe' that 'Bertha is Hiding'. The translation is given as a conjunction of the presupposition that Bertha is hiding together with the assertion that Anna has come to believe this. I have used a similar translation scheme for E166:

E166 a. Anna does not realise that Bertha is hiding.
   b. $\lnot (\partial bih$ AND $cb_{a}.bih)$
   c. $\lnot (\partial bih$ AND $cb_{a}.bih) \models bih$

In these translations I have been intentionally naive with respect to the lexical semantics of 'realises', and I would not wish to defend a general strategy of dividing the meaning of a mentalistic factive verb into one presupposed proposition and one asserted proposition about someone's mental state. The same strategy seems particularly problematic in the case of the verb 'regret', a verb so intrinsically intensional that it is almost impossible to isolate a purely mental component for it in ordinary English. The best I could manage was the strange circumlocution 'negative vibes arising from belief.' But for the moment it will be helpful to assume this division of meaning, as it will make the logical behaviour of presuppositions transparent. In the next chapter I will offer an approach to the lexical semantics of factives which is more abstract, and does not rest upon this assumption.

Crucially, both the entailments in E165c and E166c are valid in Partial Update Logic. If a formula presupposes (in the technical sense of definition D31) that Bertha is hiding,
then (i) the formula entails that Bertha is hiding, and (ii) the negation of the formula entails that Bertha is hiding. In fact a negated formula always carries precisely the same presuppositions (i.e. the set of propositions which are presupposed in the above technical sense) as its positive counterpart. Thus PUL preserves the characteristic behaviour of presuppositions under negation.

Since an understanding of this behaviour is essential to the remainder of the thesis, I will go through the entailments in E165c and E166c in detail. Given a PUL term model restricted to the two propositions $bih$ and $cb\_a\_bih$, information states will be subsets of the following four worlds:

$$A = \{bih, cb\_a\_bih\}$$
$$B = \{bih\}$$
$$C = \{cb\_a\_bih\}$$
$$D = \emptyset$$

Firstly let us consider the denotation of the first sub-formula in E165b, $\partial bih$. From definition D25, the meaning of $bih$ is given by:

$$\text{(7)} \quad \sigma[bih] \tau \iff \tau = \{w \in \tau \mid bih \in w\}$$

Definition D35 allows us to calculate from this the denotation of $\neg bih$:

$$\text{(8)} \quad \sigma[\neg bih] \tau \iff \tau = \sigma \land \sigma[bih] \sigma \iff \tau = \sigma \land \forall w \in \sigma \ bih \in w$$

This relation is equivalent to a set of pairs of states, where each state is expressed in terms of the four worlds $A - D$:

$$\text{(9)} \quad \llbracket \partial bih \rrbracket \equiv \{(\{A, B\}, \{A, B\}), (\{A\}, \{A\}), (\{B\}, \{B\}), (\bot, \bot)\}$$

Utilising definition D25 again, we can calculate the denotation of the whole formula:

$$\text{(10)} \quad \sigma[\partial bih \land cb\_a\_bih] \tau \iff \exists u \sigma[\partial bih] u [cb\_a\_bih] \tau$$
$$\iff \forall w \in \sigma \ bih \in w \land$$
$$\tau = \{w \in \sigma \mid cb\_a\_bih \in w\}$$

Again this can be written as an equivalent set of pairs:

$$\text{(11)} \quad \llbracket \partial bih \land cb\_a\_bih \rrbracket \equiv \{(\{A, B\}, \{A\}), (\{A\}, \{A\}), (\{B\}, \bot), (\bot, \bot)\}$$
It can be easily verified that the formula entails both $bih$ and $cb\text{.}a\text{.}bih$, since both of the possible output states (i.e. the right-hand members of the pairs in 11) $\{A\}$ and $\perp$ are fixed points of $bih$ and $cb\text{.}a\text{.}bih$. Similarly we can calculate the denotation of the formula in E166b:

$$\sigma[\text{NOT}(\partialbih \text{ AND } cb\text{.}a\text{.}bih)]\tau \iff \exists v \, \sigma[\partialbih \text{ AND } cb\text{.}a\text{.}bih]v \text{ and } \tau = \sigma \setminus v$$

Expressing this as a set of pairs, in 13 below, shows that the only information states that can result from updating with E166b are $\{B\}$ and $\perp$:

$$\partialbih \text{ AND } cb\text{.}a\text{.}bih \equiv \{(A, B), \{B\}, (A, \perp), (\perp, B), (\perp, \perp)\}$$

These two output states are once again fixed points of $bih$, so the entailment that $bih$ is preserved. Indeed, all the possible input states (i.e. the left-hand members of the pairs in 13) are also fixed points of $bih$, which shows that the formula in E166b presupposes $bih$ as well as entailing it. The same holds for E165b — all the possible inputs of the formula are fixed points of $bih$. However, E166b differs from E165b in that the output states of E166b are not fixed points of $cb\text{.}a\text{.}bih$, but of $\text{NOT} \, cb\text{.}a\text{.}bih$. Thus E166a) is seen to presuppose that Bertha is hiding, and assert that Anna has not come to believe this.

Before proceeding to the remaining examples, a comment is in order about the translations in E165b and E166b. Both of these translations involved an asymmetric conjunction, and derivation of the correct presuppositional behaviour depended crucially on the ordering of the conjuncts. This seems unnatural, for it is not obvious why there should be any preferred ordering of these conjuncts which essentially derive from the lexical semantics of a single verb rather than from any surface ordering of lexical items.

However, it is quite possible to introduce a second, static conjunction into PUL:

**Definition D37 (Static Conjunction)**

$$\sigma[\phi \& \psi]\tau \iff \exists \rho, v \, \sigma[\phi]\rho \text{ and } \sigma[\psi]v \text{ and } \tau = v \cap \rho$$

If the dynamic conjunctions in E165b and E166b were replaced with this static conjunction, the same presuppositional behaviour would result, and the ordering of the conjuncts would be irrelevant. With this additional connective, a sensible strategy might be to translate surface occurrences of 'and', 'but' and sentence sequencing in terms of the dynamic conjunction, and to make all other conjunctions static. My reasons for not pursuing this strategy here are pedagogical — one type of conjunction is enough for current purposes.

Let us now consider some more examples from §7.1. The entailments in E169c, E170c and E171c) show that in PUL if a formula contains a presupposed proposition embedded
within the antecedent of a conditional, or within an operator of epistemic possibility, or within another presuppositional construction, then the formula as a whole will entail the presupposed proposition. Thus the PUL analysis correctly predicts that all the three examples entail that Bertha is hiding.

In the translations, \( awfb \) is the proposition that 'Anna Will Find Bertha', and \( \text{NVB}_B \) is the proposition that 'Bertha' has 'Negative Vibes' arising from her 'Belief' that Anna realises that Bertha is hiding:

E169 a. If Anna realises that Bertha is hiding, then she will find her.
   b. \( (\partial \text{bih} \text{ AND cb}_a \text{.bih}) \text{ IMPLIES } awfb \)
   c. \( (\partial \text{bih} \text{ AND cb}_a \text{.bih}) \text{ IMPLIES } awfb \models \text{ bih} \)

E170 a. Anna might realise that Bertha is hiding.
   b. \( \text{MIGHT}(\partial \text{bih} \text{ AND cb}_a \text{.bih}) \)
   c. \( \text{MIGHT}(\partial \text{bih} \text{ AND cb}_a \text{.bih}) \models \text{ bih} \)

E171 a. Bertha regrets that Anna realises that she is hiding.
   b. \( \partial(\partial \text{bih} \text{ AND cb}_a \text{.bih}) \text{ AND NVB}_B \) \( (\partial \text{bih} \text{ AND cb}_a \text{.bih}) \)
   c. \( \partial(\partial \text{bih} \text{ AND cb}_a \text{.bih}) \text{ AND NVB}_B \) \( (\partial \text{bih} \text{ AND cb}_a \text{.bih}) \models \text{ bih AND cb}_a \text{.bih} \)

As a final illustration of PUL, we consider two examples where an embedded presupposition is not projected. Example E172 shows the standard weak predictions of the CCP model with respect to presuppositions embedded in the consequent of a conditional. As shown in E172c, the conditional in E172a does not entail that Bertha is in the attic. We only have the weaker entailment shown in E172d, that if Bertha is not in the kitchen then she is in the attic. Note that whilst this behaviour seems appropriate for the conditional in E172a, similar CCP predictions for other conditionals have often been criticised. In a later chapter it will be shown how this aspect of the CCP model can be defended.

E172 a. If Bertha is not in the kitchen, then Anna realises that Bertha is in the attic.
   b. \( (\text{NOT bc}) \text{ IMPLIES } (\partial \text{ba} \text{ AND cb}_a \text{.ba}) \)
   c. \( (\text{NOT bc}) \text{ IMPLIES } (\partial \text{ba} \text{ AND cb}_a \text{.ba}) \neq \text{ ba} \)
   d. \( (\text{NOT bc}) \text{ IMPLIES } (\partial \text{ba} \text{ AND cb}_a \text{.ba}) \models (\text{NOT bc}) \text{ IMPLIES } \text{ ba} \)

Any PUL information state will admit E174b, so that the sentence as a whole carries no presupposition. The reason should by now be familiar. The second clause is evaluated in the context set up by previous evaluation of the first clause. Since updating with the sub-formula \( \text{bih} \) results in a context containing only worlds in which Bertha is hiding, and
since the sub-formula corresponding to the second clause, $\partial bih \land cb_a bih$ is defined on all such contexts, the whole formula will be admitted by any incoming context. And if a formula is admitted by any context, then it has no presuppositions.

E174  a. Perhaps Bertha is hiding and Anna realises that Bertha is hiding.
     b. $\text{MIGHT}(bih \land \partial bih \land cb_a bih)$
     c. $\text{MIGHT}(bih \land \partial bih \land cb_a bih) \not\models bih$

The N-bird Problem

The main aim of this chapter was to provide sufficient introduction to dynamic semantics for the reader to appreciate how that approach may be applied to presupposition and to other phenomena. Secondarily, I hope to have shown that, in principle, the theory of interpretation underlying the CCP model of presupposition can be independently motivated in terms of extraneous semantic phenomena. I have considered only one of these phenomena, namely the behaviour of epistemic modality. However, the treatment of epistemic modality is far from being the only non-presuppositional motivation for a dynamic semantics. A far better established motivation is the treatment of donkey and discourse anaphora, and in the following chapters I make some attempt at the harder “3-bird” problem, combining a treatment of presupposition and modality with a Groenendijk & Stokhof-style treatment of anaphora. An account of presupposition along the lines I have sketched also has potential for a dynamic treatment of focus, as shown in [Krif92]. But the search must continue for the semanticist's stone, that single theory of information with which we could knock any arbitrary collection of problems in the theory of Natural Language meaning straight out of the sky.
Chapter 8

A Bit Like English

8.1 Introduction

The system (ABLE) to be described in this chapter and the next, which will form the basis of the fragment to be defined in Chapter sec:lgr, brings together ideas from many sources. The account of presupposition, is a further development of that in [Bea92], which is based on the earlier work of Heim [Hei83a] and Karttunen (e.g. [Kar73]); the theory of anaphora descends from that of Kamp [Kam81], Heim [Hei82], Groenendijk and Stokhof [GS91a] and Dekker [Dek92]; the dynamic approach to quantification is based on the work of Dekker [Dek92], Chierchia [Ch92] and Groenendijk and Stokhof [GS91a, GS91b]; and the account of epistemic modality, which extends Veltman’s [Ve91] might operator as incorporated in UL and PUL systems of Chapter 7 to the predicate level, arose from collaboration with those of the above who are Amsterdam colleagues.¹

I will follow Muskens [Mus90] in preferring classical type theory over Montague’s variant IL as a formalism appropriate to the embedding of dynamic semantics in an otherwise Montagovian theory of meaning. IL is a designer logic. Montague’s aim was to build a formalism that reflected his Fregean view of meaning, and intertwined ideas from modal logic and type theory so as to reflect that view. Thus the underlying formalism Montague created is inextricably tied to the application Montague had in mind. For that very reason, any attempt to model a qualitatively different account of meaning using IL is fraught with problems. More particularly, in the last decade or so, much effort has gone into theories of meaning which are partial and/or dynamic, but the theory of meaning Montague had in mind was both total and static.

For this reason, it has been generally recognised that IL was not handed down on stone tablets, and is open to modification or replacement. Thus Muskens [Mus90] has introduced a variant of type theory to model partiality, and Groenendijk and Stokhof

¹Thus Dekker, Groenendijk, Stokhof and Veltman.
[GS91b] have utilised a variant of IL developed by Janssen in order to model dynamics. However, I think it is clear that whilst some alternative to IL is required, the needless multiplication of semantic formalisms is to be avoided. Fortunately, one does not have to look far in order to find an existing formalism adequate to my purposes: classical type theory, apart from having a much cleaner logic than IL, is well suited to modeling the dynamics of natural language.

To be more precise, the advantage of classical type theory over IL in the remainder of this thesis is as follows. I will want to reason formally about information and information states, and possible worlds will be involved in the specification of these states. However, in IL it is difficult to reason explicitly about possible worlds, since intensional objects are cloaked by special syntactic restrictions which prevent the use of objects of type s (i.e. possible worlds). In type theory, there is no restriction as to which types can be represented syntactically by constants and variables, and the full apparatus of functional abstraction and application is available over all types. Thus, in type theory, it is possible to be explicit about possible worlds (and other aspects of information states) where in IL one would have to use ad hoc and round-about trickery.

This chapter will be taken up with an initial definition of the semantics for a language A Bit Like English, or ABLE. ABLE is a first order language in the tradition of DPL, EDPL and KPL.² That is to say, it is a dynamic logic based around a language sufficiently close to English that those with imagination and faith can easily believe that formulae are compositionally derivable from the English sentences they are supposed to represent.

Definition D38 (Syntax of ABLE) Given a set of predicates P consisting of unary predicates P₁, binary predicates P₂ and attitude predicates Pₐ, a language L_{ABLE} P is given by recursion over the following set of rewrite rules, where all brackets are optional:

²These abbreviations are for, respectively, the Dynamic Predicate Logic of Groenendijk and Stokhof [GS91a], the Eliminative Dynamic Predicate Logic of Dekker [Dek93], and finally my Kinematic Predicate Logic [Bea92].
In the following sections of this chapter I will firstly discuss some general and meta-theoretical considerations, before tackling the various basic components of ABLE one at a time. In Chapter 9 it will be shown how this basic apparatus can be applied to the study of presupposition and epistemic modality. Those who lack imagination or faith will hopefully be appeased by Chapter 10, where it will be shown how ABLE can be utilised in the definition of a compositional grammar fragment.

8.2 Some Metatheory

Throughout Chapters 8-10, classical type theory will play much the same role as IL does in PTQ: it will be the vehicle for the formal expression of meaning. In Chapters 8 and 9 it will be used to specify the semantics of ABLE, and in Chapter 10 it will be used to specify the semantics of a fragment of English. The motivation for having used type theory to give the semantics of ABLE will be made clear in Chapter 10.

Definition D39 (Three Sorted Type Theory) Ty₃ is a three sorted type theory along the lines of Gallin’s Ty₂³ [Gal75], which itself is a reformulation of Russell’s Theory of Types, having the normal apparatus of abstraction, function application, existential and universal quantification over objects of every type, and standard truth functional connectives, as well as a number of distinguished constants to be introduced in the remainder of the thesis. The symbol “.”, which will be used left-associatively, will denote function application.

The types are given by the category TYPE in the following recursion, in which d, e, w and t are, respectively, the types of discourse referents, individuals, possible worlds and truth values:

\[
\text{BASIC} \implies d \mid e \mid w \mid t \\
\text{TYPE} \implies \text{BASIC} \mid \langle \text{TYPE, TYPE} \rangle
\]

³Also see [GS84] for an application of Ty₂, and yet another motivating discussion.
Definition D40 (Models) A Model $M$ for $\text{Ty}_3$ is a tuple $\langle W, E, ||.|| \rangle$ where $W$ and $E$ are non-empty. Each type $\alpha$ is associated with a domain $D_{\alpha}$, with $D_w = W$, $D_d = \mathbb{N}$ (the natural numbers), $D_e = E$, $D_t = \{\text{true, false}\}$ and domains for the complex types being built up recursively from the basic domains such that for all types $\alpha$ and $\beta$, the domain $D_{(\alpha, \beta)}$ is the set of functions from $D_{\alpha}$ to $D_{\beta}$. The interpretation function $||.||$ is a mapping from constants of type $\alpha$ to elements of the domain $D_{\alpha}$, with the distinguished constants $\top$ and $\bot$ of type $t$ mapped onto true and false respectively, and any discourse marker $i$ (where $i$ must be in $\mathbb{N}$) mapped to $i$.

Rather than independently defining models for ABLE, I will simply let $\text{Ty}_3$ models also be ABLE models. Using $\text{Ty}_3$ models for such a simple language might be felt to be overkill, but will be formally adequate.

ABLE formulae are to be interpreted dynamically, as functions from information states to information states. Following Stalnaker [St79] and Veltman [Ve91], factual information will be encoded in terms of shrinking sets of possible worlds. At a given point in a conversation the information state of a participant will be partially characterised by a set of worlds, intuitively those worlds which are compatible with everything that has been established up until that point.

A conversational participant must do a certain amount of bookkeeping in order to keep track of what is being talking about. For current purposes it will be assumed that what is being talked about — the topics of conversation — are individuals or groups of individuals, and not, for instance, properties or propositions. Further, as the reader might have expected, it will be assumed that each participant keeps track of the conversation using a set of discourse markers. On encountering a new discourse topic, for instance introduced by an indefinite noun phrase, a conversational participant assigns a discourse marker to that topic.

I will take a discourse referent or topic to be a public entity, something shared by all the conversational participants. By contrast I will treat discourse markers as personal record-keeping devices private to each conversational participant. In fact, discourse markers, as the reader may have gathered from the above definition of $\text{Ty}_3$ models, will simply be natural numbers. Thus the privacy of discourse markers amounts only to the absence of any assumption that different participants use the same markers.

The relationship between discourse markers and the objects in the model is mediated by what I will call an extended sequence, a simple development of the notion of a sequence employed by Heim: where no ambiguity is introduced I will used the terms sequence and extended sequence interchangeably. A Heimian sequence is a partial func-

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4The reader is also referred to Dekker's thesis [Dek93] and, developing these ideas further, recent work of Groenendijk, Stokhof and Veltman [GSV95, GSV:MS], for some indication of how a system like the one here presented lends itself to an account of discourse referents as public objects.
tion from discourse markers to objects. An extended sequence is just a total function from discourse markers to sets of objects, and this set will be referred to as the sequence valuation of the discourse marker by the sequence.

**Definition D41 (Extended Sequences)** Any object of type \((d, \langle e, t \rangle)\) is an extended sequence, and the type of such an object will be abbreviated as \(\sigma\).

The sequence valuation of an unused marker is the empty set, and other markers are mapped onto singleton or larger sets according to whether they represent one or many objects. Strictly I should not talk of sets of objects, since the standard interpretation of type theory would make these entities functions from individuals to truth values rather than sets, but I find that some things are best understood in terms of functions, and some in terms of sets. So I will continue to talk about sets of this or that, but when I do so I will usually mean not sets of this or that but rather the characteristic function corresponding to a set of this or that, that is a function from the domain of this or that into the domain of truth values. There will be similar latitude in my use of the word relation: commonly what I will refer to as relations between this and that will be coded up in type theory as functions from this into functions from that to truth values.

Extended sequences can be equated with the states of Groenendijk and Stokhof’s Dynamic Montague Grammar (DMG) [GS91b] or Muskens’ Logic of Change (LoC) [Mus90], since in these systems the only function of an information state is to keep track of the values of discourse markers. A first difference to note is that in DMG and LoC, states are total assignment functions, whereas here extended sequences are to be interpreted as partial assignment functions (i.e. a marker being mapped onto the empty set is interpreted as equivalent to the marker not being in the domain of the function) albeit encoded in a space of total functions. A second significant difference is that in both DMG and LoC states rather than discourse markers are ontologically primary: there is a basic domain of states in the models (type \(s\)), and discourse markers are interpreted as functions from states to individuals (type \(\langle s, e \rangle\), although in DMG this is the intensional type corresponding to a discourse marker, and its extension simply has type \(e\)).

The motivation for states being basic in DMG and LoC does not appear to be philosophical but technical, and stems from consideration of anaphora to objects of types other than that of individuals, although neither DMG nor LoC provides any treatment

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5In Heim’s work there is some variation as to whether partial or total assignments are used. In [Hei82] and [Hei83a] she uses total assignments, but with a distinguished referent to indicate which entities are already in the domain of discourse, whilst in [Hei83b] she uses partial assignment functions. Dekker [Dek93] recognised that using partial assignments could be of technical benefit in defining logics which deal both with anaphora and modality, and so my use of partial assignments could be traced to his work as much as to Heim’s.
of non-individual-typed anaphora. If discourse markers are basic, then, in order to keep track of information about high-typed objects in a completely general way, the notion of a state has to be complicated so as to provide mappings from discourse markers to the high-typed domains. Indeed, if we really want the potential to keep track of arbitrarily typed objects, then there will be no single type of state up to the job, since the type hierarchy is infinite but individual types are finite.

The solution suggested by Janssen [Ja84], which is to assume a basic type of states, and make discourse markers do the work, is quite general. In order to keep track of objects of some type \( \alpha \), simply use discourse markers having (intensional) type \( (s, \alpha) \). That such an elegant sleight of the mathematical hand should yield an immediate and general solution to the problem seems to good to be true, and, indeed, it is. A number of technical difficulties arise, most of which can be solved using some relatively simple constraints on the structure of the space of information states, but the most serious of which requires rather unappetising (although not unreasonable) restrictions to be placed on the typed models. The problem is that in a standard (generalised) model for type theory (or Montague's IL or Janssen's DIL) there will not in general be enough states to store information about arbitrarily typed domains. A simple example is the domain of objects of type \( \langle s, \langle s, t \rangle \rangle \): in a non-trivial model, the cardinality of this domain must be greater than the cardinality of the domain of states, \( s \) (as can be seen from a standard diagonalisation argument). Thus for any discourse marker \( d \) of type \( \langle s, \langle s, \langle s, t \rangle \rangle \rangle \), there will always be an object \( S \) of type \( \langle s, \langle s, t \rangle \rangle \) such that there is no state \( s \) having the property \( d(s) = S \). Clearly this is undesirable, since the whole point of discourse markers is that they can stand for any given object in the relevant domain. The solution offered by Janssen [Ja90] involves the construction of canonical models. Although there is a sense in which discourse markers will still not be able to take arbitrary values, this becomes irrelevant for most purposes, since for any assignment of a value to a discourse marker expressible in the language, there will always be an appropriate state in a canonical model.

But is such a mathematically general approach really motivated for the analysis of natural language? I know of no argument to the effect that there are anaphoric expressions in natural language of arbitrary type, and neither Janssen nor Muskens have provided evidence for this assumption. I can conceive of only a handful of relevant types, and any claim for the existence of anaphora other than to individuals, properties or propositions stands in need of defence. Yet even if the number of anaphoric domains was thus limited, some might still prefer Janssen's approach, since otherwise an information state combining separate mappings of discourse markers into even three distinct domains would have a somewhat messy type.
\[ T^x \varphi \psi \delta \gamma = \varnothing \]  

(1)

**Meaning Postulate M1**

Indicated with square brackets, put the scope of quantifiers, but the scope of lambda abstractions will always be determined by the scope of quantifiers; put the scope of quantifiers. In case of ambiguity, I will use round brackets for instance, \( \overline{E} = \psi \) (\( \psi \) is not a set variable). In the presence of quantifiers, and are taken to reinforce constructions so that

\[ \text{some} \]  

that that of individuals (or else of then) are involved.

One way or another, I think it possible, and are all just different types of individuals. One way or another, I think it possible, an.

provided the whole question is common property. In such theories, there is a.

condition the individual to all others. On the other hand, in property.
\[(2) \quad \text{domain}(\sigma, (d,t)) = \lambda f_\sigma \lambda D_d [\exists x. f.D.x]\]

\[(3) \quad \succeq_{\sigma, (\sigma,t)} = \lambda f_\sigma \lambda g_\sigma [\forall D_d \text{domain}(\sigma, (d,t)). g.D \rightarrow g.D = f.D]\]

This meaning postulate, and those that follow, should be thought of as constraints which Ty\textsubscript{3} models should obey if they are to suit the purpose of defining a semantics for ABLE. Note that whereas meaning postulates are Ty\textsubscript{3} formulae, and Ty\textsubscript{3} validity of these formulae constrains Ty\textsubscript{3} models, other definitions (which are labelled Dxy rather than MPxy) are not Ty\textsubscript{3} formulae (although they may contain Ty\textsubscript{3} formulae). A model which obeys all the meaning postulates to be given I will call a suitable model. Thus in any suitable model \(\circ\) is interpreted as the empty sequence, that is the function mapping every discourse marker onto the empty set of individuals, and \(\text{domain}(\sigma, (d,t))\) is a function from any sequence onto the set discourse markers in its domain. Thus we have that \(\text{domain.}\circ = \lambda D_d [\bot]\). The constant \(\succeq_{\sigma, (\sigma,t)}\) can be thought of as a binary relation: using infix notation, \(f \succeq g\) can be read as "\(f\) is an extension of \(g\)", meaning that for all discourse markers in the domain of sequence \(g\), the two sequences give the same interpretation, but \(f\) may also assign values to some additional discourse markers.

Since infix form often makes binary relations more perspicuous, I will adopt the following convention: if \(+\) is a binary operation of type \(\langle\alpha, (\alpha, t)\rangle\), where \(\alpha\) is any type except \(t\), and if \(A\) and \(B\) are of type \(\alpha\), then \(A + B\) will be used to mean \(+.A.B\). Furthermore, iteration will be allowed, such that if \(C\) is also of type \(\alpha\) then \(+.A.B + .B.C\) may be abbreviated as \(A + B + C\). Sometimes the notation \(A{+}B{+}C\) will be used to show that \(+\) is being treated as an infix operator. For the truth-functional connectives in Ty\textsubscript{3}, as for the ABLE connectives, the standard notational conventions will be assumed, which is the reason for the exceptive clause saying that \(\alpha\) cannot be \(t\).

Information states are defined as mappings from possible worlds onto sets of extended sequences. Ignoring for a moment the differences between my definition of an extended sequence and other authors' definitions of sequences, assignments or partial assignments, there remains only a superficial difference between making information states into sets of pairs of worlds and sequences as in Heim's work, or into functions from worlds to sets of extended sequences as here, or into functions from worlds to total assignments as in van Eijck and Ceparello's [EC92]. The additional slight variation on Heim's original notion of a context is of course just a by-product of the use of functional type theory as a meta-language. In fact I will sometimes prefer to talk of a pair of a world \(w\) and an extended sequence \(f\) being contained in some state \(I\), rather than saying that \(I\) maps \(w\) to a set containing \(f\), and I will sometimes talk of one such pair being an extension of another, meaning that each pair involves the same possible world, but that the sequence in one pair is an extension of the sequence in the other pair.
Definition D42 (Information States in ABLE) Any object of type \(\langle w, \langle \sigma, t \rangle \rangle\) is an information state, and the type of such objects will be abbreviated as \(i\).

It might be of mathematical interest to explore the structure of the space of ABLE information states in detail, but I refrain from doing so here since my intention is to provide only as much formal theory as is necessary for the applications which are to follow.\(^6\) However, I will introduce some constants which make ABLE information easier to manipulate:

Meaning Postulate MP

\[(1) \quad \cap_{i, (t, i)} = \lambda I_i \lambda I_j \lambda w_i \lambda f_{\sigma} [I.w.f \land J.w.f]\]
\[(2) \quad \cup_{i, (t, i)} = \lambda I_i \lambda I_j \lambda w_i \lambda f_{\sigma} [I.w.f \lor J.w.f]\]
\[(3) \quad \setminus_{i, (t, i)} = \lambda I_i \lambda I_j \lambda w_i \lambda f_{\sigma} [I.w.f \land \neg J.w.f]\]
\[(4) \quad \subseteq_{i, (t, i)} = \lambda I_i \lambda I_j [\forall w \forall f J.w.f \rightarrow I.w.f]\]
\[(5) \quad \overline{\top} = \lambda w_i \lambda f_{\sigma} [f_{\sigma} = \emptyset]\]
\[(6) \quad \bot_i = \lambda w_i \lambda f_{\sigma} [\bot]\]
\[(7) \quad \omega\text{-set}_{i, (w, t)} = \lambda I_i \lambda w_i [\exists f_{\sigma} I.w.f]\]
\[(8) \quad p\text{-domain}_{i, (d, t)} = \lambda I_i \lambda D_d [\exists w_{\omega} \exists f_{\sigma} I.w.f \land \text{domain}_{(d, t)}(I.w.f) = D]\]
\[(9) \quad t\text{-domain}_{i, (d, t)} = \lambda I_i \lambda D_d [\forall w_{\omega} \forall f_{\sigma} I.w.f \rightarrow \text{domain}_{(d, t)}(I.w.f) = D]\]

The interpretation of the first four constants, for which infix notation will be used, should be obvious: thinking of states as sets of world-sequence pairs, \(\cap\), \(\cup\), \(\setminus\) and \(\subseteq\) are just the standard set-theoretic operators. The constants \(\overline{\top}\) and \(\bot\) represent respectively the zero information state, which may be thought of as the established common ground at the beginning of a conversation, and the absurd information state, which is reached whenever an information state is updated with contradictory propositions. The function \(\omega\text{-set}\) associates with each information state a set of possible worlds, intuitively those worlds which are compatible with all the information up to that point in the discourse. Thus \((\omega\text{-set}.\overline{\top})\) is the set of all possible worlds, and \((\omega\text{-set}.\bot)\) is the empty set of worlds — for there are no possible worlds that are compatible with contradictory information.

The constants \(p\text{-domain}_{i, (d, t)}\) and \(t\text{-domain}_{i, (d, t)}\) are analogues of \(\text{domain}_{(d, t)}\), which was introduced above. Given a state \(I\), \(p\text{-domain}.I\) denotes the set of discourse markers which are at least partially defined in \(I\). If \(D\) is a discourse marker, then \(p\text{-domain}.I.D\) will hold just in case there is some world associated with a sequence which has \(D\) in its domain. The total domain of \(I\), the set of discourse markers which have a

---

\(^6\)But the reader may wish to refer to Dekker's discussion in [Dek93], where the algebra of a closely related state space, that of Dekker's EDPL, is examined more fully.
value in every world-sequence pair in $I$, is given by $t$-domain.$I$. This thesis will mostly concern itself with totally-defined discourse markers, so that regarding most information states which arise in examples, the partial and total domains will be identical. However, the possibility of partially defined discourse markers do arise.

Following Heim, I will call the denotation of an ABLE formula a context change potential:

**Definition D43 (Context Change Potentials)** Any object of type $(t, (t, t))$ is a context change potential (CCP), and the type of such objects will be abbreviated as $\pi$.

ABLE formulae are thus relations between information states, and have denotations of the form $\lambda I \lambda J [p]$. Such expressions have the by now obvious interpretation that $\lambda I$ is an abstraction over possible input states, and $\lambda J$ over possible outputs. If an ABLE formula has denotation $F$ and it holds for some $I$ and $J$ that $I\{F\}J$, we say that in state $I$ the formula provides an update to state $J$. In fact it will hold that no ABLE formula denotes more than one update from a given input state, but it will be argued that expressions of natural language should be thought of as having such relational meanings, for the ambiguity and underspecificity of natural language often means that there is more than one way in which a given expression could be used to update an information state.

In effect this non-determinism will be built into the translation from natural language into ABLE formulæ, so that ABLE could be viewed as, in that favourite phrase of the last few decades "a disambiguated language of logical form". However, this thesis has certain Montagovian pretensions: a type logic is used here, as for Montague, in order to make the business of defining a compositional semantics easier, and not because type theory is claimed to bear any special relationship to any language of mental representation. Similarly, the disambiguated LFs of ABLE are not intended to be thought of as mental representations (although, equally, none of the formal developments here preclude ABLE being thought of in this way). The motivation for putting the non-determinism into the translation function, or more properly the translation relation, is methodological. Keeping ABLE denotations deterministic permits the definition of a relatively clean logic over the ABLE language, and thus facilitates the process of turning what I think is the semanticist's primary source of data, namely natural language entailments, into intuitions about how natural language expressions must be translated. So we now turn to the problem of defining a notion of entailment for a language which claims to describe not facts about the world, but information change in agents. Consider a notion of entailment discussed by Veltman in [Ve91]:

A formula entails another if after updating any state with the first, updating with the second adds no new information.
In ABLE, as in Heim's File Change Semantics and Dekker's Eliminative Dynamic Predicate Logic, there are two ways in which information can grow. Firstly extra constraints can be learnt concerning the interpretation of predicates and whatever discourse markers are in use, and secondly new discourse markers may be added. It seems that it is the first type of information and not the second which is relevant to our intuitive notion of natural language entailment, as is shown in the following example discourse:

E181 a. There are three frogs, and exactly two are in the water.
   b. Therefore one of the frogs is out of the water.
   c. Obviously, it is not swimming.

The argument from a to b is valid, since in any world in which the first is true the second will also be true. However, b also introduces a new discourse referent. This is shown by considering a followed directly by c, a strange discourse indeed. It is clear that a does not by itself license the pronoun in c. However, the full discourse of a+b+c is quite natural. Thus b augments the context set up by a even though b is entailed by a, and more generally we must conclude that an entailed sentence can introduce certain types of new information. A suitably modified version of Veltman's entailment, which is essentially that used by Groenendijk and Stokhof in [GS91a] and by Dekker in [Dek92], is thus:

A formula entails another if after updating with the first, updating with the second adds no new information except for the possible introduction of new discourse referents.

The formalisation of this notion hinges on the possibility of differentiating between different types of information. The following postulate defines a notion of closure with respect to anaphoric potential. The anaphoric closure of a CCP \( F \), written \( \downarrow F \), is a purely eliminative CCP: it may remove some of the states in the input, but it will not introduce any new discourse markers. Given states \( I \) and \( J \), and a CCP \( F \), the formula \( I\{\downarrow F\}J \) will be true if and only if some state \( K \) can be obtained by updating \( I \) with \( F \), and the output, \( J \), is that subset of the world-sequence pairs in the input, \( I \), which have extensions in \( K \):

**Meaning Postulate MP3**

\[
\downarrow_{\langle\pi,\pi\rangle} = \lambda F\pi \lambda I\pi \lambda J\pi; [\exists K \{F\}K \land
\]

---

7 This discourse is reminiscent of Partee's marble examples.

8 It does not matter to the current discussion whether the argument from a to c is sound. Even if it is not sound, it remains the case that (1) anaphora between premises and conclusions in arguments is possible, and (2) a notion of consequence which fails to take anaphoric information into account is inadequate for modeling argument in natural language.
Such a notion of closure does not make sense for arbitrary CCPs. For instance, if a CCP were to denote a *downdate* (a loss of information), such that the \( \omega\)-set of the output could be a strict superset of the \( \omega\)-set of the input, then the anaphoric closure of the CCP would not preserve this property: the anaphoric closure of a CCP always denotes an update, whereby the output \( \omega\)-set is a (not necessarily strict) subset of the input \( \omega\)-set. Thus, to be sure that the notion of anaphoric closure is appropriate to ABLE CCPs, in Appendix A the following fact will be proved:

**Fact F4 (Eliminativity)** For any ABLE formula, \( \phi \), and states \( I \) and \( J \), \( I[\phi]J \) if and only if \( J \) contains only extensions of world-sequence pairs in \( I \).

If updating a context with the closure of some CCP would have no effect, then the context will be said to *satisfy* the CCP:

**Meaning Postulate MP4**

\[
satisfies_{I, (\pi, t)}(\omega) = \lambda I \lambda F [I[\downarrow F]I]\]

Infix notation will be used for relations between states and CCPs, producing formulae like \( I \) satisfies \( F \). It is now simple to define a binary relation *entails* which holds between two CCPs just in case any update with the first produces a state which satisfies the second, and in terms of this constant to define an entailment relation holding directly between ABLE formulae, as opposed to their denotations.

**Meaning Postulate MP5**

\[
entails_{(\pi, (\pi, t))} = \lambda F_x \lambda F_y [\forall I \forall J, I[F]J \rightarrow J satisfies F']
\]

**Definition D44 (Entailment in ABLE)**

\[
\phi \models_{\text{able}} \psi \iff [\phi][entails][\psi] \text{ is valid on the class of suitable models.}
\]

This thesis is largely concerned with the entailments of presupposing formulae, and it is to the notion of presupposition that we now turn. The denotations of ABLE formulae are functions from input states to sets of output states, and there may be some input states which are mapped onto the empty set of output states. For such inputs, the ABLE formula provides no update. Those states for which an ABLE formula does provide an
update will be said to admit the formula, and in terms of this property presupposition is defined. One ABLE formula presupposes another when the only states from which the first formula provides an update are those in which the second is satisfied:

**Meaning Postulate MP6**

1. \( \text{admits}_{\pi,(\pi,t)} = \lambda I \lambda F \pi [\exists J, I \{F\} J] \)
2. \( \text{presupposes}_{\pi,(\pi,t)} = \lambda F \lambda F' [\forall I, I \text{ admits } F \rightarrow I \text{ satisfies } F'] \)

**Definition D45 (Presupposition)** An ABLE formula \( \phi \) presupposes an ABLE formula \( \psi \) if and only if \( \text{presupposes}_{\pi,(\pi,t)}(\phi) = \psi \) is true in every suitable model.

The ABLE notions of presupposition and entailment are logically independent, in the sense that not all presuppositions of a formula are entailments, and not all entailments are presuppositions. However, for the class of non-modal formulae (i.e. the sub-language of ABLE not involving \( \text{MIGHT} \)), the presuppositions of a formula will form a strict subset of the entailments: this is shown in appendix A.

To finish this section I will introduce one last meta-theoretic notion, consistency. MP7 says that a state \( I \) is consistent-with a CCP \( F \) just in case it is possible to update \( I \) with \( F \) and not end up in the absurd information state. A consistent formula is just one for which there is some state which it is consistent-with.

**Meaning Postulate MP7**

1. \( \text{consistent-with}_{\pi,(\pi,t)} = \lambda I \lambda F \pi [\exists J, I \{F\} J \land \neg(J = \bot)] \)
2. \( \text{consistent}_{\pi,t} = \lambda F \pi [\exists I \text{ consistent-with}_{\pi,(\pi,t)}(\phi) I F] \)

**Definition D46 (Consistency)** An ABLE formula \( \phi \) is consistent if and only if \( \text{consistent}_{\pi,t}(\phi) \) is true in every suitable model. An ABLE formula \( \phi \) is consistent with an information state \( I \) if and only if \( \text{consistent-with}_{\pi,(\pi,t)}(\phi) I \) is true in every suitable model.

Having now introduced all the major types of objects to be used, sequences, states and so forth, the presentation can be simplified by ceasing to decorate every variable and constant with its type, and instead using a simple set of typing conventions: these are given in Table 8.1. For all the symbols given in the table, it will be assumed that the same symbol with subscripted numbers or superscripted dashes is of the same type.

The following sections will be concerned with providing a semantics for ABLE formulae. But this semantics will be given only indirectly, via an embedding into the language of Ty3, the embedding function being denoted \([\_]\). For instance, the embedding rule for
Table 8.1: Types of Meta-variables

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Type</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d )</td>
<td>( d )</td>
<td>discourse markers</td>
</tr>
<tr>
<td>( D )</td>
<td>( d )</td>
<td>variables over discourse markers</td>
</tr>
<tr>
<td>( a, b, c )</td>
<td>( e )</td>
<td>individual constants</td>
</tr>
<tr>
<td>( x, y, z )</td>
<td>( e )</td>
<td>variables over individuals</td>
</tr>
<tr>
<td>( A, B, C )</td>
<td>( e )</td>
<td>variables over groups</td>
</tr>
<tr>
<td>( X, Y, Z )</td>
<td>( e )</td>
<td>variables over worlds</td>
</tr>
<tr>
<td>( f, g, h )</td>
<td>( \sigma )</td>
<td>variables over extended sequences</td>
</tr>
<tr>
<td>( I, J, K )</td>
<td>( \iota )</td>
<td>variables over information states</td>
</tr>
<tr>
<td>( F )</td>
<td>( \pi )</td>
<td>variables over the denotations of ABLE formulae (CCPs)</td>
</tr>
<tr>
<td>( P )</td>
<td>( \rho )</td>
<td>variables over dynamic properties</td>
</tr>
<tr>
<td>( Q )</td>
<td>( \rho, \pi )</td>
<td>variables over dynamic Generalised quantifiers</td>
</tr>
</tbody>
</table>

an ABLE expression \( \text{Exp}_0 \) might be given as follows: \( \llbracket \text{Exp}_0 \rrbracket = f(\llbracket \text{Exp}_1 \rrbracket, \ldots, \llbracket \text{Exp}_n \rrbracket) \). Here the expression on the right hand side is not itself an expression of Ty3, but recursive replacement of the expressions \( \llbracket \text{Exp}_1 \rrbracket, \ldots, \llbracket \text{Exp}_n \rrbracket \) according to the definition of \( \llbracket \cdot \rrbracket \) will eventually yield a Ty3formula. For any ABLE formula \( \phi \) I will refer to the corresponding Ty3formula \( \llbracket \phi \rrbracket \) as the semantics of \( \phi \).

8.3 Predication and Identity

What will be presented in this section is not so much a theory of predication as a place for such a theory to go. ABLE predicates will be interpreted directly as higher order constants of Ty3, mapping arguments onto CCPs, and without additional constraints on Ty3 models these CCPs do not reflect any intuitive notion of the information which is given by a predication.

It will be assumed that for every unary ABLE predicate there is a corresponding identically named Ty3 constant which has the type of a function mapping the denotation of an ABLE discourse marker to the denotation of an ABLE formula. Constants of this type will be called dynamic unary predicates. ABLE discourse markers correspond directly to Ty3 discourse markers, that is to say objects of type \( d \), and ABLE
formulae denote CCPs, which have the type \( \langle d, \pi \rangle \). The symbol \( P^1 \) will be used as a metavariable over ABLE unary predicates and the corresponding Ty3 dynamic unary predicate constants. Similarly, every binary ABLE predicate (metavariable \( P^2 \)) will correspond to a dynamic binary predicate of type \( \langle d, \langle d, \pi \rangle \rangle \), and every ABLE attitude predicate (metavariable \( P^a \)) will correspond to a dynamic attitude predicate of type \( \langle d, \langle \pi, \pi \rangle \rangle \) — a function from discourse markers to a function from the denotation of an ABLE formula to the denotation of an ABLE formula. The clauses for the semantics of ABLE predications are thus trivial, and given by the following schema:

**Definition D47 (Semantics of Predication in ABLE)**

\[
\begin{align*}
[P^1.i] & = P^1.i \\
[P^2.i.j] & = P^2.i.j \\
[P^a.i.] & = P^a.i.[\phi]
\end{align*}
\]

It is arguable that such a minimalist approach to the interpretation of predications leaves a bit too much unsaid, for ABLE predicates are intended in the first place to correspond to the predicates of natural language, and there are many functions in the denotation spaces of the dynamic predicate constants which have no intuitive interpretation in terms of natural language. As things stand, it is possible for an ABLE predication to denote, for example, an information downdate, or the introduction of every prime numbered discourse marker into the context, or perhaps some sort of complement operation leaving only worlds in the \( \omega \)-set of the output which were not in the \( \omega \)-set of the input. To exclude such possibilities I will put some constraints on the denotations of dynamic predicates, although the job of creating particular entries within the remaining denotation space I see as the role of the lexical semanticist, and I will do no more in that respect than give a couple of examples.

The following set of meaning postulates radically restricts the behaviour of ABLE predications by relating the dynamic predicate constants to objects of lower, static types such as might be found in a more conventional Montague grammar. The traditional, Fregean intension of a one place predicate is, of course, a function from worlds to a function from individuals to truth values, which in the current framework would be an object of type \( \langle w, \langle e, t \rangle \rangle \). Similarly, the Fregean intension of a two-place predicate can be correlated with a Ty3 object of type \( \langle w, \langle e, \langle e, t \rangle \rangle \rangle \). Given that ABLE concerns not only individuals (type \( e \)) but also groups of individuals (type \( \varepsilon \)), it will come as no surprise that in the first of the following clauses dynamic unary predicate constants are related to objects of type \( \langle w, \langle \varepsilon, t \rangle \rangle \), whilst in the second clause dynamic binary predicate constants are related to objects of type \( \langle w, \langle \varepsilon, \langle \varepsilon, t \rangle \rangle \rangle \). I will call objects of these types
static unary predicates and static binary predicates, respectively.

The first postulate says that for every dynamic unary predicate constant there must be some static unary predicate, such that whenever the CCP obtained by application of the constant to a discourse marker provides an update from some input state, the input state must have the discourse marker in its domain, in which case the output state can be calculated in terms of the static predicate. In particular, the output must be the set of world-sequence pairs in the input such that the extension of the static predicate at the world includes the sequence valuation of the discourse marker.

The second postulate follows the first closely, and presumably requires no further explanation. The third postulate concerns dynamic attitude predicates, but it does not simply relate them to Fregean intensions of attitude verbs, the reason being that it is difficult to find a single such postulate appropriate to the needs of both factive and non-factive attitude verbs. Instead, for any given formula which serves as the propositional complement of a dynamic attitude predicate, the combination of the dynamic attitude predicate and the denotation of the formula is related to a static unary predicate. In effect the translations of complete verb phrases involving attitude verbs, like 'realises that she is surrounded' and 'doubts that Shakespeare will ever write another best-seller', are constrained to behave like intransitive verbs. Note the use in the following postulates of the type $e$, which is simply an abbreviation for $(e, t)$:

---

9Note that the formula in the postulate is not strictly a formula of $T_3$ but a schema over such formulae, since $P^1$ is not a constant of $T_3$ but a metavariable over dynamic unary predicate constants. Similar comments apply to other meaning postulates.
Meaning Postulate MP8
If \( P^1 \) is a dynamic unary predicate constant then:

\[
\exists V_{(w,(e,t))} \quad \forall \forall \forall D \ I\{P^1.D\} J \rightarrow \\
\quad \text{t-domain}.I.D \land \\
\quad J = \lambda w \lambda f [I.w.f \land V.w.(f.D)]
\]

If \( P^2 \) is a dynamic binary predicate constant then:

\[
\exists V_{(w,(e,t))} \quad \forall \forall \forall D \forall D' \ I\{P^2.D.D'\} J \rightarrow \\
\quad \text{t-domain}.I.D \land \text{t-domain}.I.D' \land \\
\quad J = \lambda w \lambda f [I.w.f \land V.w.(f.D).(f.D')]
\]

If \( P^a \) is a dynamic attitude predicate constant, and \( F \) of type \( \pi \) is the denotation of some ABLE formula, then:

\[
\exists V_{(w,(e,t))} \quad \forall \forall \forall D \forall D' \ I\{P^a.D.F\} J \rightarrow \\
\quad \text{t-domain}.I.D \land \\
\quad J = \lambda w \lambda f [I.w.f \land V.w.(f.D)]
\]

On the basis of these postulates some general characteristics of unary and binary ABLE predications can be given. If an ABLE formula \( \phi \) is of the form \( P^1.i \) or \( P^2.i.j \) then the following will hold:

**Partiality** There may be some states from which \( \phi \) does not produce an update. For instance \( \phi \) will only provide an update from states which have the discourse markers in \( \phi \) in their domain. That is to say:

\[ \exists I \neg (I \text{ admits } [\phi]) \]

**Determinism** \( \phi \) is functional, so that for every input state there is at most one possible output state. Formally:

\[ \forall \forall J (I[\phi] J \rightarrow \neg(\exists K I[\phi] K \land \neg(K = J))) \]

**Distributivity** An update with \( \phi \) can be calculated pointwise on the individual world-sequence pairs in the input, a property which is discussed in [GS90]. Thus if \( (w,f) \) denotes the singleton information state having only one sequence-world pair in it, namely the pair consisting of \( w \) and \( f \), then for any states \( I \) and \( J \):

\[ I[\phi] J \rightarrow J = \lambda w \lambda f \left[ \exists K \exists w' \exists f' I.w'.f' \land ((w', f'))[\phi] K \land K.w.f \right] \]

**Eliminativity** If \( I[\phi] J \) then \( J \) contains only a subset of the world-sequence pairs in \( I \). This property, which, like distributivity, is discussed in [GS90], excludes the
possibility of \( \phi \) being a downdate, and also excludes the possibility of \( \phi \) introducing new discourse markers. If \( I \) and \( J \) are states then:

\[
I[\phi] J \text{ iff } \forall w \forall f. J.w.f \rightarrow I.w.f
\]

**Relevance**

Only the discourse markers mentioned in \( \phi \) are relevant to the calculation of an update with \( \phi \). More formally, if \( I - k \) denoted a state differing from \( I \) only by the sequences in \( I \) being shrunk so as not to give a value to \( k \), then we would have for any states \( I, J \) and \( K \):

\[
(I[\phi] J \land (I - k)[\phi][K]) \rightarrow J = \lambda w \forall f. [I.w.f \land \exists g f \geq g \land K.w.g]
\]

**Alpha-invariance**

The names of the discourse markers in \( \phi \) do not matter, except to the extent that they determine sequence valuations. Let us say that \( \phi[i/k] \) denotes the formula obtained by substituting \( k \) for \( i \) in \( \phi \), and that \( I[i/k] \) denotes the information state obtained by swapping the values given to \( i \) and \( k \) in every sequence in \( I \). Then if \( k \) is not mentioned in \( \phi \) and is not in the domain of the state \( I \) we would have that for any states \( I \) and \( J \):

\[
I[\phi] J \leftrightarrow I[i/k][\phi[i/k]][J[i/k]]
\]

These six properties could, of course, have been stated as meaning postulates in the first place, and would have replaced the above postulate MP8. However, I have not attempted any proof that this alternative would yield precisely the same denotation space for the dynamic predicate constants.

To illustrate the working of ABLE predication, I will firstly consider the definition of ABLE predicates \textsc{male}^u, \textsc{female}^u, \textsc{animate}^u, \textsc{neuter}^u, \textsc{singular}^u and \textsc{plural}^u. We begin with a simple theory about the meaning of the predicates, expressed in terms of simple static predicates:

**Meaning Postulate MP9**

\[
\forall w \forall x \text{ male}_{\omega_1}(e_1, e_2).w.x \rightarrow \neg \text{ female}_{\omega_1}(e_1, e_2).w.x
\]

\[
\forall w \forall x \text{ female}_{\omega_1}(e_1, e_2).w.x \rightarrow \neg \text{ male}_{\omega_1}(e_1, e_2).w.x
\]

\[
\forall w \forall x \neg (\text{neuter}_{\omega_1}(e_1, e_2).w.x) \rightarrow (\text{male}_{\omega_1}(e_1, e_2).w.x \lor \text{female}_{\omega_1}(e_1, e_2).w.x)
\]

\[
\forall w \forall x \text{ male}_{\omega_1}(e_1, e_2).w.x \rightarrow \text{ animate}_{\omega_1}(e_1, e_2).w.x
\]

\[
\forall w \forall x \text{ female}_{\omega_1}(e_1, e_2).w.x \rightarrow \text{ animate}_{\omega_1}(e_1, e_2).w.x
\]

\[
\forall X \text{ singular}_{e_1}(e_2).X \rightarrow \exists x (X.x \land \forall y (X.y \rightarrow x = y))
\]

\[
\forall X \text{ plural}_{e_1}(e_2).X \rightarrow \exists x \exists y (X.x \land X.y \land \neg (x = y))
\]

I take it that these postulates do not require much explanation, save for noting that I allow objects to be both neuter and sexed. This might be appropriate in the case of
animals, for instance, since we can refer to an animal as 'it' even though it has a sex.\textsuperscript{10}

In the following postulate, dynamic unary predicates are defined in terms of the static predicates introduced above. Note that the first postulate defines distributive predicates, so that a group of individuals can only be animate if all the members of the group are animate. It would, however, make no sense for the predicates SINGULAR\textsuperscript{u} and PLURAL\textsuperscript{u} to be distributive.

**Meaning Postulate MP10**

If \( P^1 \) is one of the dynamic unary predicate constants MALE\textsuperscript{u}, FEMALE\textsuperscript{u}, NEUTER\textsuperscript{u} and ANIMATE\textsuperscript{u} then:

\[
P^1 = \lambda D \lambda I \lambda J [t\text{-domain}.I.D \land J = \lambda w \lambda f [I.w.f \land \forall x (f.D.x \rightarrow P_{(w,\langle e,\tau \rangle)}.w.x)]]
\]

If \( P^1 \) is either of the dynamic unary predicate constants SINGULAR\textsuperscript{u} and PLURAL\textsuperscript{u} then:

\[
P^1 = \lambda D \lambda I \lambda J [t\text{-domain}.I.D \land J = \lambda w \lambda f [I.w.f \land P_{(e,\tau)}(f.D)]]
\]

Identity, although not introduced as one of the predication clauses syntactically, behaves as a two place predicate, and would obey a version of postulate MP8, above. The following semantic clause makes an ABLE identity statement \( i \) is \( j \) provide an update only if both discourse markers are in the domain of the input state, in which case the output is the set of world-sequence pairs from the input such that the sequence valuation of the two discourse markers is identical. Note that identity is effectively a collective predicate, since the condition for identity of two groups is not that every member of the first group is equal to every member of the second, but that the two groups consist of the same set of individuals.

\textsuperscript{10}Of course, in many languages it would be completely inappropriate to conflate sex and grammatical gender. For such a language it would be more sensible to class the discourse markers themselves as having gender, number and so on, so that the number and sex of the discourse referents could be irrelevant for the purposes of the grammar. There are several ways in which ABLE could be extended so as to allow for this possibility, for instance by using predicates like \( \text{female}_{(d.t)} \) to provide a permanent sortal structure on the domain of discourse markers, or by using predicates like \( \text{female}_{\text{num.}(d.t)} \) to make the sortal categorisation of discourse markers contingent; in this case an information state could be updated with the fact that a given discourse marker fell into a particular grammatical category. And of course, mixed strategies are also possible, whereby gender and number of markers is related to sex and number of referents, but this relation is not one-to-one. I will not pursue any of these possibilities any further here since for the fragment of English with which I will be concerned, there is a systematic relation between grammatical and semantic categorisations.
Definition D48 (Semantics of Identity in ABLE)

\[ [i \equiv j] = \lambda I \lambda J \left[ t\text{-domain}.I.i \land t\text{-domain}.I.j \land J = \lambda w \lambda f \left[ I.w.f \land f.i = f.j \right] \right] \]

So far we have only seen ABLE predicates which provide an update on a context whenever the context has the predicated discourse markers in its domain. However, a predicate may involve more complex presuppositions. Presuppositions may be stated as meaning postulates explicitly stating what formulae must be satisfied by the input state of a predication in order for the predication to provide an update. For instance the following postulate would restrict the predicate \textsc{walk}\textsuperscript{a} so that it could only apply to markers the values of which were already established to be animate entities:

\[ \forall I \forall D \exists J I(\textsc{walk}\textsuperscript{a}.D)J \leftrightarrow I \text{ satisfies } (\textsc{animate}\textsuperscript{a}.D) \]

Here a selectional restriction on a predicate is encoded as a presupposition. I will not argue the point as to whether all such selectional restrictions may appropriately be considered presuppositional, although, from a technical perspective, it is attractive that we have this option. As indicated earlier, I will concentrate on the presuppositions of factive verbs like those found in all the presuppositional examples considered in the previous chapter of this thesis, verbs like 'realise' and 'regret'. These may be understood as two place verbs which place selectional restrictions on both arguments: the subject argument must be animate (a restriction which will be ignored), and the object must be a proposition which is satisfied in the local context. To ensure that the corresponding ABLE predicates \textsc{regret}\textsuperscript{a} and \textsc{realise}\textsuperscript{a} have appropriate presuppositional properties, it is simply stipulated that they have the property of factivity, defined below:\textsuperscript{11}

Definition D49 (Factivity) A dynamic attitude predicate denoted by the constant \textsc{att}\textsuperscript{a} is factive if

\[ \forall I \forall D \forall F I \text{ admits } \textsc{att}\textsuperscript{a}.D.F \leftrightarrow t\text{-domain}.I.D \land I \text{ satisfies } F \]

Meaning Postulate MP11 The dynamic attitude predicate constants \textsc{regret}\textsuperscript{a} and \textsc{realise}\textsuperscript{a} are factive.

In what follows, it will often be useful to ignore presupposition altogether. For this purpose a class of distributive, and almost presupposition-free predicates is now introduced:

\textsuperscript{11}As discussed earlier in footnote 1, I do not divide between factive and semifactive verbs, the evidence which originally motivated Karttunen to make such a distinction being best explained in terms of conversational implicature than in terms of a bifurcation of lexical semantics.
Definition D50 (Simple Predicates)

A dynamic unary predicate denoted by the constant $p^u$ is simple if there is a constant $P$ of type $\langle w, \langle e, t \rangle \rangle$ such that:

$$\forall I \forall J \forall D \quad I\{p^u.D\}J \leftrightarrow$$

$$(t\text{-domain}.I.D \land J = \lambda w \lambda f \quad [I.w.f \land \forall x (f.D.x \rightarrow P.w.x)])$$

A dynamic binary predicate denoted by the constant $P^b$ is simple if there is a constant $P$ of type $\langle w, \langle e, \langle e, t \rangle \rangle \rangle$ such that:

$$\forall I \forall J \forall D \forall D' \quad I\{P^b.D.D'\}J \leftrightarrow$$

$$(t\text{-domain}.I.D \land t\text{-domain}.I.D' \land J = \lambda w \lambda f \quad [I.w.f \land \forall x \forall y ((f.D.x \land f.D'.y) \rightarrow P.w.x.y)])$$

The reason for calling the simple predicates almost presupposition free is that there remains a presupposition that the predicated discourse markers are in the domain of the input. However, if this condition is met in some input state, then it is clear that the definition of a simple predicate defines what the output is, so a predication involving a simple predicate is guaranteed to provide an update. Note that on the above definition, all the predicates $\text{MALE}^u$, $\text{FEMALE}^u$, $\text{ANIMATE}^u$ and $\text{NEUTER}^u$ are simple.

8.4 Connectives

The definitions of the connectives given below are natural generalisations of those presented in the first part of this thesis, and furthermore differ little from the definitions found in [Dek92] and [Bea92], which in turn are close to those of [Hei83a].

Conjunction is defined as relational composition, and negation is defined as a set complement operation, although the anaphoric closure of the negated formula is taken so as to avoid problems which would be caused by the introduction of discourse markers in the negated formula. Implication is defined using the same standard equivalence as in Chapter 7, and disjunction is also defined using a standard equivalence.
Definition D51 (Semantics of Connectives in ABLE)

\[
\begin{align*}
\llbracket \phi \land \psi \rrbracket & = \lambda I \lambda J \left[ \exists K \ 1[\phi]K[\psi]J \right] \\
\llbracket \neg \phi \rrbracket & = \lambda I \lambda J \left[ \exists K \ 1 \downarrow \llbracket \phi \rrbracket K \land J = I \setminus K \right] \\
\llbracket \phi \implies \psi \rrbracket & = \llbracket \neg (\phi \land \neg \psi) \rrbracket \\
\llbracket \phi \lor \psi \rrbracket & = \llbracket \neg (\neg \phi \land \neg \psi) \rrbracket
\end{align*}
\]

8.5 Determiners

The treatment of determiners to be given is related to earlier dynamic accounts such as Heim's [Hei82, Hei83b], Chierchia's [Ch92] or Groenendijk and Stokhof's [GS91a, GS91b]. I hope I will be able to demonstrate that within a dynamic setting an analysis of determiners is possible which begins to parallel that of Barwise and Cooper [BC81] in its uniformity, and yet encompasses a much broader view of meaning than is found in their by now standard account of Generalised quantifiers.

Historically we could attribute to Russell the first serious attempt at a uniform analysis of determiners [Ru05]. But although the tradition of quantificational analysis extending from his work has fared well in the treatment of relatively exotic determiners, it has faced much criticism closer to home, in particular regarding the treatment of the humblest determiners of all, 'the' and 'a'. The two best known philosophical challenges to the uniform quantificational analysis of determiners, namely that of Strawson [St50] with respect to definites, and that of Geach [Gea62] with respect to the interaction of indefinites and definites, have both been met with non-quantificational solutions in the work of Heim and Kamp [Kam81] in the early eighties. It is not until Groenendijk and Stokhof's [GS91a], building on the work of Barwise [Ba87] and Rooth [Roo87], that the possibility of bringing indefinites back into the quantificational fold — given a suitably broad conception of quantification — became readily apparent. Chierchia [Ch92] showed a way to extend the approach developed by Groenendijk and Stokhof in [GS91a] and [GS91b] to a wider range of determiners, and this dissertation is intended to further this line of development, attempting to bring out the similarities between the dynamic analyses not only of indefinites and paradigmatically quantificational determiners, but also of definites.
8.5.1 Indefinites

The semantics of the ABLE determiner SOME will follow the standard dynamic analysis, which holds that the meaning of an indefinite resides in its ability to introduce a new referent into the discourse context. When an indefinite is used in a conversation the speaker may or may not intend to refer to a particular object which he or she has in mind. But regardless of whether the indefinite is being used specifically or non-specifically, it will not generally be the case that the other conversational participants are able to pinpoint a particular object to which the indefinite refers, and thus each participant's information state must leave the reference of the indefinite phrase underspecified. In fact, given that in this chapter an information state is being conceived of only as a model of the common ground, and that other aspects of a participant's knowledge are being ignored, even the speaker's information state must leave the reference of an indefinite noun-phrase underspecified. The process of updating an information state so as to incorporate an underspecified referent begins with the assignment of a completely underspecified value to a previously unused discourse marker, which is defined in terms of the constants \textit{add} and \textit{+}:

\textbf{Meaning Postulate MP12}

\begin{align*}
(1) \quad \text{add} & = \lambda D \lambda f \lambda g \left[ \exists x \ g = \lambda D' \lambda y \left[ D' = D \rightarrow x = y \wedge \left( \neg D' = D \rightarrow f.D'.y \right) \right] \right] \\
(2) \quad + & = \lambda D \lambda I \lambda J \left( \neg \text{p-domain}.I.D \wedge \right. \\
& \left. J = \lambda w \lambda f \left[ \exists g \ I.w.g \wedge g\{\text{add}.D\}f \right] \right]
\end{align*}

A $\text{Ty}_3$-formula $f\{\text{add}.i\}g$ says that sequences $f$ and $g$ agree on the values they assign to all discourse markers apart from $i$, and that $g$ maps $i$ to a set containing one, arbitrary object. The constant $+$ is defined in terms of $\text{add}$, and says something similar at the level of information states instead of sequences. A formula $I\{+.i\}J$, "$J$ is an \textit{arbitrary extension} of $I$ with a value for $i$", means that $i$ is not in the domain of state $I$, and that any pair of a world and a sequence in $J$ differs from some pair in $I$ only by the sequence being extended with some arbitrary valuation for $i$.

In ABLE, determiner clauses are of the form $\textit{det.dm.form.form}$. I will call the discourse marker at the head of such a clause the \textit{determined} marker, and, following standard conventions, I will call the first subformula the \textit{restrictor} and the second the \textit{scope}. The interpretation of a formula $\text{SOME}.i.\phi.\psi$ can be thought of procedurally as an instruction to perform the following sequence of modifications to an input state: arbitrarily extend the input with a value for the determined marker, then add firstly the information in the restrictor, and secondly the information in the scope. This is captured by the following definition:
Definition D52 (Semantics of SOME: first version)

\[
{\text{SOME}.i.(\phi.\psi)} = \lambda I. \lambda J. [\exists I_{in} \exists I_{res} \{ (+:i) I_{in}, \phi \} I_{res}, \psi] J
\]

In E182 a simple example of an indefinite sentence and its ABLE translation is given. The translation defines (the relational equivalent of) a partial function from input states which give no valuation for the determined marker 1, to output states containing only world-sequence pairs which are defined on 1 and map it onto an individual from the domain which is a walking woman in the given world. The output pairs are thus extensions of elements of that subset of the input pairs involving a world where there is a walking woman.

E182 a. A woman is walking.

b. SOME.1.(WOMAN^u.1).(WALK^u.1)

8.5.2 Definites

In [Bea92] I analysed definite descriptions in a similar way to [Hei83a]. It was assumed that a definite description presupposed the existence of an object which satisfied the description. However, the analysis I will present for the ABLE determiner THE will be closer to the treatment given in Heim's thesis [Hei82]. The difference is as follows: in her thesis Heim argues not that a definite presupposes the existence of an object satisfying the description, but that it presupposes there to be a salient discourse marker whose value satisfies the description. Thus a formula THE.i.(WOMAN^u.i).(WALK^u.i) will provide an update only from those contexts in which the determined marker is established to refer to a woman.

The analysis has the advantage, as will be seen later, that it can yield a uniform treatment of various sorts of definite noun phrase — the semantics of proper names and pronouns will all be defined in terms of the semantics of THE. On the other hand, there are many uses of definite descriptions for which the analysis given here will be inadequate, for instance when the referent has been introduced but the fact that it satisfies the given description has to be be inferred, or when the referent has not been explicitly introduced but only implicitly made salient by its relevance to other topics of conversation. I believe, in agreement with views expressed by Heim, that such uses of definites should be dealt with by supplementing the theory with an account of accommodation — the process whereby a context which does not satisfy the presuppositions of a sentence is adjusted so as to allow update with that sentence. In Chapter 10 I will show how such a mechanism may be formalised.

The following definition restricts input states from which a formula [THE.i.(\phi.\psi)] provides an update to those which both have the determined marker in their
domain and support the proposition in the restrictor. If these conditions are met then the output state is simply the input updated sequentially with the restrictor and the scope.

**Definition** D53 (Semantics of THE)

\[
\langle \text{THE}.i.\phi.\psi \rangle = \lambda I\lambda J \left[ t\text{-domain}.I.i \land I \text{satisfies} [\phi] \land \exists K I[\phi]K[\psi]J \right]
\]

Examples E183–E185, which provide an inane continuation to E182, illustrate the effects of the above definition:

**E183**

a. The woman is talking.

b. THE.1.(WOMAN\textsuperscript{u}.1).(WALKING\textsuperscript{u}.1)

**E184**

a. Butch is barking.

b. THE.2.(NAMED-BUTCH\textsuperscript{u}.2).(BARK\textsuperscript{u}.2)

**E185**

a. She regrets that he is barking.

b. THE.1.(FEMALE\textsuperscript{u}.1 AND SINGULAR\textsuperscript{u}.1).

\[(\text{REGRET}\textsuperscript{u}.1.(\text{THE}.2.(\text{MALE}\textsuperscript{u}.2 AND SINGULAR\textsuperscript{u}.2).(\text{BARK}\textsuperscript{u}.2)))\]

The scene was set, the reader will recall, by the introduction of a walking woman, for whom a corresponding ABLE discourse marker 1 was introduced. Such a context can be updated with E183b, which provides an update from any input state in which 1 is determined to be a woman, to that subset of input world-sequence pairs in which the referent of 1 is talking. Occurrences of discourse markers outside of the syntactic scope of the quantificational determiner that introduced them, as in E183b, will sometimes be referred to as being *unquantified*. E184b does not provide an update from all possible outputs of the conjunction of E182b and E183b, but only from those in which the unquantified marker 2 is totally defined, and only takes *butch* values. Supposing this condition — which will be turned to in a moment — is met, E184 adds the information that 2 is barking, and thus provides an appropriate input to E185b, which is only defined on contexts in which 1 is female, 2 is a barking male. Of course, this input will only be fully appropriate on the assumption that being named Butch entails masculinity.

**Meaning Postulate** MP13

1. NAMED-BUTCH\textsuperscript{u} is simple

2. \( \forall w \forall x \text{named-butch}_{(w,(\varepsilon,r))}.w.x \rightarrow \text{male}_{(w,(\varepsilon,r))}.w.x \)
Failing the existence of such a postulate, accommodation would be needed, enabling the addition of the information that 2 was male as and when this was presupposed. Accommodation should also provide the answer to the problem of satisfying the presupposition of E184, that the discourse marker 2 corresponded to Butch. However, as will be seen in Chapter 9, even without describing a general mechanism of accommodation, presupposing formulae like E184b can still play an interesting role in the logic of ABLE.

To end this section on definites, here are some obvious abbreviations for ABLE formulae:

\[
\begin{align*}
\text{SHE}.i.\phi &= \text{THE}.i.\text{(SINGULAR}^{u}.i \text{ AND FEMALE}^{u}.i).\phi \\
\text{HE}.i.\phi &= \text{THE}.i.\text{(SINGULAR}^{u}.i \text{ AND MALE}^{u}.i).\phi \\
\text{IT}.i.\phi &= \text{THE}.i.\text{(SINGULAR}^{u}.i \text{ AND NEUTER}^{u}.i).\phi \\
\text{THEY}.i.\phi &= \text{THE}.i.\text{(PLURAL}^{u}.i).\phi
\end{align*}
\]

8.5.3 Dynamic Generalised Quantifiers

According to the standard Generalised quantifier analysis of [BC81], the quantificational determiners of natural language correspond to binary quantifier relations, a binary quantifier relation being able to combine with some set to produce a Generalised quantifier, which is a property of sets. With which sets should the quantifier relations combine? The restrictor and scope of the quantifier appear to be properties of individuals, so that the sets are simply the sets of individuals in the extensions of the properties. But here the classic problem of donkey anaphora arises, as discussed by Geach in [Gea62]. Consider the following examples:

E186 a. Every farmer owns a donkey.
   b. \( \text{EVE} \text{RY}.i. \text{(FARMER}^{u}.i). \text{(SOME}.j.\text{(DONKEY}^{u}.j).\text{(OWN}^{b}.i.j)) \)

E187 a. Every farmer who owns a donkey beats it.
   b. \( \text{EVE} \text{RY}.i. \text{(FARMER}^{u}.i \text{ AND SOME}.j.\text{(DONKEY}^{u}.j).\text{(OWN}^{b}.i.j)) \text{(IT}.j.\text{(BEAT}^{b}.i.j)) \)

The Generalised quantifier analysis for E186 will be familiar: the quantifier relation corresponding to every must hold between the set of farmers and the set of donkey owners. Equally familiar will be the difficulties that the analysis faces with E187. The denotation of the restrictor is obvious — it is just the property of being a farmer who owns a donkey. But the scope does not seem to denote an absolute property: the
property of being a *beater of it* must be relativised to some interpretation for the pronoun *it*.

This problem concerns the *internal dynamism* (cf. [GS91a]) of quantificational constructions, by which is meant the fact that the dynamic effects of updating with the restrictor of a quantifier help determine the context of evaluation for the scope. To begin with I will present an analysis which makes quantifiers internally dynamic, but makes them *externally static*. The final ABLE semantics, however, will make quantifiers both internally and externally dynamic, thus both allowing for internal anaphoric links within the quantificational construction, and allowing whole quantificational constructions to introduce discourse markers so that they may support later anaphoric reference.

To introduce internally dynamic quantifiers, it will be helpful to consider a restricted set of worlds: in this way the relevant information states become more easily visualisable. In figure 8.1 the characteristics of eight individuals (*a* - *h*) in four different worlds (*W1* - *W4*) are pictured. It should be clear that in world *W2*, for instance, one farmer who owns three donkeys beats only one of them, whilst the two other farmers own one donkey each, and in each case beat it. Note that I am also assuming the predicates *FARMER*\(^u\), *DONKEY*\(^u\), *OWNS*\(^b\) and *BEATS*\(^b\) to be *simple* (i.e. both presupposition free and distributive) in the sense of §8.3 above, thus allowing the denotation of a predication involving one of these to be calculated in terms of underlying lower typed predicates like *farmer*\(_{(w,(e,t))}\): it is the denotation of these lower typed predicates which is pictured in figure 8.1.

Suppose that we wish to update the information state which has only these four worlds in its *\(\omega\)-set*, and which has no discourse markers in its domain, with the formula in E187b, the ABLE representation of the classic donkey sentence. We begin the analysis of the universal in the same way as for an indefinite, calculating the effect of introducing the determined marker *i* into the input context, and then updating with the restrictor followed by the scope. Let us call the information state in which the discourse marker *i* has been initialised *init*, the state reached after updating with the restrictor *res*, and the state reached after updating with the scope-*sco*.

Given that, for the moment, it is being assumed that the complete quantificational construction introduces no discourse markers, the output state must contain a subset of the world-sequence pairs in the input, namely all those world-sequence pairs which are compatible with the information in the quantificational formula. To discern whether a given world-sequence pair from the input should be in the output, it is necessary to look at the extensions of that pair in the contexts *res* and *sco*, and check what values the determined discourse marker takes in these extensions.

Let us represent sequences in such a way that, for example, a sequence in which the discourse marker *i* is mapped onto the single individual *a*, and the discourse marker *j*
The four worlds pictured in fig.8.1 do not vary with respect to the denotation of the underlying predicates \textit{farmer}, \textit{donkey} and \textit{owns}: in each world there are five ways in which \(i\) and \(j\) can be mapped respectively onto a farmer and a donkey which that farmer owns. This means that update of the input state with the restrictor introduces only five different sequences, so that \(res\) consists of each of the four worlds paired with

\[
\begin{align*}
W1 & (\emptyset) \\
W2 & (\emptyset) \\
W3 & (\emptyset) \\
W4 & (\emptyset)
\end{align*}
\]
each of the five sequences:

\[
\begin{align*}
& W_1 \{i \mapsto \{a\}, j \mapsto \{d\}\} \\
& W_1 \{i \mapsto \{a\}, j \mapsto \{e\}\} \\
& W_1 \{i \mapsto \{a\}, j \mapsto \{f\}\} \\
& W_1 \{i \mapsto \{b\}, j \mapsto \{g\}\} \\
& W_1 \{i \mapsto \{c\}, j \mapsto \{h\}\} \\
& W_2 \{i \mapsto \{a\}, j \mapsto \{d\}\} \\
& W_2 \{i \mapsto \{a\}, j \mapsto \{e\}\} \\
& W_2 \{i \mapsto \{a\}, j \mapsto \{f\}\} \\
& \vdots \quad \vdots
\end{align*}
\]

For a given world-sequence pair \(wf\) from the input and a given \(res\), let us call the set of sequences which extend \(f\), and are paired with \(w\) in \(res\), the restrictor sequence set of \(wf\). Formally the restrictor sequence set will be given by the expression: \(\lambda h [h \supseteq f \land res.w.h]\). A similar expression can be used to determine the scope sequence set of a given world-sequence pair from the input.

The state \(sco\), which includes not only information about farmer-donkey ownership, but also about cruelty to animals, associates different sets of sequences with each world:

\[
\begin{align*}
& W_1 \{i \mapsto \{a\}, j \mapsto \{d\}\} \\
& W_1 \{i \mapsto \{a\}, j \mapsto \{e\}\} \\
& W_1 \{i \mapsto \{a\}, j \mapsto \{f\}\} \\
& W_1 \{i \mapsto \{b\}, j \mapsto \{g\}\} \\
& W_1 \{i \mapsto \{c\}, j \mapsto \{h\}\} \\
& W_2 \{i \mapsto \{a\}, j \mapsto \{d\}\} \\
& W_2 \{i \mapsto \{b\}, j \mapsto \{g\}\} \\
& W_2 \{i \mapsto \{c\}, j \mapsto \{h\}\} \\
& W_3 \{i \mapsto \{a\}, j \mapsto \{d\}\} \\
& W_3 \{i \mapsto \{a\}, j \mapsto \{e\}\} \\
& W_3 \{i \mapsto \{a\}, j \mapsto \{f\}\}
\end{align*}
\]

Here \(W_1\) is the only world which is paired with the same sequences as in \(res\), all three farmers being hideously cruel in this world. By contrast, each of \(W_2\) and \(W_3\) remain paired with only three of the original five sequences, whilst the utopian \(W_4\) is no longer even in the \(\omega\)-set of \(sco\): there are no beatings in \(W_4\).

One way to define internally dynamic quantifiers would be to preserve in the output those world-sequence pairs from the input for which the restrictor sequence set and scope sequence set are satisfied by the given quantifier relation. Thus, in the current example concerning the quantifier \(\text{every}\), only the pair \(W_1()\) would be in the output, since this is the only pair for which every extension in \(res\) is also in \(sco\). However, as is
shown in [Roo87], this sequence counting approach will not work in general. Consider
the following simple variation on E187:

E188 a. Most farmers who own a donkey beat it.
   b. MOST.i.
      FARMER\textsuperscript{u}.i AND (SOME.j.(DONKEY\textsuperscript{u}.j).(
      OWNS\textsuperscript{b}.i.j)).
      (IT.j.(BEATS\textsuperscript{b}.i.j))

Intuitively this sentence should be true only in worlds W1 and W2, since in both of
the other worlds less than half of the farmers are malign. However, counting sequences
would leave not only W1 and W2 in the output, but also W3. In this world, one farmer
happens to own and beat three donkeys whilst the other two farmers do not beat their
single donkeys, which means that three out of the five sequences from res are preserved
in sco, and, 3/5 being more than half, this in turn means that the quantifier relation most
is satisfied. The conclusion to be drawn is that the quantification involves counting
farmers and not sequences.

It is easy to calculate the relevant set of individuals from a given restrictor sequence
set or scope sequence set. Suppose that for some pair \(wf\) the restrictor sequence set is
\(G_{\text{res}}\), and the determined marker is \(i\). Then the set of values of \(i\) in res extensions of \(wf\)
will be: \(\lambda x [\exists h G_{\text{res}}.h \land h.i.x]\), and similarly for the scopal values.

In the example we have been looking at, each of the four world-sequence pairs in
the input will yield the same set of values for the determined marker in the restrictor,
namely the set: \{a, b, c\}. The scopal values will of course differ. The input pairs \(W1()\)
and \(W2()\) yield values \{a, b, c\}, whilst \(W3()\) yields the set \{a\}, and \(W4()\) yields the
empty set. Clearly using these sets will be an improvement in the case of E188, since
it will only hold for the pairs \(W1()\) and \(W2()\) that over half the members from the
restrictor extensions are still in the scope extensions.

There are other consequences of the decision to count individuals rather than se-
quences, to which we will come shortly. Firstly let us see what the upshot of the above
discussion is in terms of a general semantics for internally dynamic quantificational de-
terminers. In the definition below, \(init\), \(res\) and \(sco\) are assumed to be variables of type
\(\iota\), and have the same interpretation as in the above discussion. The output state \(J\)
is obtained by calculating for each \(wf\) in the input state the corresponding restrictor and
scope sequence sets \(G_{\text{res}}\) and \(G_{\text{sco}}\), which are each of type \(\langle \sigma, t \rangle\), then calculating from
these the sets of values taken by the determined marker in the restrictor and the scope
(these sets being given by \(X_{\text{res}}\) and \(X_{\text{sco}}\), each of type \(\epsilon = \langle e, t \rangle\)), and finally checking
whether the relevant underlying quantifier relation holds between two sets. The defini-
tion we arrive at is essentially the same as that of [Ch92], although tailored to ABLE's
needs:
Definition D54 (Non-existential Determiners - First Version)

If $\mathcal{D}$ is one of EVERY, MOST, FEW or NO, and $\mathcal{D}'$ is a corresponding quantifier relation of type $\langle (e,t), (e,t), t \rangle$, then:

$$[\mathcal{D}.i.\phi.\psi] = \lambda I \lambda J$$

$$[\exists res \exists sco]$$

$$I[+i]init[\phi]res[\psi]sco \land$$

$$J = \lambda w \lambda f$$

$$[\exists G_{res} \exists G_{sco} \exists X_{res} \exists X_{sco}]$$

$$I.w.f \land$$

$$G_{res} = \lambda h [h \geq f \land res.w.h] \land$$

$$G_{sco} = \lambda h [h \geq f \land sco.w.h] \land$$

$$X_{res} = \lambda x [\exists h G_{res}.h \land h.i.x] \land$$

$$X_{sco} = \lambda x [\exists h G_{sco}.h \land h.i.x] \land$$

$$[\mathcal{D}'.X_{res}.X_{sco}]$$

The following postulate ensures that the underlying quantifier relations have appropriate denotations. The definitions for EVERY' and NO' should be clear. I have made the simplifying assumption that MOST' means *more than half*, and FEW' means *less than half*. The definition for MOST' says that two sets of individuals $X$ and $Y$ stand in the relation MOST' if and only if there is no function with domain $Y \setminus X$ (i.e. the set containing only members of $Y$ that are not in $X$) and range $X \cap Y$: this implies that the cardinality of $X \cap Y$ is greater than the cardinality of $X \setminus Y$, and thus that more than half of the elements of $X$ are in $Y$. The definition for FEW' runs along similar lines.

Meaning Postulate MP14

$$\text{EVERY}' = \lambda X \lambda Y [\forall x (X.x \rightarrow Y.x)]$$

$$\text{NO}' = \lambda X \lambda Y [\neg \exists x (X.x \land Y.x)]$$

$$\text{MOST}' = \lambda X \lambda Y [\neg \exists F_{e,e}$$

$$\forall x ((X.x \land Y.x) \rightarrow \exists y (X.y \land \neg Y.y \land F.y = x))]$$

$$\text{FEW}' = \lambda X \lambda Y [\neg \exists F_{e,e}$$

$$\forall x ((X.x \land \neg Y.x) \rightarrow \exists y (X.y \land Y.y \land F.y = x))]$$

Before going on to consider externally dynamic quantifiers, we will briefly consider how the account given so far bears on the problem of weak and strong readings for donkey sentences. In the case of E187, the strong reading — that obtained on Lewis's
case-quantification analysis, in Kamp’s DRT and in Heim’s FCS — is the reading where every donkey owning farmer beats every donkey he owns. This is strong in the sense that it entails the so-called weak reading, where every donkey owning farmer beats at least one of the donkeys he owns. There is, to the best of my knowledge, as yet no conclusive evidence as to which reading is appropriate for E187, or whether, indeed, both readings are present. However, intuitions are clearer with regard to donkey sentences involving right downward monotone quantifiers such as ‘no’:

E189 a. No farmer who owns a donkey beats it.
   b. NO.i.
      (FARMER^u.i AND SOME.j.(DONKEY^u.j).(OWNS^b.i.j)).
      (IT.j.(BEAT^b.i.j))

Regarding this example it is difficult to justify the existence of a weak reading — the reading where no farmer beats all of his donkeys although some farmers may beat some of their donkeys. It seems that the existence of a single example of a farmer owning a donkey but not beating it would be enough to falsify the statement in E189.

In general the definition in D54 yields existential readings. Thus E187 comes to have the weak reading where every donkey-owning farmer beats at least one donkey, and E189 comes to have the strong reading (which as I have indicated seems to be the only plausible one) where no farmer beats any of his or her donkeys. However, both [Ch92] and [Kan93] indicate that it is best to allow for the possibility of right upward monotone quantifiers having both strong and weak readings. To this end I note that we at least have the option of making the upward monotone quantifiers ambiguous, simply by introducing two new ABLE quantifiers EVERY* and MOST*. I leave it to the reader to verify that the following definitions would introduce strong right upward monotone determiners:

Definition D55 (Strong upward monotone determiners)

\[
\begin{align*}
\mathrm{EVERY^*} . i. \phi. \psi &= [\mathrm{NO}.i.\phi. (\mathrm{NOT} \ \psi)] \\
\mathrm{MOST^*} . i. \phi. \psi &= [\mathrm{FEW}.i.\phi. (\mathrm{NOT} \ \psi)]
\end{align*}
\]

One final point about the limits of the current strategy for defining internally dynamic quantifiers: as discussed by Chierchia, the strategy is only appropriate for conservative quantifiers. This property holds of a quantifier, roughly speaking, if in calculating the truth of a quantificational statement, it is only necessary to consider how many of the individuals satisfying the restrictor also satisfy the scope, and it is irrelevant whether individuals not satisfying the restrictor satisfy the scope. Thus in evaluating whether it is true that ‘every girl is hiding’, it is irrelevant whether boys and other non-girls are
hiding. It seems possible to maintain that this property holds for all English determiners: the only putative exception with which I am familiar is the word 'only'. However, it is quite plausible that 'only' is not a determiner at all, since, from the point of view of a naive syntactician, it has a quite different distribution than would be expected. For instance, 'only' can modify not only nouns, but also noun-phrases, as in 'only me', as well as commonly occurring outside of noun-phrases. On the other hand, even if 'only' is not a determiner, this does not excuse us from the task of giving it a semantics. But my feeling is that this job belongs within a more general theory of the dynamics of focus-sensitive constructions (see, for instance, [Krif92]) and not here.

We now move on to the external dynamics of quantifiers. Consider the following examples:

E190 a. Most farmers own a donkey.
   b. MOST.i. (FARMER^u.i). (SOME.j.(DONKEY^u.j).(OWN^b.i.j))

E191 a. They ride to the pub.
   b. THEY.i.(RIDE-TO-PUB^u.i)

E192 a. Most of the donkeys are alcoholic.
   b. MOST.k. (THE.j.(DONKEY^u.j AND PLURAL^u.j).(OF^b.i.j)) (ALCOHOLIC^u.k)

The fact that E190a–E192a constitute a coherent discourse shows that it is a simplification to assume that quantificational determiners have no anaphoric potential. It seems that E190a introduces at least two new topics of conversation: the set of donkey owners, and the set of donkeys that people own. In fact, there may well be other new topics, such as the set of all farmers and the set of all donkeys, but I will ignore these, although it seems plausible that the approach that will be described could be extended appropriately.

To make matters more tangible, the desired dynamic effects of E190b can be relativised to the example input state used earlier, consisting of the four worlds from fig.8.1 paired with empty sequences. Given this input, the following update would leave (in each world) i mapped to the set of all farmers, and j mapped to the set of donkeys owned by farmers:

\[
\begin{align*}
\{W1\} & \quad \{W2\} \\
\{W3\} & \quad \{W4\}
\end{align*}
\]

\[\text{[E190b]}\]

\[
\begin{align*}
W1 & \langle \rangle \\
W2 & \langle \{a, b, c\}, j \mapsto \{d, e, f, g, h\}\rangle \\
W3 & \langle \{a, b, c\}, j \mapsto \{d, e, f, g, h\}\rangle \\
W4 & \langle \{a, b, c\}, j \mapsto \{d, e, f, g, h\}\rangle
\end{align*}
\]
In the earlier definition of internally dynamic quantifiers, D54, extensions of a given input pair compatible with both the restrictor and scope were collected in the scope sequence set. Let us say that some input pair $wf$ has a scope sequence set $G_{sc}$, and that $wf$ would have been in the output state according to the earlier definition of internally dynamic quantifiers, D54. Then the output of an externally dynamic quantifier might be expected to contain the pair consisting of $w$ together with the sequence $g$ defined as follows:

$$g = \lambda D\lambda x [\exists h G_{sc}.h \land h.D.x]$$

This formula defines $g$ as a function which maps each discourse marker onto the set of all values which it is assigned by the member sequences of the scope sequence set: we can call this the scope sequence union. However, although this approach would lead to an acceptable definition for E190b above, it would not be appropriate for right downward monotone quantifiers. For instance, the input world-sequence pairs which satisfy a quantification with the determiner ‘few’ may sometimes be associated with empty scope sequence sets since, on standard assumptions, there being few farmers who own a donkey is compatible with there being none at all. Thus calculating output sequences from scope sequence sets alone will sometimes yield only empty output sequences, and so fail to preserve the anaphoric information present in the input state.

One strategy for avoiding this problem would be to use different definitions for the semantics of upward and downward right monotone quantifiers. However, a single definition for both upward and downward monotone quantifiers is possible, provided the output sequence corresponding to a given input pair is calculated in terms of a combination of the input sequence and the scope sequence set. For a given input pair $wf$, having a scope set $G_{sc}$, the relevant output sequence associated with $w$ will be calculated using the following formula:

$$g = \lambda D\lambda x \left[ f.D.x \lor \exists h G_{sc}.h \land h.D.x \right]$$

This makes the output sequence map each discourse marker onto the set of values which it is assigned either by the initial sequence $f$, or by member sequences of the scope sequence set. The problem is thus avoided, since if the scope sequence set is empty, the output sequence will be the same as the input, and if the scope sequence set is nonempty, then the output sequence will just be the scope sequence union, as before. We thus finally arrive at a semantics for non-existential determiners which incorporates both internal and external dynamism:
Definition D56 (Non-existential Determiners - Second Version)

If $D$ is one of EVERY, MOST, FEW or NO, and $D'$ is a corresponding quantifier relation of type $\langle (e,t), (e,t), t \rangle$, then:

$$[D.i.\phi.\psi] = \lambda I \lambda J$$

$$I \{+i\} init[\phi] res[\psi] sco \land$$

$$J = \lambda w \lambda g$$

$$\exists f \exists G_{res} \exists G_{sco} \exists X_{res} \exists X_{sco}$$

$$I.w.f \land$$

$$G_{res} = \lambda h [h \geq f \land res.w.h] \land$$

$$G_{sco} = \lambda h [h \geq f \land sco.w.h] \land$$

$$X_{res} = \lambda x [\exists h G_{res}.h \land h.i.x] \land$$

$$X_{sco} = \lambda x [\exists h G_{sco}.h \land h.i.x] \land$$

$$D'.X_{res}.X_{sco} \land$$

$$g = \lambda D \lambda x \left[ f.D.x \lor \exists h G_{sco}.h \land h.D.x \right]$$

In the recent semantic literature, indefinites have typically been distinguished from other quantificational determiners by virtue of their anaphoric potential. Having given, in D56, a definition that forms externally dynamic ABLE determiners from arbitrary quantifier relations, an obvious question arises: is the same definition appropriate for indefinites? Consideration of the following two classic examples shows that although extending D56 to include indefinites opens up a promising vista, it is not tenable:

E193 a. ?There is a doctor in London. He is Welsh.

b. SOME.i.(DOCTOR$^u.i$).(IN-LONDON$^u.i$)
AND HE.i.(WELSH$^u.i$)

E194 a. If a farmer owns a donkey then he beats it.

b. SOME.i.(FARMER$^u.i$).(SOME.j.(DONKEY$^u.j$).(OWNS$^b.i.j$))
IMPLIES HE.i.(IT.j.BEATS$^b.i.j$)

It has been argued, in [Ev77], that E193a is odd or incoherent, and that this oddity arises from the fact that the first sentence does not uniquely pick out a single individual to which the singular pronoun in the second sentence can refer. The judgement that the discourse is incoherent is not clear cut: there is at least one reading of E193a on which it is coherent, namely the so-called specific reading, as would be more obvious if
the noun phrase 'a doctor' were replaced with 'a certain doctor'. However, let us assume for the moment that E193a is indeed incoherent, and consequently that explaining this incoherence would be a desirable goal for a semantic theory. Extending D56 to include the determiner SOME, and translating E193a in the obvious way as E193b, would immediately satisfy this desideratum. The translation of the first sentence would no longer introduce a discourse marker which ranged over alternative individual doctors in each world, but instead would introduce a marker assigned to the set of all London doctors in each world. Since an ABLE formula HE.i.ϕ carries the presupposition that i is established to be both male and singular, presupposition failure would follow unless the input state guaranteed that there was only one London doctor, and that this individual was male.

Regarding E193a, then, the extension of D56 to indefinites would have acceptable, or even desirable results. However, the same strategy would have significant effects for the treatment of quantificational donkey sentences, and disastrous effects for the treatment of conditional donkey sentences like E194a. In the case of quantificational donkey sentences like the prototypical E187a, the important consequence would be the introduction of a presupposition that every farmer had at most one donkey. But the translation of E194a would carry a far stronger uniqueness presupposition, namely that there was only one farmer, and that this farmer had only one donkey. In effect Lewis's insight that the donkey conditional can be seen in terms of case-quantification would have been lost, since under such an analysis there could only be one case. Furthermore, it does not seem that an alternative semantics for the pronouns in E194a could save the analysis of indefinites. The translation of 'a farmer owns a donkey' would introduce markers corresponding to the complete set of farmers and the complete set of donkeys owned by farmers, and it would not preserve information about which farmers owned which donkeys. Clearly extensive further modifications would be needed in order to provide a compositional semantics for the conditional which entailed only that farmers beat their own donkeys.

Thus it seems that it would be wiser to stick with the original semantics for indefinites given in D52 than to use the same semantics as for the non-existential determiners. However, it remains somewhat disconcerting that D52 bears so little resemblance to D56, and I think it reasonable to wonder whether the ABLE determiner SOME, as defined in D52, in any sense qualifies as a quantifier. I will now show that D52 can be reformulated in more obviously quantificational terms, and this reformulation will have the advantage that it is also appropriate to other related quantifiers like EXACTLY-ONE.12

12 In natural languages, quantifiers are sometimes not lexicalised, but still expressible. 'Exactly one' may be such a quantifier. In common with much recent literature on generalised quantifiers (see e.g. Keenan's [Kee92]), it is assumed here that we require logical analysis of both lexicalised and non-lexicalised quantifiers, and that an analysis which is mathematically uniform across both classes is
In terms of internal dynamics, the following definition for the existential quantifiers SOME and EXACTLY-ONE is identical to D57, which defined the semantics of EVERY, MOST, FEW and NO. However, the external dynamics is quite different. Instead of collecting together members of the scope sequence set to produce group referents, as in D56, the following definition puts all members of relevant scope sequence sets directly into output world-sequence pairs:

**Definition D57 (Existential Determiners: second version)**

*If D is SOME or EXACTLY-ONE, and D' is a corresponding quantifier relation of type \( \langle (e, t), (e, t), t \rangle \), then:*

\[
[D.i.\phi.\psi] = \lambda l.AJ
\]

\[
\exists \text{init} \exists \text{res} \exists \text{SCO}
\]

\[
I(+.i) \text{init}[\phi] \text{res}[\psi] \text{SCO} \land
\]

\[
J = \lambda w.\lambda g
\]

\[
\exists f \exists \text{res} \exists \text{SCO} \exists X_{\text{res}} \exists X_{\text{SCO}}
\]

\[
I.w.f \land
\]

\[
G_{\text{res}} = \lambda h [h \geq f \land \text{res.w}.h] \land
\]

\[
G_{\text{SCO}} = \lambda h [h \geq f \land \text{SCO.w}.h] \land
\]

\[
X_{\text{res}} = \lambda x \exists h G_{\text{res}.h \land h.i.x} \land
\]

\[
X_{\text{SCO}} = \lambda x \exists h G_{\text{SCO}.h \land h.i.x} \land
\]

\[
D'.X_{\text{res}}.X_{\text{SCO}} \land
\]

\[
G_{\text{SCO}}.g
\]

I leave it to the reader to verify that the new definition is equivalent to D52 with regard to the determiner SOME. We can now say in precisely what sense the definition for SOME is quantificational: the output of a formula SOME.i.\phi.\psi is determined in terms of an underlying quantifier relation. In particular, the set of input pairs of which extensions survive in the output of such a formula can be determined by checking whether the relation SOME holds between the set of values taken by i after update with the restrictor and the set of values taken by i after update with the scope. It remains only to specify the quantifier relations relevant to the determiners SOME and EXACTLY-ONE:

**Meaning Postulate MP15**

\[
\text{SOME}' = \lambda X \lambda Y [\exists x (X.x \land Y.x)]
\]

\[
\text{EXACTLY-ONE}' = \lambda X \lambda Y [\exists x (X.x \land Y.x \land \forall y ((X.y \land Y.y) \rightarrow x = y))]
\]

desirable.
8.6 PUL and ABLE

It will now be demonstrated that ABLE preserves the intuitions of the propositional system, PUL, introduced in the previous chapter. Although it would be possible to define a direct embedding of PUL in ABLE, this would be messy, and instead I will introduce a slight variant of ABLE, ABLE+$. This variant is just ABLE but with the addition of atomic propositions and a $\partial$ operator:

**Definition D58 (ABLE+$\partial$)**

- ABLE+$\partial$ has the same syntax as ABLE, but with two additional classes of formulae, firstly a set of atomic formulae, $\mathcal{P}$, and secondly formulae of the form $\partial \text{form}$.
- For every $p \in \mathcal{P}$, there is a corresponding $T_{3}$ constant $p$ of type $\langle w, t \rangle$.
- $[p] = \lambda I \lambda J [J = \lambda w \lambda f [I.w.f \land p.w]]$
- $[\partial \phi] = \lambda I \lambda J [I = J \land I \text{ satisfies } \phi]$
- Remaining semantic clauses and definition of entailment, which will be written $\models_{\text{ABLE}+\partial}$, are as for ABLE.

It can now be shown that regarding the intersection of the PUL and ABLE+$\partial$ languages, PUL and ABLE+$\partial$ entailment are identical. The proof will be sketchy in places, but then the result only concerns the internal consistency of this thesis, and is presumably not of general interest. Note that in this section $\sigma$ will not be used as an abbreviation for the type of ABLE extended sequences, but as a variable over PUL information states.

**Lemma L1**

1. If $p \in \mathcal{P}$, then $\forall I \ I \text{ admits }[p]$. That is to say, the atomic formulae have no presuppositions.

2. If $p \in \mathcal{P}$, then $\downarrow [p] = [p]$. That is, atomic formulae do not introduce new discourse markers.
3. If \( \phi \) is a formula of PUL and of ABLE+\( \partial \), then \( I \models [\phi]_P = [\phi]_A \). That is, the PUL fragment of ABLE+\( \partial \) does not introduce discourse markers.

4. If \( \phi \) is a formula of PUL and of ABLE+\( \partial \), then for states \( I, I', J, J' \) such that \( \omega \text{-set}.I = \omega \text{-set}.I' \) and \( \omega \text{-set}.J = \omega \text{-set}.J' \), it holds that:

\[
I \models [\phi]_J \iff I' \models [\phi]_{J'}
\]

Proofs of (1) and (2) follow directly from the ABLE+\( \partial \) semantics for atomic propositions above and the definitions of admits and \( I \). Proof of (3) follows by inspection of the semantic clauses in the PUL fragment of ABLE+\( \partial \), and verification that, in each case, if the subformulae introduces no discourse markers, then neither does the compound formula. (4) follows from (3) and another induction over formulae of ABLE+\( \partial \).

**Definition** D59 (Information Injection) It is simple to define a mapping from PUL models to ABLE models: we simply identify the set of worlds, \( W \), in a PUL model with the domain \( D_w \) in the ABLE (Ty 3 ) model, and the PUL interpretation function with the ABLE (Ty 3 ) interpretation function. This allows the definition of an injective mapping, \( + \), from PUL information states to ABLE information states. In the following, the variable \( \omega \) of type \((w, t)\) stands for a PUL information state:

\[
\omega^+ = \lambda w \lambda f \, [\omega.w \land f = \circ]
\]

**Lemma** L2 Semantic Correspondence For any formula \( \phi \) in the PUL fragment of ABLE+\( \partial \), and any two PUL information states \( \omega_1, \omega_2 \),

\[
\omega_1 [\phi]_{PUL} \omega_2 \iff \omega_1^+ [\phi]_{ABLE+\partial} \omega_2^+
\]

Proof: An induction over formula complexity. I will consider only a few cases:

**Conjunction** It must be shown that if the result holds for \( \phi \) and \( \psi \), then:

\[
\omega_1 [\phi \land \psi]_{PUL} \omega_2 \iff \omega_1^+ [\phi \land \psi]_{ABLE+\partial} \omega_2^+
\]

This follows trivially from the fact that in both systems conjunction is defined as relational composition.

**Negation** The semantic clauses for PUL and ABLE+\( \partial \) run as follows:

\[
\sigma [\not \phi]_{PUL} \tau \iff \exists v \, \sigma [\phi]_{PUL} v \land \tau = \sigma \setminus v
\]

\[
[\not \phi]_{ABLE+\partial} = \lambda I \lambda J \, [\exists K \, I \upharpoonright [\phi]_{ABLE+\partial} K \land
\]

\[
J = I \setminus K]
\]
By lemma L1, we are only interested in $\phi$ for which $\downarrow [\phi]_{\text{ABLE+\partial}} = [\phi]_{\text{ABLE+\partial}}$. Thus we can ignore the closure operation in the ABLE+\partial semantics. It remains only to show that for any three PUL information states $\sigma, \nu$ and $\tau$, $\sigma \setminus \nu = \tau$ iff $\sigma^+ \setminus \nu^+ = \tau^+$, which is clear from the definitions of $+$ and $\setminus_{(\omega, \sigma, \partial)}$.

**Epistemic Possibility** Here are the definitions:

$$\sigma [\text{MIGHT}\phi]_{\text{PUL}} \tau \text{ iff } \exists \upsilon [\phi]_{\upsilon} \land$$

$$\left( \upsilon \neq \bot \rightarrow \tau = \sigma \right) \land$$

$$\left( \upsilon = \bot \rightarrow \tau = \bot \right)$$

$$[\text{MIGHT}\phi]_{\text{ABLE+\partial}} = \lambda I \lambda J \exists K \left[ [\phi]_{K} K \land$$

$$\left( \neg (K = \bot) \rightarrow J = I \right) \land$$

$$\left( K = \bot \rightarrow J = \bot \right)$$

It suffices that $+$ maps the PUL absurd information state onto the ABLE absurd information state.

The following fact sums up the close relation between the logics of PUL and ABLE+\partial:

**Fact F5** If $\phi$ and $\psi$ are formulae of PUL involving no atomic presuppositions except for those in $\mathcal{P}$, then:

$$\phi \models \text{PUL} \psi \text{ iff } \phi \models \text{ABLE+\partial} \psi$$

**Proof:** It must be shown that for formulae not introducing discourse markers, the two notions of entailment are equivalent. For single premise arguments, the PUL notion of entailment (defn. D26) may be written:

$$\phi \models \text{PUL} \psi \text{ iff } \forall \omega_0, \omega_1$$

$$\omega_0 [\phi]_{\text{PUL}} \omega_1 \rightarrow \omega_1 [\psi]_{\text{PUL}} \omega_1$$

In ABLE+\partial, (collapsing the two parts of the definition from earlier in this chapter) entailment is defined as:

$$\phi \models \text{ABLE+\partial} \psi \text{ iff } \forall \forall J$$

$$I[\phi]_{\text{ABLE+\partial}} J \rightarrow J \text{ satisfies } [\psi]_{\text{ABLE+\partial}}$$

From lemma L1.4, we know that if $I'$ and $J'$ have an empty domain, and $\omega\cdot\text{-set.} I = \omega\cdot\text{-set.} I'$ and $\omega\cdot\text{-set.} J = \omega\cdot\text{-set.} J'$, then:

$$I[\phi]_{\text{ABLE+\partial}} J \text{ iff } I'[\phi]_{\text{ABLE+\partial}} J'$$
It follows that in considering $ABLE+\partial$ we only need to consider states with an empty domain, so that the above definition is equivalent to:

$$\phi \models_{ABLE+\partial} \psi \iff \forall \omega_0 \forall \omega_0' \omega_0^+ [\phi]_{ABLE+\partial} \omega_0^+ \rightarrow \omega_1^+ \text{satisfies} [\psi]_{ABLE+\partial}$$

Given lemma L2, we have that for sentences of $PUL$:

$$\omega_0[\phi]_{PUL} \omega_1 \iff \omega_0^+ [\phi]_{ABLE+\partial} \omega_1^+$$

Thus it is only necessary to prove that:

$$\omega_1 [\psi]_{PUL} \omega_1 \iff \omega_1^+ \text{satisfies} [\psi]_{ABLE+\partial}$$

The definition of satisfaction given above in this chapter is:

$$\text{satisfies} = \lambda \lambda F [I\{\downarrow F\}I]$$

From lemma L1.2 and another application of the semantic correspondence lemma, the result follows.
Chapter 9

Presupposition and Modality in ABLE

In this chapter the dynamic framework developed so far will be adapted to allow for an adequate treatment of presupposition and epistemic modality. I will begin, in §9.1, by detailing the projection behaviour of presuppositions in the system as so far defined, showing some advantages of the approach, but also revealing a problem in the interaction between presupposition and quantification. The problem is one familiar from Heim's development of the CCP model, as found in her 1983 paper [Hei83a]. §9.2 will be spent ignoring the problem, and instead concentrating on the semantics of epistemic modality. It will be shown that essentially the same definition of the might operator as was given for the system UL, in Chapter 7, is also appropriate for ABLE, and that this definition leads immediately to a satisfying treatment of modal identity. However, it will also be shown that difficulties occur when quantifiers outscope the might operator, and this will motivate some tinkering with definitions from Chapter 8. Having dealt appropriately with the semantics of epistemic modality, we will return, in §9.3, to presupposition, only to find that the earlier problems concerning the interaction of presupposition and quantification have miraculously vanished.

9.1 Presupposition projection in ABLE

It will now be shown that, regarding the projection of presuppositions, the system developed in Chapter 8 is comparable with Heim's 1983 account¹. It will also be shown, in §9.1.2, that the ABLE treatment of proper names and definite, combined with ABLE's

¹That is to say, the system is comparable with the more formally developed aspects of Heim's 1983 account, since including Heim's informal description of a mechanism of accommodation would lead to a quite different set of predictions. In Chapter 10, I will consider ways in which the process of accommodation might be formalised.
presupposition projection properties, lead to DRT-like anaphoric accessibility.

9.1.1 Projection from connectives

With regard to the connectives, ABLE manifests much the same presupposition projection behaviour as PUL, or any other CCP account. This is most obviously seen by considering ABLE translations of the simple examples from Chapter 7, for which the following abbreviations will be useful:

\[
\begin{align*}
\text{BERTHA.D.}\phi &= \text{THE.D.}(\text{NAMED-BERTHA}^{u}.D).\phi \\
\text{ANNA.D.}\phi &= \text{THE.D.}(\text{NAMED-ANNA}^{u}.D).\phi \\
nbh &= \text{BERTHA.7.(HIDING}^{u}.7)
\end{align*}
\]

We begin with simple cases of presuppositions embedded under negations or in the antecedents of conditionals:

E195 a. Bertha is hiding
b. \(nbh\)

E165' a. Anna realises that Bertha is hiding
b. \(\text{ANNA.3.}(\text{REALISES}^{a}.3.\text{nbh})\)

E166' a. Anna does not realise that Bertha is hiding.
b. \(\text{NOT}(\text{ANNA.3.}(\text{REALISES}^{a}.3.\text{nbh}))\)

E169' a. If Anna realises that Bertha is hiding, then she will find her.
b. \((\text{ANNA.3.}(\text{REALISES}^{a}.3.\text{nbh}))\text{ IMPLIES (FIND}^{b}.3.7)\)

It is easily shown that all of E165b, E166b and E169b presuppose (and also entail) E195b. Indeed, these are just instances of the following general property:

**Fact F6** If \(\phi\) presupposes \(\psi\), then \(\text{NOT }\phi, \phi \text{ AND } \chi\) and \(\phi \text{ IMPLIES } \chi\) all presuppose \(\psi\).

**Proof:** Simple. Firstly show by inspection of the clause for \(\text{NOT } (\text{AND}, \text{ IMPLIES})\) that if I admits \(\text{NOT }\phi\) (etc.) then I admits\(\phi\). Thus the set of contexts which admit \(\phi\) is a superset of those which admit \(\text{NOT }\phi\) (etc.), and if all contexts which admit \(\phi\) satisfy \(\psi\) (the condition for \(\phi\) to presuppose \(\psi\)), then it must be the case that all contexts which admit \(\text{NOT }\phi\) (etc.) satisfy \(\psi\).

Next a case of one presuppositional construction embedded within another:

E171 a. Bertha regrets that Anna realises that she is hiding.
b. BERTHA.7.(regrets a.7.(ANNA.3.(realises a.3.bih))

E171b presupposes E195b, and once again this is an instance of a more general principle:

**Fact F7** If \( \phi \) presupposes \( \psi \), and \( \psi \) presupposes \( \chi \), then \( \phi \) presupposes \( \chi \).

**Proof:** The result follows directly from the definitions of presupposition and satisfaction. If \( \phi \) presupposes \( \psi \) then every context which admits \( \phi \) must satisfy \( \psi \), but \( \psi \) can only be satisfied in contexts which admit it, and all these contexts satisfy \( \chi \). Thus every context which admits \( \phi \) satisfies \( \chi \).

What really marks the CCP model out from other theories is the occurrence of Conditionalised presuppositions, as in the following case:

E172 a. If Bertha is not in the kitchen, then Anna realises that Bertha is in the attic.

b. NOT (BERTHA.7.(IN-KITCHEN’u.7)) IMPLIES
   (ANNA.3.(REALISES a.3.(IN-ATTIC’u.7))

c. NOT (BERTHA.7.(IN-KITCHEN’u.7)) IMPLIES
   (IN-ATTIC’u.7)

As might be expected, E172b presupposes the conditional E172c, i.e. that if Bertha is not in the kitchen, then she's in the attic. Conditionalised presuppositions are probably the single most controversial aspect of the CCP model, and the paradigmatically CCP behaviour of ABLE in this respect, as witnessed by the following fact, will be explored in more detail in Chapter 10.

**Fact F8** If \( \phi \) presupposes \( \psi \), then \( \chi \) and \( \phi \) and \( \chi \) IMPLIES \( \phi \) each presuppose \( \chi \) IMPLIES \( \psi \).

**Proof:** It suffices to show that under the assumption that \( I \) admits \( (\chi \ AND \ \phi) \) (or \( I \) admits \( (\chi \ IMPLIES \phi) \)) it follows that \( I \) satisfies \( [\chi \ IMPLIES \psi] \). Here is the reasoning for the conjunctive case, that for implication being similar:

\[
\begin{align*}
I \text{ admits } (\chi \ AND \ \phi) & \quad \text{assumption} \\
\exists J I[\chi]J \land J \text{ admits } [\phi] & \quad \text{defs. of AND, admits} \\
\exists J I[\chi]J \land J \text{ satisfies } [\psi] & \quad \text{ass., defn. of } \gg \\
\exists J I[\chi]J \downarrow [\psi]J & \quad \text{defn of satisfies} \\
\exists J I[\chi]J[\ NOT \ \psi] & \quad \text{defn of NOT} \\
I[\chi \ LAND \ NOT \ \psi] & \quad \text{defn of AND} \\
I[\ NOT \ (\chi \ AND \ NOT \ \psi)]I & \quad \text{defn. of NOT} \\
I[\chi \ IMPLIES \psi]I & \quad \text{defn. of IMPLIES} \\
I \text{ satisfies } [\chi \ IMPLIES \psi] & \quad \text{defn. of satisfies}
\end{align*}
\]
9.1.2 The Projection of Proper Names

The projection of presuppositions from embedded contexts, combined with the fact that proper names presuppose the presence of an appropriately named discourse marker, leads to proper names having very special anaphoric properties. Consider the following example:

E196 a. If Butch is happy, then he is barking. He is happy. Therefore he is barking.

b. \( \text{THE.2.(NAMED-BUTCH^u.2).(HAPPY^u.2) IMPLIES} \)
\( \text{THE.2.(MALE^u.2 AND SINGULAR^u.2).(BARK^u.2)} \)

c. \( \text{THE.2.(MALE^u.2 AND SINGULAR^u.2).(HAPPY^u.2)} \)

d. \( \text{THE.2.(MALE^u.2 AND SINGULAR^u.2).(BARK^u.2)} \)

e. \( \text{E196b,E196c} \models_{\text{able}} \text{E196d} \)

On its most obvious reading, that in which all the pronouns refer to Butch, E196a is an intuitively valid argument involving one application of *modus ponens*. Correspondingly, the ABLE entailment in E196e holds — that is to say, the translations of the first two sentences (i.e. E196b,c) entail the translation of the third (i.e. E196d). Yet E196b carries the same presupposition as E184b (‘Butch is barking’), namely that 2 is established to refer to an individual named Butch. So even without specifying ways in which contexts satisfying the presuppositions of a formula may be generated, the formula can still appear in the premises of arguments. This is because the notion of entailment given in D44 does not depend on the premises being defined on all contexts, but only says that for all *suitable* contexts which satisfy the presuppositions of the premises, updating with the premises yields a state which satisfies the consequent.

It may surprise some readers that the ABLE translations of E196 can sustain the anaphoric linkage between the proper name Butch, which occurs within the antecedent of a conditional, and later pronouns beyond the scope of the conditional. In effect, ABLE manifests accessibility conditions for proper names and other definites reminiscent of Kamp’s Discourse Representation Theory. The fact that a presuppositional analysis for proper-names can account for their distinctive anaphoric properties is discussed in [Ze91], but this awkward detail of Kamp’s work had generally been ignored in earlier reformulations of DRT, such as [GS91b] and [Mus90].

Looked at from a slightly different angle, the projection behaviour of definites in ABLE tells us something about the relationship between DRT and File Change Semantics. The special mechanism for dealing with accessibility of names appears to be one of the few ways in which DRT and FCS differ significantly. Yet the ABLE analysis of definites, which I have shown leads to DRT-like anaphoric accessibility conditions, is basically that found in the original manifestation of FCS, namely Heim’s thesis. Thus
FCS contains the core of an idea that not only reproduces, but also explains the special behaviour of definites and names in DRT. One can imagine adding an ABLE-like notion of entailment to FCS, something along the lines of "one sentence entails another if after updating a file set with the file change potential (FCP) of the first, updating with the FCP of the second has no effect". It remains a matter of speculation whether such a line of research could lead to a more concrete demonstration that DRT-like behaviour of definites can be derived within FCS.

9.1.3 Quantificational Projection: A Problem

The following example shows that a problem which appeared in Heim’s 1983 paper (and discussed in §5.3) has resurfaced:

E197
a. A man discovered that he owned a priceless Modigliani.
 b. SOME.i.(MAN^u.i).(DISCOVER^a.i.(OAPM^u.i))

E198
a. Every man owned a priceless Modigliani.
 b. EVERY.j.(MAN^u.j).(OAPM^u.j)

After updating with the restrictor of E197b, a state would be reached in which i was known to be a man. Assuming that the dynamic attitude predicate DISCOVER^a is constrained to be factive, this state could only be updated with the scope of E197b if the formula OAPM^u.i was satisfied. However, this formula would only be satisfied if all the values of i were already established to correspond to priceless Modigliani owners. Thus a state could only be updated with E197b if in that state every man was established to be a priceless Modigliani owner. Thus E197b presupposes E198b. The problem is that E197a certainly does not presuppose (or entail) E198a.

Heim’s account also predicts overly strong, universal presuppositions from existential sentences, and for essentially the same reason: quantified markers represent arbitrary objects. Thus at the point where the factive verb in E197b is reached, the discourse marker plays the role of an arbitrary man, and updating can only continue if it is established that any arbitrary man satisfies the factive’s presupposition. The provisional semantics for ABLE’s determiner’s presented in Chapter 8 is uniform in this respect, so that the semantics for all the quantificational determiners is based on the same approach as for indefinites, which can be summed up as “add an arbitrary object, and check what effect updating with the restrictor and scope will have”. Thus the occurrence of universal presuppositions is not restricted to indefinites alone, as is shown by the following examples:

E199
a. Every woman regrets that she is married
b. \( \text{EVERY}.i.(\text{WOMAN}^u.i).((\text{REGRET}^a.i.\text{MARRIED}^u.i)) \)

E200 a. No woman regrets that she is married
b. \( \text{NO}.i.(\text{WOMAN}^u.i).((\text{REGRET}^a.i.\text{MARRIED}^u.i)) \)

E201 a. Every woman who regrets that she is married is sane
b. \( \text{EVERY}.i.((\text{WOMAN}^u.i \text{AND REGRET}^a.i.\text{MARRIED}^u.i)).(\text{SANE}^u.i) \)

E202 a. Every woman is married
b. \( \text{EVERY}.i.(\text{WOMAN}^u.i).\text{MARRIED}^u.i \)

According to the Chapter 8 semantics, each of E199b, E200b and E201b presuppose E202b. This may be defensible for E199a, though I will later argue that even in this case the universal presupposition is inappropriate, it seems harder to justify the presence of such a strong presupposition for E200a and E201a. Intuitively, E200a is true just in case there are no married women who regret being married, and E202a is true just in case every married woman who regrets being married is sane. Such truth conditions are, of course, compatible with the existence of unmarried women.

The following fact summarises the problem with the arbitrary object analysis of quantification, showing that, as things stand, whenever a presupposing construction is bound within the scope of a quantifier, a universal presupposition arises:

**Fact F9** If \( \phi \gg \psi \), \( \mathcal{D} \) is any quantificational determiner (i.e. not THE), and true of type \( \pi \) is interpreted as the trivial CCP \( \lambda x \lambda y[I = J] \), then under the temporary definitions D57 and D56 the following hold:

1. \( \mathcal{D}.i.\phi.\chi \gg \text{EVERY}.i.\text{true.}\psi \), and
2. \( \mathcal{D}.i.\chi.\phi \gg \text{EVERY}.i.\chi.\psi \)

**Sketch of proof for (1):** inspection of definitions D57 and D56 shows that if \( I \) admits \( \models \mathcal{D}.i.\phi.\chi \) then \( \exists J I(\langle i \rangle J \land J \text{admits} \models \phi ; \) and thus that \( \exists J I(\langle i \rangle J \land J \text{satisfies} \models \psi \). By the definition of satisfaction it follows that \( (\langle i \rangle \circ \models \psi \) (where \( \circ \) is relational composition) is satisfied in \( I \), and thus that \( (\langle i \rangle \circ \text{true} \circ \models \psi \) is satisfied in \( I \), from which the result follows by the semantics of EVERY. The reasoning is similar for (2).

The above problem motivates an alteration to the semantics of quantification, but, as will be seen in the next section, such a change can to some extent be motivated by independent considerations.
9.2 Epistemic modality

9.2.1 Modal Identity Problems

In this section I will provide a semantics for the unary ABLE operator \textsc{might}, extending the development of Veltman's [Ve91] account of epistemic modality in the first part of this thesis, which concerned a propositional language, to the first-order language of ABLE. In fact the PUL definition D25 requires little modification, except for adjustment to the Ty3 format of ABLE semantic clauses:

**Definition D60 (Epistemic Modalities in ABLE)**

$$
\begin{align*}
[MIGHT\phi] & = \lambda I\lambda J \exists K I[\phi] K \land \\
& \quad (\neg(K = \bot) \rightarrow J = I) \land \\
& \quad (K = \bot \rightarrow J = \bot) \\
[MUST\phi] & = I[\neg(MIGHT(\neg\phi))] 
\end{align*}
$$

This definition for \textsc{might}, which clearly preserves the intuitions behind the operator in Veltman's original work and in the UL/PUL systems from Chapter 7 of this thesis, is essentially that used by Dekker in [Dek92, Dek93]. In discussing the general properties of these operators, it is helpful to introduce Veltman's notion of a \textit{test}. Tests are a special class of formulae which have, in terms of ABLE, the following property:

**Definition D61 (Tests)** A formula \(\phi\) is a test if and only if:

$$
\forall I \forall J I[\phi] J \rightarrow (J = I \lor J = \bot)
$$

This means that updating with a test is either uninformative, the output state being the same as the input state, or \textit{over-informative}, the output being the absurd state. It is now possible to get a formal grasp on the interpretation of ABLE's epistemic modalities:

**Fact F10** The \textsc{might}-operator defines a consistency test. Thus for any non-absurd state \(I\) which admits an ABLE formula \(\phi\):

1. \(MIGHT\phi\) is a test.
2. \(I[MIGHT\phi]I \iff I\text{ consistent-with }[\phi]\)
3. \(I[MIGHT\phi] \bot \iff I\text{ consistent-with }[\phi]\)

\textit{Proof of (1):} Simply observe that if a pair \(I, J\) are in the denotation of \(MIGHT\phi\), then by the definition of \(MIGHT\) there must be some state \(K\) for which \(I[\phi]K\) holds, and since \(K\) either is or is not the absurd state, at least one of the two conditional antecedents
Presupposition and Modality in ABLE

in the definition must be met, from which it follows that at least one of the consequents holds, namely that \( J = I \) or \( J = \bot \), but this is the condition for test-hood.

Proof of (2) and (3). The condition for \( I[MIGHT\phi] \) is that updating with \( \phi \) does not lead to the absurd state, and this is also the condition for consistency of \( \phi \) with \( I \). On the other hand, the condition for \( I[MIGHT\phi] \perp \) is that updating with \( \phi \) does lead to the absurd state, and this is both necessary and sufficient for consistency to fail.

The following lemma helps to clarify the interpretation of the \textbf{MUST}-operator:

**Lemma L3** For any state \( I \) which admits a formula \( \phi \):

\[
I \text{ satisfies } [\phi] \iff \neg I \text{ consistent-with } [\text{ NOT } \phi]
\]

Proof: By the definition of satisfaction, \( I \) satisfies \( [\phi] \) iff \( I[\phi] I \), and by the definition of negation this holds iff \( I[\text{ NOT } \phi] \perp \), which in turn, by the definition of consistency, holds iff \( \neg I \text{ consistent-with } [\text{ NOT } \phi] \).

**Fact F11** The \textbf{MUST}-operator defines a test for satisfaction. Thus for any non-absurd state \( I \) which admits an ABLE formula \( \phi \):

1. \textbf{MUST}\( \phi \) is a test.
2. \( I[MUST\phi] I \) iff \( I \) satisfies \( [\phi] \)
3. \( I[MUST\phi] \perp \) iff \( \neg I \) satisfies \( [\phi] \)

Proof of (1): Since for any \( \phi \), \textbf{MIGHT}\( \phi \) is a test, it follows that \textbf{MIGHT}(\text{ NOT } \phi) is also a test, and so if a pair \( I, J \) is in the denotation of \textbf{MIGHT}(\text{ NOT } \phi), then \( J = I \) or \( J = \bot \). From the semantics of negation it follows that if \( I[MIGHT(\text{ NOT } \phi)] I \) then \( I[\text{ NOT } (MIGHT(\text{ NOT } \phi))] \perp \), and if \( I[MIGHT(\text{ NOT } \phi)] \perp \) then \( I[\text{ NOT } (MIGHT(\text{ NOT } \phi))] I \), and since this is exhaustive, the result follows.

Proof of (2):

\[
I \text{ satisfies } \phi \iff \neg (I \text{ consistent-with } [\text{ NOT } \phi]) \quad \text{lemma L3}
\]
\[
\iff I[MIGHT(\text{ NOT } \phi)] \perp \quad \text{defn. of consistency}
\]
\[
\iff I[\text{ NOT } (MIGHT(\text{ NOT } \phi))] I \quad \text{defn. of negation}
\]

Proof of (3): Similar to (2).

I will not discuss the \textbf{MUST} operator in detail here, but the reader is invited to verify that it behaves appropriately with respect to the hide-and-seek examples from Chapter 7. Even without examining the applications of the \textbf{MUST} operator, it is at least comforting that the dual of a consistency test should be a test for satisfaction. On the basis of this interpretation, one could think of \textbf{MUST} as corresponding not to the English
Presupposition and Modality in ABLE

modal 'must', but as one of the sentential operators which could be called argument connectives, for example 'so' or 'therefore'. It is also comforting that the definitions of the modalities preserve the standard CCP presupposition inheritance properties of modals:

**Fact F12** If \( \phi \) presupposes \( \psi \), then (1) MIGHT\( \phi \) and (2) MUST\( \phi \) also presuppose \( \psi \).

**Proof:** From the definition of MIGHT it can be seen that I admits MIGHT\( \phi \) only if I admits\( \phi \), from which the first result follows by the definition of presupposition. The same result for MUST follows from the proof of (1), the duality of the modalities, and the inheritance properties of NOT (Fact F6).

To see that in general this projection behaviour is appropriate — counter-examples being discussed in Chapter 10 — recall the following example from Chapter 7, which was argued to presuppose that Bertha is hiding:

E170 Anna might realise that Bertha is hiding.

Although there remain some problems, to which we will turn shortly, the account of epistemic possibility already has significant applications. In particular, some intuitive results arise when epistemic uses of the English modals 'possible' and 'perhaps' are translated using the MIGHT operator, as will now be shown with reference to examples E203–E206:

E203 a. It is possible there is a happy farmer, but, then again, it is possible that there are no happy farmers.
   
   b. MIGHT.(SOME.i.(FARMER^i.i).(HAPPY^u.i)) ∧ MIGHT.(NO.i.(FARMER^u.i).(HAPPY^u.i))

E204 a. No farmer is happy.
   
   b. NO.i.(FARMER^u.i).(HAPPY^u.i))

E205 a. Perhaps the spy is the president.
   
   b. MIGHT.(THE.i.(SPY^u.i).(THE.j.(PRESIDENT^u.j).(i = j)))

E206 a. The spy is not the president.
   
   b. THE.i.(SPY^u.i).(THE.j.(PRESIDENT^u.j).(NOT.(i = j)))

If my proposal for research into depression amongst farmers were to include E203a, and if, having obtained funding and done the relevant fieldwork, I were later to produce a report including E204a, I think I would feel justified in taking a certain pride in the fruitfulness of my research. In spite of an initial dearth of data, I would have
made an important discovery about the distribution of agriculture-related emotional disorders. On the other hand, if my research proposal were to include E204a, and my final conclusions included E203a, I think DARPA would be justified in questioning whether I deserved further funding. For, quite apart from the limited military potential of my results, it would seem that my proposals tended to be rather unreliable. The conclusion that some farmer might be happy is simply inconsistent with the previous assertion that no farmer is happy.

The contrast between E203a followed by E204a on the one hand, and the E204a followed by E203a on the other, is mirrored by the fact that the ABLE formula E203b AND E204b is consistent, whereas the formula E204b AND E203b is not. This would hold if definition D60 was used just as it will with the final definition for the semantics of MIGHT to be given below. The predictions with regard to examples E205 and E206 are similarly sensible on either definition. In particular, the formula E205b AND E206b is consistent, but swapping the order of the two conjuncts would yield an inconsistent formula. The success of ABLE in dealing with this last pair of examples shows that a serious problem with another recent attempt to combine DPL and update semantics, namely the Dynamic Modal Predicate Logic (DMPL) of van Eijck and Cepparello [EC92], has been circumvented. In DMPL it would make little sense to give a definition like D60. The reasons are technical: the DMPL notions of entailment and negation require that DMPL formulae can be calculated pointwise over assignment functions (recall the property of distributivity introduced in §8.3). However, D60 cannot be calculated pointwise over assignment functions (sequences) as consistency of a formula is evaluated with respect to the full set of world-sequence pairs in the input. To give a flavour of the DMPL semantics for epistemic possibility, it must be translated into ABLE terms. This is not difficult given the technical proximity of DMPL to ABLE:

Definition D62 (Epistemic Possibility: DMPL version)

\[
[MIGHT\phi] = \lambda I \lambda J \exists K [I \downarrow [\phi]K \land J = \lambda w \lambda f [I.w.f \land \exists w' K.w'.f]]
\]

This definition can be seen as being in terms of equivalence classes of world-sequence pairs in the input: each class contains only pairs involving the same sequence. Given some input state \( I \), the (anaphorically closed) output of MIGHT\( \phi \) will be the union of all the equivalence classes for which at least one world-sequence pair would survive in an update with \( \phi \). Put another way, a given world-sequence pair in the input will survive just in case there is some relevant world where the values that the sequence gives to discourse markers are consistent with \( \phi \). Unfortunately the use of a definition like D62
means that in DMPL the following entailment is valid:

$$\exists x \exists y (x = y) \models x = y$$

Thus if the above examples were to be translated into DMPL with 'the spy' as $x$ and 'the president' as $y$, then the validity of the pattern above would mean that learning that the spy might be the president would be equivalent to learning that the spy was the president. Furthermore, there is no obvious alternative translation which would solve the problem. For instance, translating the definite descriptions as constants would not help, since in DMPL updating with $\text{MIGHT}(a = b)$ for separate constants $a$ and $b$ always yields absurdity. Besides this, the problem is not restricted to definite descriptions but applies also to pronouns:

E207 a. The first lady is not spying  
   b. $\text{THE.}(\text{FL}.j).(\text{NOT SPYING}^n.j)$

E208 a. I can see a woman in the Whitehouse  
   b. $\text{SOME.i.(WOMAN}^n.i).(\text{ISEEIN\text{-}W}^n.i)$

E209 a. She might be spying.  
   b. $\text{SHE.i.MIGHT(SPYING}^n.i)$

E210 a. However, she might be the first lady.  
   b. $\text{SHE.i.(THE.j.(FL}.j).\text{MIGHT(i IS}j))$

This discourse consisting of E207a–E210a seems perfectly consistent, and, indeed, the formula ($E207b \text{ AND } E208 \text{ AND } E209 \text{ AND } E210$) is consistent in ABLE. Yet there is a single discourse referent which might be spying, and also might be the first lady, even though the first lady is apparently above suspicion. Thus the sub-formula $\text{MIGHT(SPYING}^n.i)$ in E209b should not — as would be the case in DMPL — remove sequences which map $i$ onto the first lady, even though such sequences considered in isolation lead to values which are inconsistent with the assertion of spyhood.

It should be realised that identity problems like those found in DMPL are not altogether new, and not essentially dynamic in nature: many static systems of modal predicate logic have comparable properties. Indeed Kripke has argued persuasively [Krip:MS] that identity is a non-contingent property, so that if two things are identical then they are necessarily so. Given the logical duality of necessity and possibility, it immediately follows that things which might not be identical are not identical, and thus

---

^2Note that in DMPL, as in DPL or ABLE, quantifiers take non-standard scope, so that a quantifier in the premise of an argument is able to bind a variable in the conclusion.
that things which might be identical are identical. However, it is clear that Kripke’s arguments (which anyway have remained controversial) could not be applied in defence of the properties of DMPL, for those arguments concern metaphysical contingency within total logics, and do not extend into the epistemic realm.3

In summary, whilst DMPL could well be an interesting object of logical investigation, it lacks any intuitive interpretation. In most respects the $\Diamond$ operator behaves comparably to Veltman’s $\textit{MIGHT}$: learning what is the case restricts what merely might be. However, with regard to questions of identity the $\Diamond$ behaves not as a $\textit{Veltmanesque}$ epistemic modality but as a $\textit{Kripkean}$ metaphysical modality: learning that things might be identical determines that they are identical. The reason is technical. An epistemic operator must be sensitive to all aspects of a referent’s identity, but DMPL’s entailment (and also negation) is not compatible with such a sensitive operator. On the other hand, the entailment used in ABLE or in Dekker’s EDPL (from which much of the inspiration for ABLE derives), is compatible with the definition of an identity-sensitive operator. Finally, and without going into details, I should point out the sequence semantics for DPL introduced by Vermeulen [Ver93] provides yet another route to an identity-sensitive modality, but, as with DMPL and ABLE, would also be technically compatible with a DMPL-style insensitive operator.

9.2.2 The dynamics of quantifying-in.

Unfortunately, the definition of $\textit{MIGHT}$ given in D60, does not interact appropriately with quantificational determiners. For example, it is clear that E211a and E212a express quite different propositions, but the formula E211b differs from E212b only in terms of its anaphoric potential. That is to say, E211b and E212b are satisfied by exactly the same set of states.

E211 a. Most politicians might be spying.

3Kripke's main argument, which I again must stress is applied within a metaphysical rather than an epistemic setting, involves consideration of which pairs of objects could falsify the open formula $\Diamond(x = y) \rightarrow x = y$. Substituting distinct values for $x$ and $y$ into the implication would falsify the antecedent, whereas substituting one value for both $x$ and $y$ would validate the consequent. But, if the formula is not falsifiable, then $\Diamond$ is not metaphysically contingent. Within an epistemic setting, the argument loses its force, for although it is impossible to simultaneously locate a pair of objects in the world which would obviously falsify the implication, it seems that the objects of thought and discourse, which may not even have a location in the exterior world, behave differently. There is manifestly no contradiction in positing that the objects referred to by two different descriptions, perhaps even in different languages, are in fact identical, nor is there necessarily any certainty about the question of whether a person seen in one guise is the same person as was separately seen in another guise. This is why it is consistent, albeit unwise in the wrong company, to assert that ‘the spy might be the president’, and still allow that they may later be discovered to be different. So even if data like that discussed here is reminiscent of data discussed in the philosophical literature, it is even so distinct data, since it concerns epistemic and not metaphysical modality, and the Kripkean arguments would seem to be irrelevant to a defence of the DMPL account of quantifying-in.
b. \textsc{most}.i.(\textsc{politician}^n.i)(\textsc{might}(\textsc{spying}^u.i))

E212 a. Some politician might be spying.

b. \textsc{some}.i.(\textsc{politician}^n.i)(\textsc{might}(\textsc{spying}^u.i))

The definition for the semantics of ABLE's quantificational determiners is based on the principle of adding a discourse marker which in different sequences takes each single object as its value, and then checking what new information about the possible value of the marker is obtained by updating with the restrictor and scope. Thus the privileged marker stands proxy for the entire set of objects onto which it is mapped by different sequences associated with the various intermediate information states involved in updating with a quantificational formula. However, the interpretation of discourse markers outside of the scope of their introductory quantifier is quite different. An unquantified marker may also have several alternative valuations, but the interpretation is essentially disjunctive rather than conjunctive. Rather than standing proxy for the entire set of objects which it can take as values, an unquantified marker is understood as standing for just one of the alternative values: the presence of alternative values represents not multiplicity but underspecificity.

This difference in interpretation becomes crucial when quantifying into a modal context. \textsc{might} must check separately for consistency of its argument with regard to each of the alternative values for the quantified markers. Thus in E211b and E212b, where $i$ is a quantified discourse marker, the effect of updating with $\textsc{might}(\textsc{spying}^u.i)$ should be to remove all those sequences which map $i$ onto an object which is known not to be spying. However, the same sub-formula should have a quite different effect when it occurs in E209b, above.

In order to define a notion of epistemic possibility which preserves the positive traits of that given in D60, and also gives a sensible account of the behaviour of quantified markers which distinguishes appropriately between E211b and E212b, the semantics of ABLE quantification will be altered. Note, however, that this is not the only way to proceed: in an earlier version of this chapter (appearing as [Bea93a]), a special mechanism was added for keeping track of quantified markers, and the semantics of \textsc{might} was altered so as to interact differently with quantified and unquantified markers. Here I will formulate things more along the lines taken by my colleagues, Groenendijk, Stokhof and Veltman, in recent talks [GSV95]. The motivation, however, differs somewhat from theirs, in that my reason for following their formulation is not simply that it eliminates the above-mentioned problem with epistemic modality, but that it will also lead to an elegant solution of some longstanding problems in the treatment of presupposition.

As I have said, under the semantics above, a quantified marker at some point in a formula stands proxy for the entire set of individuals which are compatible with the
formula up to that point. So, although the semantics of MIGHT is only appropriate when multiplicity of interpretations represents underspecificity of reference, this is not the case with a quantified marker. The solution is to alter the semantics of quantification so that rather than considering all the different possible values of the quantified marker simultaneously, they are considered one at a time. By evaluating the restrictor and scope of a quantification separately for each value of the quantified marker, operators in the restrictor and scope become insensitive to the multiplicity of different values. To this end, a special way of interpreting a formula is defined. Given an ABLE formula, \( \phi \), and a discourse marker, \( d \), \( [\phi]_d \) will define that relation between information states such that after splitting up the first state with respect to each different value of the marker \( d \), and updating each of the resulting states using the normal interpretation \( [\phi] \), the outputs can be recombined to yield the second state. \( [\phi]_d \) is defined using a constant distribute, where distribute.\( F.d \) will be referred to as the distributive re-interpretation of the CCP \( F \) with respect to the marker \( d \).

In the definition below, distribute is defined in two stages. Firstly a function unfold is given, where unfold.\( F.d \) defines a relation between two information states holding just in case there is a value of \( d \) such that after restricting the input to sequences which give that valuation, updating with \( F \) yields the output. Here, the process of restricting an information state with respect to values taken by some discourse marker is easily defined: if \( I \) is a state and \( d \) is a marker, then the restriction of \( I \) with respect to the value \( X \) for \( d \) is given by \( \lambda w \lambda f [I.w.f \land f.d = X] \). The constant distribute is then defined such that a formula distribute.\( F.d \) denotes a CCP having as inputs only states which admit unfold.\( F.d \), and having as output the union of the outputs of \( [\phi]_d \). Here, given an input \( I \), the union of the outputs of a CCP \( F \) is just \( \lambda w \lambda f [\exists K I\{F\} K \land K.w.f] \).

Meaning Postulate MP16

\[
\begin{align*}
\text{(1) } \text{unfold} & = \lambda F \lambda D \lambda I \lambda J [\exists X (\lambda w \lambda f [I.w.f \land f.d = X])\{F\} J] \\
\text{(2) } \text{distribute} & = \lambda F \lambda D \lambda I \lambda J [I \text{ admits (un}folds.\{F.D\}) \land \\
& \quad J = \lambda w \lambda f [\exists K I\{un}folds.\{F.D\} K \land K.w.f] \\
\end{align*}
\]

Definition D63 (Distributive Interpretation of ABLE Formulae)

\[ [\phi]_d = \text{distribute.}[\phi].d \]

The definition allows for a relatively conservative alteration to the quantifiers. It can be seen from the following two properties that the use of \( [\phi]_d \) in place of \( [\phi] \) will only have an effect in a very restricted range of circumstances:
Fact F13 If \( \phi \) does not contain the discourse marker \( d \) then \([\phi]_d = [\phi]\)

Sketch of proof: ABLE predicates have the property of relevance (cf. §8.3), which is easily shown (by induction over formula complexity) to be preserved by all ABLE's operators. Thus we need only consider states \( I, J \) which do not have \( d \) in their domain. For such states it is clear that \( I[\phi]J \iff I[\phi]_dJ \) (Note that the definition of \([\phi]_d\) is compatible with the input state only giving \( d \) the value \( \lambda x [\lambda] \).

Fact F14 If \( I \) admits \([\phi]\) and \( \phi \) does not contain any occurrences of the epistemic modality, MIGHT, then \( I[\phi]J \iff I[\phi]_dJ \)

Sketch of proof: ABLE predicates have the property of distributivity (cf. §8.3), and this can be shown (another induction) to be preserved by ABLE's connectives and determiners. Thus we need only consider singleton states (those with only one world-sequence pair), and for these the result is trivial, since distributing over a single world-sequence pair has no effect.

Having given a definition of \([\phi]_d\), amending the semantics of quantifiers is easy. Where before the passage from input to output was mediated by a sequence of updates \( I[+i].init[\phi]res[\psi]sco \), we simply replace the occurrences of \([\cdot]\) by \([\cdot]_d \) to get the new definitions, given below for completeness:

Definition D64 (Non-existential Determiners - Final Version)

If \( D \) is one of EVERY, MOST, FEW or NO, and \( D' \) is a corresponding quantifier relation of type \( \langle (e,t),(e,t),t \rangle \), then:

\[
[D.i.\phi.\psi] = \lambda x J
\]

\[
\exists f \exists G_{res} \exists G_{sco} \exists X_{res} \exists X_{sco} I[+i].init[\phi]res[\psi]sco \land \\
J = \lambda w \lambda g \\
\exists f \exists G_{res} \exists G_{sco} \exists X_{res} \exists X_{sco} I.w.f \land \\
G_{res} = \lambda h [h \succ f \land res.w.h] \land \\
G_{sco} = \lambda h [h \succ f \land sco.w.h] \land \\
X_{res} = \lambda x [\exists h G_{res}.h \land h.i.x] \land \\
X_{sco} = \lambda x [\exists h G_{sco}.h \land h.i.x] \land \\
D'.X_{res}.X_{sco} \land \\
g = \lambda D \lambda x [f.D.x \lor \\
\exists h G_{sco}.h \land h.D.x] 
\]
Definition D65 (Existential Determiners: final version)

If $D$ is SOME or EXACTLY-ONE, and $D'$ is a corresponding quantifier relation of type $\langle\langle e, t\rangle, \langle\langle e, t\rangle, t\rangle\rangle$, then:

$$[[D.i.\phi.\psi]] = \lambda I \lambda J \exists i \exists res \exists sco \langle +.i \rangle \text{init}[\phi], res[\psi], sco \wedge$$

$$J = \lambda w \lambda g$$

$$\exists f \exists G_{res} \exists G_{sco} \exists X_{res} \exists X_{sco} \langle \cdot \cdot w.f \wedge$$

$$G_{res} = \lambda h [h \bar{\geq} f \wedge res.w.h] \wedge$$

$$G_{sco} = \lambda h [h \bar{\geq} f \wedge sco.w.h] \wedge$$

$$X_{res} = \lambda x [\exists h G_{res}.h \wedge h.i.x] \wedge$$

$$X_{sco} = \lambda x [\exists h G_{sco}.h \wedge h.i.x] \wedge$$

$$D'.X_{res}.X_{sco} \wedge$$

$$G_{sco}.g$$

What is the reason for the definedness condition, $I \text{ admits (unfold}.F.D)$ in postulate MP16? It is easy to see that with no such condition some rather strange effects would arise. For instance, if the formula $F$ was the denotation of a simple predication $P(d')$ for some discourse marker $d'$ not in the domain of the input, then unfold.$F.D$ would have no outputs, and the effect of taking the union of the outputs would be to produce the absurd state. Whereas, for such an input, $[P(d')_]$ would not define an output, $[P(d')]_d$ would define an update to the absurd state. Thus undefinedness would cease to propagate through formulae involving distributive updates, leading to a rather strange inconsistency, and consequently strange logic. So some definedness condition is needed. However, it would be possible to use a stronger definedness condition, say insisting not just that there is at least one value of the distributed marker for which there is an update, but requiring that for every value an update was defined. Such a condition would still be sensible as far as the interaction of quantification and modality was concerned, but would lead to different predictions regarding the projection of presuppositions from quantified contexts. In fact, such a strong definedness condition would yield the same predictions for presupposition projection as were obtained with the earlier definitions of quantification in Chapter 8. However, in the next section it will be shown that whereas the earlier definitions of quantification give the same undesirable presupposition projection effects as plagued the system in [Hei83a], the new definitions provide quite attractive projection properties.
9.3 Presupposition in ABLE revisited

Fact F9, which says that universal presuppositions arise from the Chapter 8 definitions of quantification, would fail under the amended definitions. To see this it suffices to give a counter-example, so constructing an information state which admits E197b but does not satisfy E198b will be adequate:

**E197**

- A man discovered that he owned a priceless Modigliani.
- SOME.i.(MANu.i).(DISCOVERa.i.(OAPMu.i))

**E198**

- Every man owned a priceless Modigliani.
- EVERY.j.(MANu.j).(OAPMu.j)

Let us assume MANu and OAPMu to be simple, and DISCOVERa to be factive. Here is a definition of an appropriate information state in a simplified model:

- The state $I$ contains the single world sequence pair $w, ()$
- The domain $D_e$ consists of only two individuals, $a$ and $b$
- $w$ is a man's world (i.e. both $a$ and $b$ are men)
- In $w$, $a$ owns a priceless Modigliani and has discovered that he owns it, whilst $b$ does not own a priceless Modigliani. Note that ownership of a priceless Modigliani is assumed to be a basic property, so that the object corresponding to the Modigliani can be ignored.

That $I$ does not satisfy E198b should be clear, since in $I$ there is a man who does not own a priceless Modigliani. Inspection of the semantics for existential determiners D65 shows that the condition for $I$ to admit E197b is that there exist states $J, K, L$ such that:

$I\{+.i\}J[(\text{MAN}u.i)]K[D\text{ISCOVER}a.i.(\text{OAPM}u.i)]L$

It is easy to find the appropriate values for $J$ and $K$. $J$ must consist of two world sequence pairs mapping $i$ onto $a$ and $b$, respectively. Furthermore, since both $a$ and $b$ are men, we should expect the update with $[(\text{MAN}u.i)]$, not to remove any world-sequence pairs from $J$: since MANu is simple, $J$ must admit $[(\text{MAN}u.i)]$, and F14 means that we can ignore the fact that MANu.i is evaluated distributively with respect to $i$. Thus:

$J = K = \{w: (i \mapsto a), w: (i \mapsto b)\}$
By the definition of distributive update D63, \( L \) is calculated by taking the union of all the states \( M \) (provided that there is at least one state \( M \)) such that:

\[
K\{\text{unfold.}[\text{DISCOVER}^a.i.(\text{OAPM}^u.i)].i\}M
\]

Since of the two values for \( i \), only \( a \) corresponds to an individual satisfying the factive presupposition of \( \text{DISCOVER}^a.i \), there will only be one appropriate \( M \):

\[
M = \{ w : (i \mapsto a) \}
\]

The existence of this single value for \( M \) is enough to guarantee that E197b is admitted in \( I \), as required. Further, it can be seen that \( L \) will take the same value, i.e. the singleton set containing the pair of \( w \) and a sequence mapping \( i \) onto the individual \( a \), and, by inspection of the semantics for \( \text{SOME} \), that this will also be the output of the whole formula E197b.

So given that Fact F9 is no longer appropriate, what are the presuppositions of quantificational formulae involving a bound presuppositional expression? The following fact shows that under the new semantics for quantifiers, although the universal presuppositions associated with the Chapter 8 definitions have gone, there remain at least existential constraints:

**Fact F15** If \( \phi \gg \psi \), \( D \) is any quantificational determiner (i.e. not \( \text{THE} \)), and true of type \( \pi \) is interpreted as the trivial CCP \( \lambda \alpha \lambda J [I = J] \), then under the final definitions D64 and D65:

1. \( D.i.\phi.\chi \gg \text{SOME}.i.\text{true.}\psi \), and
2. \( D.i.\chi.\phi \gg \text{SOME}.i.\chi.\psi \)

**Sketch of proof for (1):** From D64 and D65, \( D.i.\phi.\chi \) is admitted by any state for which:

\[
\exists \text{init} \exists \text{res} \exists \text{sco} \ I^{\{+.i\}\text{init}[[\phi]],[\text{res}][\chi],\text{sco}}
\]

Given the assumption that \( \phi \) presupposes \( \psi \), it follows that a necessary condition for the formula to be admitted by \( I \) is that there is some state \( \text{init} \) such that:

\[
I^{\{+.i\}\text{init} \land \text{init admits}[[\phi]],}
\]

From the definition of distributive update, it follows that:

\[
\text{init admits}[[\phi]] \iff \text{init admits (unfold.}[[\phi].i])
\]

Using the definition of \( \text{unfold} \) gives us:

\[
\text{init admits}[[\phi]] \iff \exists X ((\lambda w. f [\text{init}.w.f \land f.d = X]) \text{ admits }[[\phi])]}
\]
Presupposition and Modality in ABLE

But by assumption \( \phi \) presupposes \( \psi \), so:

\[
\text{init admits } \llbracket \phi \rrbracket \text{ iff } \exists X ((\lambda w. f \ [\text{init}. w. f \land f.d = X]) \text{ satisfies } \llbracket \psi \rrbracket)
\]

Thus for \( I \) to admit \( D.i.\phi_X \), it must be the case that there is some individual in a singleton set \( X \) such that if \( I \) is extended with the marker \( i \), and is then restricted by removing all the world sequence pairs where \( i \) is not mapped onto \( X \), the resulting state satisfies \( \psi \). This existential requirement is enough to force the required condition, that \( \text{SOME}.i.\text{true}.\psi \) is satisfied in the input, but I omit the remaining details. The proof of (2) would be similar.

So whilst E197b does not presuppose E198b, it does presuppose E213b:

\[\begin{align*}
\text{E213} & \quad \text{a. Some man owned a priceless Modigliani.} \\
& \quad \text{b. } \text{SOME}.j.(\text{MAN}^a.j).\text{OAPM}^a.j
\end{align*}\]

Similar results are obtained for the other quantificational examples considered earlier, and it will now be argued that these results are not only reasonable, but represent an advance over previous analyses. Firstly a case with a presupposition in the restrictor of a quantifier:

\[\begin{align*}
\text{E201} & \quad \text{a. Every woman who regrets that she is married is sane} \\
& \quad \text{b. } \text{EVERY}.i.(\text{WOMAN}^a.i \text{ AND REGRET}^a.i.\text{MARRIED}^a.i).(\text{SANE}^a.i)
\end{align*}\]

\[\begin{align*}
\text{E214} & \quad \text{a. Some woman is married} \\
& \quad \text{b. } \text{EVERY}.i.(\text{WOMAN}^a.i).\text{MARRIED}^a.i
\end{align*}\]

As pointed out earlier, the prediction of a universal presupposition for E201 is quite clearly inappropriate. On the current analysis, E201b presupposes E214b, and this existential presupposition is certainly to be preferred. Although there can be no question that analyses\(^4\) which predict a universal presupposition for this case are inappropriate, there must remain a question as to whether the existential analysis is correct. I think it quite possible to defend a view that such sentences should have no presuppositions, but only implicate that there are some individuals which satisfy the restrictor conditions, and thus implicate that there are some individuals which satisfy the restrictor’s presuppositions. It could be that variations on the definition of distributive update used within the semantics of quantifiers would lead to such results, but in this thesis I will not attempt any further development along these lines.\(^5\)

\(^4\) e.g. Heim’s [Hei83a] without her suggested addition of a mechanism for accommodation.

\(^5\) Yet another option would be to produce a defeasible presupposition, along the lines to be sketched for the propositional case in Chapter 11.
Much more controversial are cases where a presupposition occurs in the scope of a quantificational construction. Unfortunately, there is, to my knowledge, no intellectually rigorous analysis of such sentences in the existing presuppositional literature, and most authors have been happy to advocate whatever claims about the data pose fewest problems for their own theory. There is thus no consensus concerning the presuppositions of the following examples, the first being repeated from above:

E199' a. Every woman regrets that she is married
   b. $\text{EVERY}.i.(\text{WOMAN}^u.i) . (\text{MARRIED}^u.i)$

E215 a. Every man discovered that he owned a priceless Modigliani
   b. $\text{EVERY}.i.(\text{MAN}^u.i) . (\text{DISCOVER}^u.i . (\text{OAPM}^u.i))$

I find the lack of consensus surprising, since there is no reason why standard presuppositional tests should not be applied in order to resolve the issue. I will only consider one such test, embedding under negation.

Firstly consider E216a below, which I consider to be true: there are women who do not regret that they are married. Given that not every woman is married, we have a situation (the real world) where E216a is true, but it is false that every woman is married. It is clear that this contradicts any claim that E216a entails that every woman is married. Now although the negation in E216a is syntactically realised within the subject NP, there is good reason to analyse it as semantically having scope over the whole sentence. Thus, it seem reasonable to accept that E216a functions semantically as the negation of E199a, and thence that E199a does not presuppose that every woman is married.

E216 a. Not every woman regrets that she is married.
   b. $\text{NOT} (\text{EVERY}.i.(\text{WOMAN}^u.i) . (\text{REGRETS}^u.i . (\text{MARRIED}^u.i)))$

Now consider E217a, which I take to be a slightly strange thing to say unless there is previous reason to think that there have been some discoveries of Modigliani ownership. I think it indisputable that the sentence could be true in worlds where some, but not...
all, men had discovered that they owned Modigliani's. This is sufficient to show that E215 does not presuppose universal Modigliani ownership.

E217  

a. Not every man discovered that he owned a priceless Modigliani.

b. \(\text{NOT} \ (\text{EVERY}.i.(\text{MAN}^u.i).(\text{DISCOVER}^a.i.(\text{OAPM}^u.i)))\)

These observations are completely in line with the predictions obtained within ABLE: none of the above four examples (E199a, E215a, E216a, E216a) carry universal presuppositions. However, this is not the end of the story. My intuition is that, knowing that not all women are married, E199a is a slightly strange thing to say, and, given that not every man owns a Modigliani, E215 is an even stranger thing to say. However, even knowing that not all women are married, E200 does not seem in the least odd. My intuition is that it is simply false, since there are married women who regret being married:

E200  

a. No woman regrets that she is married

b. \(\text{NO}.i.(\text{WOMAN}^u.i).(\text{REGRET}^a.i.(\text{MARRIED}^u.i))\)

Similarly, E218 is quite compatible with there being some non-Modigliani owners:

E218  

a. No man discovered that he owned a priceless Modigliani

b. \(\text{NO}.i.(\text{MAN}^u.i).(\text{DISCOVER}^a.i.(\text{OAPM}^u.i))\)

In ABLE these contrasts are explained straightforwardly. Although none of the above regretful-wife examples are predicted to presuppose that every woman is married, E199 is predicted to entail that every woman is married, although this is not the case with E216 or E200. Likewise, none of the Modigliani-discovery examples are predicted to presuppose that every man owns a Modigliani, although E215 is predicted to entail this proposition. Thus, what seems to me rather a strong contrast between occasions in which 'every' and 'no' are appropriate\(^8\), is explained not in terms of any difference in presupposition, but in terms of entailments.

It may seem that the contrasts are too vague to merit the sharp semantic division which ABLE predicts. However, I believe that the apparent blurriness of the contrasts is explained solely by the failure to specify the context of utterance in sufficient detail.

\(^7\)Some would doubtless go further, and say that E218 is compatible with their being no Modigliani owners. As mentioned above, I think this is a tenable viewpoint, although I will not attempt to adjust ABLE's semantics accordingly. Note also that although there may be effects with this type of example related to Karttunen's division of verbs into factives and semifactives (as indicated to me by Gerald Gazdar) the conclusions I draw are made on the basis of examples with both types of verbs (a factive in E200 and a semifactive in E218), and therefore should be safe.

\(^8\)Hans Kamp drew attention to this contrast at a recent DYANA workshop in Amsterdam.
which means that regarding the above examples it is not *a priori* clear exactly which sets of individuals are being talked about. The contrasts become much stronger when more of the preceding context of utterance is given:

E219 Ten girls were playing hide-and-seek.

E220 Every girl discovered that she could hide in the attic.

E221 No girl discovered that she could hide in the attic.

Suppose that Clothilde, one of the ten girls mentioned in E219 is too fat to fit through the trap-door to the attic, and thus cannot possibly hide there. The discourse consisting of E219 followed by E220 is not appropriate: it simply contradicts the assumption about Clothilde’s breadth. On the other hand, E219 followed by E221 could conceivably be true under the assumptions about Clothilde. The contrast is very sharp. Once again, it will be explained quite readily in ABLE, since E220 will entail that every girl could hide in the attic, but E221 will not entail this.

However, this contrast is not predicted by either Heim or van der Sandt’s accommodation based theories of presupposition, as described in Chapter 6. In both of these theories, presuppositions bound in either the scope of a quantificational construction are able to trigger restriction of the quantificational domain through the mechanism of *intermediate accommodation*. But, as was shown in §6.5, intermediate accommodation produces incorrect predictions, and the problems discussed there resurface in the case of the above discourses.

Let us see how van der Sandt’s account fares with E220. Assume that previous knowledge of Clothilde and the explicit addition of E219 have produced the following simplified DRS:

<table>
<thead>
<tr>
<th></th>
<th>c</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>clothilde(c)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>¬</td>
<td>could-hide-in- attic(c)</td>
<td></td>
</tr>
<tr>
<td>ten-girls-playing(S)</td>
<td></td>
<td>c ∈ S</td>
</tr>
</tbody>
</table>

*Presupposition and Modality in ABLE*
On parsing E221, and assuming resolution of 'she' to the locally quantified marker, an initial DRS like the following would presumably be produced:

Global accommodation of the presupposition is not possible because of van der Sandt's *trapping* constraint, which prevents markers in presuppositions from becoming unbound in the accommodation process. So the preferred option will be the next most global form of accommodation, intermediate accommodation of the presupposition in the restrictor of the quantifier⁹:

---
⁹I am not positive whether van der Sandt would wish to term this an instance of resolution or accommodation, but resolution of the presupposition would produce identical results so the choice is not relevant to current concerns.
This DRS would be true (embeddable) in a model just in case Clothilde could not hide in the attic, ten girls of whom Clothilde was one were playing, and every one of those girls who could hide in the attic had discovered that she could. But this set of requirements is consistent. Therefore no anomaly is predicted for the discourse ending in E220 (and similarly for the discourse ending in E221). I take it that Heim's earlier but less explicitly formulated account of accommodation would produce similar results.¹⁰

These examples show that presuppositions in the scope of a quantification do not automatically trigger domain restriction, and that any theory of accommodation which says they do, is wrong. Given this, and given that some women are single, does it then follow that it is never correct to say 'Every woman regrets that she is married.'? No, it does not. Rather, the contexts in which this can be appropriately uttered are those in which marriage and the set of married women are already salient, and the previous salience of the set of married women licences domain restriction, as is argued in detail in [Bea94b]. I will not attempt to describe formally a mechanism whereby previous context could trigger domain restriction, but it is clear that the basic apparatus is present in ABLE to allow such a development. In particular, the fact that

¹⁰However, the reasoning producing those results in Heim's account might be different. Global accommodation that every girl in the given set could hide in the attic would be possible in Heim's account, except that in this case it would produce inconsistency and presumably be ruled out. Likewise, purely local accommodation would be ruled out, also on pain of producing inconsistency. But intermediate accommodation would remain a feasible option, and presumably yield similar results to van der Sandt's model if it was clear exactly what material to accommodate, this not being explicitly stated in Heim's papers.
plural referents can be generated means that it would be straightforward to add a clause to the semantics of quantificational determiners to allow quantification to be restricted to a set demarcated by some existing plural referent. Neither would this be an *ad hoc* addition to the system: it is uncontroversial that some mechanism of quantificational domain restriction is present in natural language, and that domain restriction is heavily dependent on previous context\(^{11}\).

In conclusion, ABLE offers a semantic solution to many problems that arise when quantifiers and presuppositions interact, and a solution that is to some extent independently motivated by a consideration of the semantics of epistemic modality. However, there remain many outstanding problems of both an empirical and technical nature. I have indicated that I have doubts about whether even a prediction of an existential presupposition is appropriate in the above cases. But I will finish this section by noting yet another potential problem. The reader may like to verify that in the above cases where a presupposition is bound by a quantifier, ABLE predicts not only an existential presupposition, but a presupposition that there is at least one rigid designator that has the presupposed property. Thus for E200 to be admitted, there would have to be an element of the domain of individuals such that in every world in the input that individual was married. This technical artefact has little obvious empirical support, and it is clear to me that further work is required\(^{12}\).

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\(^{11}\)Further discussion of these issues is found both in [Bea94b] and in von Fintel's recent dissertation [vF94].

\(^{12}\)In fact the slight differences in the version of this work presented in [Bea93a] mean that the system given there does not suffer from the rigid-designator problem, so clearly we should not give up hope!
The language ABLE is all very well as a means for developing intuitions about the
dynamics of determiners, presuppositions and so forth, but ABLE is only a bit like
English. The aim of this chapter is to develop a more realistic model of the process
of natural language interpretation, in particular providing answers to the following two
questions:

1. How can the meaning of English sentences be derived compositionally?
2. How does world knowledge constrain utterance interpretation?

10.1 Kinematic Montague Grammar

In Chapter 7 and Chapter 9, ABLE was set up as an independent language, which
merely happened to have its semantics defined in terms of another formal language,
namely $\text{Ty}_3$. The choice of type theory as a meta-language certainly did not enhance
readability, a fact of which the reader is presumably painfully aware, and it is probably
of little consolation that using type theory in this way did not enhance writability either.
But now we come to the pay-off.

The pay-off is that it is straightforward to embed ABLE in $\text{Ty}_3$, as a mere sub-
language, and then use the apparatus of type theory to relate expressions of ABLE to
the meanings of expressions of English. I say straightforward, but what I really mean
is that Montague did the hard work, by showing how (a modified form of) type theory
can be used to specify the semantics of natural language.

10.1.1 ABLE in $\text{Ty}_3$

In this section it will be shown how to define $\text{Ty}_3$ constants $\text{AND}$, $\text{NOT}$, $\text{IMPLIES}$,
$\text{THE}$, $\text{EVERY}$, $\text{NO}$, $\text{MOST}$, $\text{FEW}$, $\text{EXACTLY-ONE}$, $\text{IS}$, $\text{MIGHT}$ and $\text{MUST}$: the embedding of
ABLE into Ty$_3$ enriched with these constants will then be unproblematic. Consider the definitions for the semantics of the ABLE connectives, given originally in Chapter 8:

\[
[\phi \text{ AND } \psi] = \lambda I \lambda J \exists K I[\phi] K[\psi] J
\]

\[
[\text{ NOT } \phi] = \lambda I \lambda J \exists K I \downarrow [\phi] K \land \\
J = I \setminus K
\]

\[
[\phi \text{ IMPLIES } \psi] = [\text{ NOT } (\phi \text{ AND } \text{ NOT } \psi)]
\]

These definitions can be used as the basis of three Ty$_3$ constants AND, NOT, and IMPLIES, of types $\langle \pi, (\pi, \pi) \rangle$, $\langle \pi, \pi \rangle$ and $\langle \pi, (\pi, \pi) \rangle$, respectively, where AND and IMPLIES will be used in infix form. In the following postulate $\phi$ and $\psi$ are taken to be Ty$_3$ variables of type $\pi$:

**Meaning Postulate MP17**

\begin{align*}
\text{AND } &= \lambda \phi \lambda \psi \lambda I \lambda J \exists K I\{\phi\} K\{\psi\} J \\
\text{NOT } &= \lambda \phi \lambda I \lambda J \exists K I\{\downarrow \phi\} K \land \\
&\quad J = I \setminus K \\
\text{IMPLIES } &= \lambda \phi \lambda \psi [\text{ NOT } (\phi \text{ AND } \text{ NOT } \psi)]
\end{align*}

The general recipe for producing an appropriate Ty$_3$ constant from an ABLE semantic clause runs as follows: firstly wipe out all the semantic brackets (but replacing formulae $[\phi]_D$ with distribute $\phi.D$), and then remove the argument expressions one at a time from left to right in the definiendum, adding them from right to left together with a preceding lambda at the beginning of the definiens. Rather than giving the by now obvious definitions for the remaining constants, the reader is referred to appendix B.

Given the enrichment of Ty$_3$ with these constants, it can be seen that any sentence of ABLE is also a sentence of Ty$_3$. Furthermore, since the postulates restricting the interpretation of the Ty$_3$ constants are based on the same definitions as for the original ABLE semantics, it is clear that for any formula, $\phi$, in the ABLE fragment of Ty$_3$, the embedding function $[.]$ will map $\phi$ onto a Ty$_3$ formula which is Ty$_3$-equivalent to $\phi$. That is, if $[\phi] = \psi$, then $\psi \equiv_{\text{Ty}_3} \phi$. 

\"
10.1.2 Indexed English in Ty₃

It will now be shown how indexed trees of English can be given a direct interpretation in type theory. Trees may be represented as bracketed strings, such that E222b is a tree for the indexed English discourse in E222a:

E222 a. A₁ woman realises John₂ owns a₃ donkey. She₁ is angry.

b. (((((a 1) woman)(realises ((John 2) (owns ((a 3) donkey)))))))₀
((she 1) is-angry)

[t]SOME.₁.(WOMANu.₁)(REALISESa.₁.(some.₁.(named-Johnu.₁).(some.₁.(donkeyu.₁).(owns₂.₁.₀))))
AND (she.₁.(is-angry².₁.₁))

Note that E222b is the tree not for a sentence of indexed English, but for a discourse, the ₀ representing a sentence sequencing operator. If words of English are defined to be constants of the right types, and if some simple notation conventions are used, such trees become formulae of type theory. We will now see how such constants can be defined, and, moreover, in a way which yields intuitively reasonable meanings — at least intuitive for those who accept that the meaning of E222a is given by the ABLE formula E222c.

Firstly, let us say that simple nouns and intransitive verbs correspond directly to the dynamic unary predicate constants that were used for ABLE’s semantics, which were constants of type \(\langle d, \pi\rangle\):

**Meaning Postulate MP18**

\[
\begin{align*}
\text{woman} &= WOMANu \\
\text{donkey} &= DONKEYu \\
\text{is-angry} &= IS-ANGRYu
\end{align*}
\]

Propositional verbs, of course, correspond to ABLE’s dynamic attitude predicate constants, except that the ABLE attitudes combined firstly with the subject marker and then with the propositional argument, whereas it will be more natural here to assume that an attitudinal predicate combines firstly with a proposition to form a verb phrase. Thus an argument swap is required:

**Meaning Postulate MP19**

\[
\text{realises} = \lambda F \lambda D [\text{REALISESa.}_D.F]
\]
In ABLE, determiners combined with a discourse marker and two propositions to form a proposition. Clearly a determiner of indexed English ought to combine with a discourse marker and two dynamic properties. Variables $P$ and $P'$, of type $\rho = (d, \pi)$, will be used to range over dynamic properties, of which dynamic unary predicates are examples.

**Meaning Postulate MP20**

\[
a = \lambda D \lambda P \lambda P' \left[ \text{SOME}.D.(P.D).(P'.D) \right]
\]

\[
\text{the} = \lambda D \lambda P \lambda P' \left[ \text{THE}.D.(P.D).(P'.D) \right]
\]

Noun phrases now correspond to Ty$_3$ formulae of type $\langle \rho, \pi \rangle$, and variables $Q$ and $Q'$ will range over objects of this type. For instance $a.1.woman$ is a formula of type $\langle \rho, \pi \rangle$. A transitive verb should combine with an noun phrase to form a verb phrase, so transitive verbs must have the type $\langle \langle \rho, \pi \rangle, \rho \rangle$. This means that in order to define constants for transitive verbs in terms of ABLE’s dynamic binary predicate constants, which have type $\langle d, \rho \rangle$, a little type raising is necessary:

**Meaning Postulate MP21**

\[
\text{owns} = \lambda Q \lambda D \left[ Q.(\text{owns}^b.D) \right]
\]

The sequencing operator $\circ$ is to be interpreted as conjunction:

**Meaning Postulate MP22**

\[
\circ = \text{AND}
\]

All that remains for the above example are pronouns and proper names. As with ABLE, these may be interpreted in terms of the determiner the:

**Meaning Postulate MP23**

\[
\text{john} = \lambda D \left[ \text{the}.D.\text{NAMED-JOHN}^a \right]
\]

\[
\text{she} = \lambda D \left[ \text{the}.D.(\lambda D \left[ \text{SINGULAR}^a.D \text{ AND FEMALE}^a.D \right]) \right]
\]

Let us adopt the following notation conventions for Ty$_3$:

**Reversability:** If $X$ is of type $\alpha$ and $Y$ is of type $\langle \alpha, \beta \rangle$ then $(XY) = (YX) = Y.X$

**Parsimony:** For any Ty$_3$ expressions $X$, $Y$ and $Z$, if $X(YZ)$ is a sentence of type theory, and $(XY)Z$ is not, then $XYZ$ will be understood as equivalent to $X(YZ)$,
and vice versa if \((XY)Z\) is a sentence of type theory but \(X(YZ)\) not. If neither or both of these bracketings are sentences of type theory, then the unbracketed string will not be understood as a sentence of type theory.

**Indexing:** If \(X\) is of type \((d,\alpha)\) and \(Y\) is of type \(d\), then \(X_Y = X.Y\).

Given the first of these conventions, \(E222b\) can be taken as a well-formed sentence of type theory. More importantly, given the above postulates, we have that \(E222b = E222c\). Here is a small part of the proof, the rest being left up to the reader:

\[
\begin{align*}
a &= \lambda \Delta \lambda \Delta' \lambda [\text{some}.D.\Delta.\Delta'.D]\\\
(a 3) &= \lambda \Delta \lambda' [\text{some}.3.\Delta.3.]\Delta'.3]\\\
donkey &= \text{DONKEY}^a\\\n((a 3)\text{ donkey}) &= \lambda \Delta' [\text{some}.3.(\text{DONKEY}^a.3).3]\\\owns &= \lambda \Omega \Delta D [\Omega.(\text{owns}^b.D)]\\\n(owns ((a 3)\text{ donkey})) &= \lambda \Omega \Delta D [\Omega.(\text{owns}^b.D)]\\\n(john) &= \lambda D [\text{the}.D.\text{NAMED-JOHN}^a]\\\nthe &= \lambda \Delta \lambda \Delta' \lambda [\text{the}.D.\Delta.\Delta'.D]\\\n(john 2) &= \lambda \Delta' [\text{the}.2.(\text{NAMED-JOHN}^a.2).3]\Delta'.2]\\\n((john 2)\text{ (owns ((a 3)\text{ donkey}))}) &= \lambda \Delta' [\text{the}.2.(\text{NAMED-JOHN}^a.2).3].\Delta'.2]\\\n(john 2)\text{ (owns ((a 3)\text{ donkey}))}) &= \text{THE}.2.(\text{NAMED-JOHN}^a.2).\text{some}.3.(\text{DONKEY}^a.3).3\text{.owns}^b.2.3)\]
\end{align*}
\]

The last two of the above notation conventions allow even more English-like structures to be interpreted in type theory. The convention of **parsimony** means that brackets are only needed where they disambiguate, and the **indexing** convention allows indices to be subscripted. Given these conventions, "(a₁ woman realises John₂ owns a₃ donkey 0) she₁ is-angry", for example, becomes a Ty₃ formula.

Next I will consider a classic example from the presupposition literature:

\(E223\) a. **Somebody managed to succeed George V on the throne of England.**
The infamous E223a appeared in an endnote to Karttunen and Peter's [KP79]. It will be recalled from part one of this thesis that the basis of Karttunen and Peters' extension to Montague Grammar is the division of meaning into two components, which for current purposes can be labelled as **assertive** and **presuppositional**, with rules of composition being separately specified for each component.

The fact that assertions and presuppositions are specified independently means that the ordinary assertive meaning of an expression places no constraints on what the presupposition might be, so that at first sight their approach appears to be of great generality. However, it is this very independence of assertion and presupposition which ultimately must force us to reject such an approach, for it makes it impossible to specify scope and binding relations between the assertion and presupposition. This is essentially what Karttunen and Peters observed with respect to E223a. They assume that the control verb 'manage' carries a presupposition (or conventional implicature) to the effect that the subject of the verb has difficulty in achieving whatever is specified by the verbs infinitival complement. Thus, for example, 'I managed to complete my thesis' should presuppose that I had difficulty in doing so. That 'manage' displays classically presuppositional behaviour can be seen from the fact that the 'difficulty' inference tends to survive from certain embedded contexts. For instance 'I might manage to complete my thesis' still seems to suggest that I would find completion difficult. Karttunen and Peters argue that it is natural to infer from E223a that the person who succeeded George V had difficulty in doing so, and that it is the fact that the successor did not in fact have any difficulty succeeding which makes the sentence seem odd to those with a little historical knowledge. Thus it seems that it is presupposed that some individual had difficulty, and asserted of the same individual that he eventually succeeded, so that there must be some binding relation between the presupposition and the assertion. To repeat: such a binding relation cannot obtain in Karttunen and Peters' system, which yields a completely independent presupposition that somebody had difficulty George V, and assertion that somebody succeeded George V. As Karttunen and Peters observe, there are many people who would have found it difficult to succeed George V, so that the predicted presupposition is trivially satisfied, and no account of the example's oddity is provided.

In this thesis I have attempted to develop an integrated account of presuppositional and assertive aspects of meaning, indeed even arguing that much of the peculiar behaviour of presupposition can be predicted from the study of extraneous semantic phenomena. Within such an integrated account, it is possible to describe binding relations between presuppositions and assertions, or *vice versa*. Let us consider a translation into

---

1C.f. the discussion of 'Somebody curtsied' in §3.2, which was shown to be problematic for Karttunen and Peters in precisely the same way as the George V example discussed above.
type theory of (the relevant aspects of) E223a:

E223 b. Somebody \textit{managed to succeed} George-V

Of course, this does not look much like a formula of type theory, but if the right constants are defined, then, according to the above notational conventions, it will be. There is a minor problem in treating this example, in that ABLE does not contain the equivalent of control verbs, although there is no obvious reason why ABLE should not be extended in this respect. For the moment it will suffice to make the crass assumption that control verbs are interpreted in terms of underlying dynamic attitude predicates. The following postulate restricts the denotation of an ABLE attitude predicate \textsc{managed} such that \textsc{managed}.D.F can only provide an update if it is established that the proposition F 'is problematic for' the individual represented by D, in which case the output is calculated by simply updating with the proposition F.

\textbf{Meaning Postulate MP24}

\begin{equation}
\forall V, \forall D, \forall I \{ \text{managed}^a. D, F \} \leftrightarrow \\
t-domain. I, D \land I \text{satisfies} \text{problematic-for}^a. D, F \land I \{ F \} J
\end{equation}

The definition of the constants appearing in E223b is now straightforward. The noun phrase\textit{somebody}, which has the same type as for a name or pronoun, is defined in terms of the ABLE determiner \textsc{some} and a unary predicate \textsc{person}^a, such that \textit{somebody} is equivalent to a person:

\textbf{Meaning Postulate MP25}

\begin{equation}
\textit{somebody} = \lambda P \lambda D [ \text{some}. D, (\text{person}^u. D), (\mathcal{P}. D)]
\end{equation}

The constants \textit{George-V}, \textit{succeed} and \textit{to} do not require much thinking about, names and transitive verbs having been dealt with above, and \textit{to} being assumed semantically trivial:

\textbf{Meaning Postulate MP26}

\begin{equation}
\textit{George-V} = \lambda D \lambda P [ \text{the}. D, (\text{named-george-v}^u. D), (\mathcal{P}. D)] \\
\textit{to} = \lambda P [P] \\
\textit{succeed} = \lambda Q \lambda D [Q. (\text{succeed}^b. D)]
\end{equation}

Finally, \textit{managed} of type (\rho, \rho) is defined in terms of the attitude predicate \textsc{managed}^a, so that for any marker D and dynamic property \mathcal{P}, \textsc{managed}.D.\mathcal{P} defines the same CCP as \textsc{managed}^a. D, (\mathcal{P}. D):
Meaning Postulate MP27

\[ \text{managed} = \lambda \varphi \lambda d \text{[managed}^a . D . (\mathcal{P} . D) \text{]} \]

Given these postulates, it is easily verified that the formulae in E223b and E223c denote the same CCP:

E223 c.  \text{SOME.6.(PERSON}^a.6). (\text{managed}^a.6. )

\((\text{THE.5.(NAMED-GEORGE-V}^a.5). (\text{SUCCEED}^6.6.5))\)

ABLE formulae of this form were discussed in §9.3, and, on the basis of that discussion, it is clear that E223c will (only) be admitted by contexts in which at least one individual is established to find his succession to the throne problematic, and it will provide an update to a context where such an individual actually has succeeded George V

Whilst the above ad hoc translations of indexed English discourses into type theory serve for illustrative purposes, it is, at least on methodological grounds, more useful to specify general procedures for the interpretation process. This will make the theory easier to extend or attack, according to taste. Here, then, is an official definition of the syntax of the language of indexed, bracketed English (Indexed Bracklish):

\textbf{Definition D66 (Syntax of Indexed Bracklish)} Given sets of names, nouns, intransitive, transitive and attitude verbs, occupying the categories Name, N, IV, TV, and AV respectively, and that the category i consists of the integers, the language of indexed bracklish is defined by the following rewrite rules:

\(^2\text{In fact I have reservations about the basic analysis of 'managed' advocated by Karttunen and Peters, and have used more or less the same analysis simply in order to bring out the difference in the binding mechanisms of the two theories. The point that the Karttunen and Peters theory does not yield an adequate account of binding could have been made with the Modigliani ownership examples which I considered only, and I refrained from doing this only because they themselves do not consider these examples. My own preference would be for analysing the presuppositions of 'managed' not in terms of a simple proposition about the subject having had difficulty, but in terms of the hearer's expectations. However, I will not attempt to justify this here.}\)
Below I present a schema for defining constants for the terminal symbols of a language of Indexed Bracklish: all underlined expressions are to be understood as replaceable by constants representing elements of the relevant categories.

**Definition D67 (Schema for constants of Indexed Bracklish)**

\[
\begin{align*}
n &= N^u \\
iv &= IV^u \\
tv &= \lambda Q \lambda D \left[ Q(TV^u) \right] \\
av &= \lambda F \lambda D \left[ ATTITUDE^u.D.F \right] \\
det &= \lambda D \lambda P \lambda P' \left[ DET.D.(P.D).(P'.D) \right] \\
name &= \lambda D \lambda P \left[ THE.D.(NAMED-NAME^u.D).(P.D) \right] \\
pronoun &= \lambda D \lambda P \left[ THE.D.(CONDITIONS ON D^u).(P.D) \right] \\
who &= \lambda P \lambda P' \lambda D \left[ P.D AND P'.D \right] \\
might &= \lambda P \lambda D \left[ MIGHT.(P.D) \right] \\
perhaps &= \lambda F \left[ MIGHT.F \right] \\
and &= \lambda F \lambda F' \left[ F AND F' \right] \\
o &= \lambda F \lambda F' \left[ F AND F' \right] \\
if &= \lambda F \lambda F' \left[ F IMPLIES F' \right] \\
do(es)n't &= \lambda P \lambda Q \left[ NOT(Q.P) \right]
\end{align*}
\]

The only definitions introduced which are not of types familiar from the earlier example derivations, are those for `who`, `might`, `perhaps`, `don't` and `doesn't`. The first of these, `who`, is defined as the conjunction of two properties. The reason for including both `perhaps` and `might` is one of convenience, in that the discussion of epistemic modality in §9.2 involves both examples where the subject noun phrase appears within the scope of a modality, and examples where the subject takes scope over the modality. It seems reasonable to assume that the sentential operator `perhaps` always takes wide scope, and this is reflected in the above definition. However, whereas the 'might' of English can appear in either scope configuration, I have assumed here that `might` is semantically
just a verb phrase modifier, so that the subject takes wide scope. The reader may like to verify that the indexed bracklish sentence \(((\textit{most 1 politicians}) \ (\textit{might be-spying}))\) (cf. E211) is given the meaning discussed in §9.2. In contrast, the constants don’t and doesn’t are both defined so as to take wide scope over the subject noun phrase in the interpretation of Indexed Bracklish. There is no fundamental reason why might should not be defined similarly, such that it also took wide scope. However, to extend the interpretation of Indexed Bracklish with a systematic way of accounting for such scope ambiguities, for instance using a type polymorphism approach as developed by Emms [Em90] or Hendriks [Hen89], would take us beyond the scope of this thesis.

10.2 A Plea for Common Sense

10.2.1 Mutual Ignorance

What is the common ground? What, for example, is the common ground between you and me? To answer this question, you need to know something about my beliefs. But what do you know about my beliefs? Can you specify even one proposition which you are sure I believe in?³

In short, no participant in a conversation knows what the common ground is. This truism ought to be rather disturbing to those who would associate the information states of dynamic semantics with the common ground of the conversational participants, a conflation which I always found attractive. But if states are identified with the common ground, and nobody knows what the common ground is, then the states could not be states of anybody, and a state which is not a state of something or someone, is not of much use to anything or anyone.

Of course, the intuition that information states be associated with some notion of common ground need not force us to accept that the relationship is one of identity. In what follows, I will try to show that once the proper relationship between information states and the common ground is established, the very intangibility and unknowability of the common ground can become a source of inspiration rather than despair.

Consider the case of a doctoral thesis: it is doubtful that there is any determinate notion of a common ground between author and readers. Unless the examiners recommend that every copy be burnt, the thesis will remain for an indefinite number of years in a dusty university archive. There always remains the possibility that somebody will accidentally stumble across the thesis, and begin to read. But if the common ground is not only unknowable by the author, but arguably completely indeterminate, then the candidate, as with any author, is forced to make assumptions about the knowledge of

³A tautology, perhaps? As it happens, I never accept tautologies, although I am quite partial to contradictions. You too? Well, then we certainly have a lot in common.
the eventual readers. These assumptions may even end up partially determining who
the readership is. Thus, although the real common ground is indeterminate, an oper-
ational definition could be given: the common ground is (modulo rewrite requests of the
examiners) whatever the candidate decides it will be.

Unfortunately, this operational definition is of only limited assistance to the reader
of a doctoral thesis, for, as has already been established, the reader knows nothing
about the author with any certainty. But the operational definition does tell us about
the nature of the problem facing the readers: the readers do not know what the author
assumed about the common ground, but they know that if they had information cor-
responding to that assumed common ground, then they could safely update this state
with the content of the thesis. In effect, readers are forced to reason abductively from
the presuppositions of the text to calculate the assumptions of the writer.

Those familiar with Stalnaker's work (e.g. [St74]) will recognise something like his
theory of pragmatic presupposition in my description up to this point of the difficulties
facing those writing and reading theses — my operational definition of the common
ground coincides with his definition of speaker's presuppositions. Now, however, I want to
ask a question that will take us beyond Stalnaker's account. Given that the readers never
know what information the author assumed them to have before they started reading,
how can we represent the information that a reader has after reading an arbitrary initial
chunk of thesis?

In terms of KMG, the obvious answer would run as follows. The relevant aspect of a
reader's information state at any point is an object of type \( t \), such that at the beginning
of the thesis the reader has information corresponding to the minimal state \( T \), and later
states are obtained by updating this state successively with the content of the thesis.
Such an answer is clearly naive, for as soon as the reader reached a presupposition which
had not been previously justified in the text, the rules of KMG would give no indication
as to how to continue updating. Could the reader simply have started with a more
informed state, and updated that? Yes, of course, but which one? Unfortunately, no
single information state is appropriate, if the term information state is taken simply
to mean the objects of type \( t \) introduced in the previous chapters. An example should
clarify the point.

Suppose that as a reader you do not know whether it was Montague or Kahlish, or
perhaps even Carnap who first started using the term pragmatics to describe what is
now commonly called indexical semantics. But you guess that the author of the thesis
will know which it was, and furthermore is quite likely to take for granted that readers
also know. Thus, as far as you know, the author may choose to use the presuppositional
phrase 'the well-known fact that Carnap first used the term 'pragmatics' to refer to what
we now call indexical semantics'. If you had simply assumed that the author's model
of the common ground involved Carnap having made the introduction, then you could safely update with a sentence containing this phrase. On the other hand, you know that the author may use the same phrase with 'Kahlish' or 'Montague' substituted for 'Carnap'. If you happen to choose the right assumption you will have no problem, but \( \nu \)-typed information states leave no room for sitting on the fence. For if you simply choose to begin reading the thesis with a disjunctive state in which it is not established who actually was responsible, then as soon as you come across the use of one of the above presuppositional phrases you will be sunk: your information state will not satisfy the presupposition, and, at least according to the theory of the last few chapters, there will be no way to continue updating.

Thus we are faced with a conundrum. If we interpret an information state realistically, in terms of the information the reader might conceivably have about the writer, then KMG will often fail to provide an update when it clearly should. On the other hand, interpreting information states as the assumed common ground of the writer is of no use in providing a realistic model of how a reader's information state develops, since a reader cannot know in advance what the writer has assumed. Conclusion? The notion of updating developed in this thesis so far is incorrect. Could the definition of an information state be preserved, but an alternative notion of update be used? If so, then the new notion will have to be defined with great caution, for I have gone to considerable lengths in this thesis to motivate a theory of information independently of presuppositional data. Thus, there is little leeway for making major changes to the relations between contexts defined by English sentences, without actually reformulating the definition of a context. Neither is it possible simply to weaken the constraints on the incoming context due to presuppositional constructions. For instance, it would not be appropriate to alter the definition of factivity such that the incoming context was only constrained to be consistent with the presupposition rather than to satisfy it, whilst making the output of a factive verb satisfy the presupposition. This would yield the right entailment patterns for unembedded factives, but would destroy projection properties: the negation of a factive would no longer entail the truth of the factive complement.

### 10.2.2 Information Sets

The Carnap/Kahlish/Montague story above is suggestive. Suppose that you maintained in parallel a number of \( \nu \)-typed states, such that in some Carnap redefined pragmatics, in others it was Kahlish or Montague, and perhaps in some it was not established that any of them redefined anything. Then you could try updating each state separately with the content of the thesis, and see which structures survived. That way, whatever assumptions the author had made concerning the redefinition of pragmatics, at least one state would satisfy the assumptions, and would thus produce an output. This idea...
is easily formalised. If states are to be maintained in parallel, then updating must be framed in terms of sets of states, where each state in the set corresponds to a possibly correct model of the speaker's assumptions about the common ground:

**Definition D68 (Information Sets)** An information set is any object of type \((\iota, \tau)\), and variables \(S, T\) will range over objects of this type.

Now a new set-update operator, \(+\), can be defined. This operator maps information sets onto functions from CCPs to information sets, and is thus of type \(\langle \langle \iota, \tau \rangle, \langle \pi, \iota, \tau \rangle \rangle\). If \(S\) is an initial information set, then the set-update of \(S\) with a CCP \(\phi\) is denoted by \(S + \phi\), which is simply the set of outputs obtained by updating (in the KMG sense) the elements of the set \(S\) with \(\phi\):

**Meaning Postulate MP28**

\[ + = \lambda S \lambda \phi \lambda I \exists J \ [S.J \land J(\phi)I] \]

To make sense of information sets, it helps to relate them to familiar logical notions. The following postulate defines a constant \(\text{supports}\). Support is an analogous notion to satisfaction, but defined at the level of information sets rather than states:

**Meaning Postulate MP29**

\[ \text{supports} = \lambda S \lambda \phi \ [\forall I \ S.I \rightarrow I \text{ satisfies } \phi] \]

In terms of \(\text{supports}\), a notion of entailment can be defined. The following fact shows that defining \(\phi\) to entail \(\psi\) as “updating the minimal information set with \(\phi\) yields an information set supporting \(\psi\)” just yields the earlier definition of entailment from Chapter 8.\(^4\)

**Fact F16 (Support-based Entailment)** If \(\phi\) and \(\psi\) are eliminative CCPs, then:

\[ \phi \models \psi \iff (\lambda I [T] + \phi) \text{ supports } \psi \]

*Proof*: By the definition of \(\text{supports}\) and \(+\), \((\lambda I [T]+\phi) \text{ supports } \psi \iff \text{ every possible output of } \phi \text{ satisfies } \psi\), but this is just the definition of entailment in MP5.

As it happens yet another standard dynamic notion of entailment in terms of information sets reduces to the same old definition:

\(^4\)Naturally, what I am calling a minimal information set is in another sense maximal, in that it contains every possible information state.
Fact F17 (Another support-based Entailment) If $\phi$ and $\psi$ are eliminative CCPs, then:

$$\phi \models \psi \iff \forall S \forall S' \ S + \phi = S' \rightarrow S' \text{ supports } \psi$$

Proof: The definition of $+$ shows that update is distributive across the component states of an information set (i.e. if $S$ is the union of $S_1, \ldots, S_n$ then $S + \phi$ is the union of $S_1 + \phi, \ldots S_n + \phi$) so that it suffices to consider singleton information sets. But for these the requirement $\forall S \forall S' \ S + \phi = S' \rightarrow S' \text{ supports } \psi$ reduces to $\forall I \forall I' \ I(\{\phi\}) I' \rightarrow I' \text{ satisfies } \psi$, which again is just the earlier definition of entailment in MP5\(^5\).

Since lifting the semantics up to the level of information sets has no effect on entailment, it is clear that the presupposition projection properties established in Chapter 9 will be maintained. The only difference is that whereas at the level of states, presuppositions can cause undefinedness, at the level of information sets a CCP always yields an update. In fact, F16 combined with the fact that (non-modal) presuppositions of CCPs are also entailments, shows that updating the minimal information set with a CCP which presupposes some (non-modal) proposition will always yield a set which supports that proposition.

Let us see how information sets help with a simple example, based on the earlier 'who redefined pragmatics?' story. Firstly, here are some constants of indexed bracklish and some relevant states:

- All constants are defined using the schema in D67, *carnap*, *montague* and 'pragmatics' as names, redefine=redefined as a TV, realised-that as a factive AV, and didn't using the same semantics as for doesn't.

- $A$ satisfies ($carnap_1 \ \text{redefined 'pragmatics'}_2$)
- $A$ satisfies ($montague_3 \ \text{didn't redefine 'pragmatics'}_2$)
- $B$ satisfies ($montague_3 \ \text{redefined 'pragmatics'}_2$)
- $B$ satisfies ($carnap_1 \ \text{didn't redefine 'pragmatics'}_2$)
- $C = A \cup B$
- $A\{montague_3 \ \text{didn't realise-that carnap}_1 \ \text{redefined 'pragmatics'}_2\}A'$

\(^5\)Both of these support based notions of entailment correspond to definitions proposed by Veltman [Ve91], who also observes that they collapse for distributive systems.
Neither $B$ nor $C$ admits the proposition $montague_3$ didn't realise-that $carnap_1$ redefined 'pragmatics'$_2$. However, representing an information set containing lotas $I, J, \ldots$ as $(I, J, \ldots)$, we have the following update:

$$(A, B, C) + montague_3$ didn't realise-that $carnap_1$ redefined 'pragmatics'$_2 = (A')$$

Thus an information state which does not support the proposition that Carnap redefined pragmatics can still be updated with a sentence in which the proposition is presupposed. More generally, by being realistic about what a hearer knows of the speaker's assumptions about the common ground, an obvious difficulty with the CCP model has been resolved. Once it is recognised that the states of ABLE and KMG semantics are not to be identified with the common ground, but are to be thought of as possible models of the common ground, the awkward fact that presuppositions can be informative becomes unproblematic.

10.2.3 Information Orderings

Information sets provide a model of an agent's uncertain knowledge of the common ground, but there is good reason to consider more sophisticated models. It is intuitively clear that not all assumptions that a writer/speaker might make are equally plausible. For instance, it is more plausible that Carnap redefined pragmatics than that Carnap's dog redefined pragmatics, and it seems reasonable that a reader's model of the writer's assumptions about the common ground should reflect this difference. But it is not appropriate merely to assume that some states are in the context and some not, since there is no obvious place to draw the line. If it is implausible that Carnap's dog redefined pragmatics, is it plausible that Carnap's grandmother redefined pragmatics?

One can imagine various more structured notions of context than simple sets of states that might be used to encode the difference in plausibility of different assumptions. For example we might consider adding a type $p$ of probabilities to the type theoretic setup, and defining a context to be a function of type $(t, p)$ — something like a vague set. The more plausible states would be mapped onto higher probabilities. Such a notion of context would allow very fine-grained distinctions between states to be made: in fact, although the vague set analysis seems intuitive to me, I do not have applications in mind which would require quite such a fine grain. It will suffice if a context provides a measure of the relative plausibility of different states, such as to answer to questions like "is state $I$ more plausible than state $J". In other words, what is needed is some sort of ordering over states:
Definition D69 (Information Ordering) Any object $O$ of type $\pi$ which has the property of transitivity is an information ordering. An initial information ordering is additionally reflexive.

**Reflexivity:** $\forall I \quad I \{O\} I$

**Transitivity:** $\forall I \forall J \forall K \ (I \{O\} J \land J \{O\} K) \rightarrow I \{O\} K$

An information ordering $O$ is a set of pairs of states, such that if a pair of states $I$ and $J$ is in the ordering relation, represented as $I \{O\} J$, then both $I$ and $J$ could correspond to the generator's assumptions, and $I$ is at least as plausible as $J$. Note that information orderings have the same type as CCPs, which are also relations between states. However, orderings and KMG CCPs are quite different slices of $\pi$. For instance, whereas KMG CCPs are never transitive relations, orderings are by definition always transitive, and whilst the only pairs in the denotation of KMG CCPs are those in which the output is more informative than the input, there is no corresponding restriction on information orderings.

The use of orderings to represent default information of various kinds is becoming well-established, as in the work of Veltman [Ve91], although Veltman's expectation frames involve orderings over possible worlds rather than over $t$-typed states. It is clear that there must be many more states than there are worlds, and thus that information orderings provide a more fine-grained notion of preference than expectation frames. Indeed, it is clear that any ordering over worlds can be expressed using an ordering over states, if we simply associate the sub-part of an information ordering containing only single-world states with the corresponding expectation frame. I will not make any attempt to account for how preference orderings over states are formed, but simply take as given that the common sense of an agent provides him or her with such an ordering. It is clear that an underlying ordering over worlds could be used to induce a (very partial) ordering over states, and thus that mechanisms like those discussed by Veltman to provide orderings over worlds could be reinterpreted so as to generate information orderings. We could then add some additional general preference criteria, like specifying that if two states differ only by one having a larger domain of discourse markers than the other, then the smaller-domained state is a priori more plausible.

Updating can be defined along similar lines to the information-set based definition, above, using a constant $*$, of type $(\pi, \langle \pi, \pi \rangle)$. The update operation proceeds by taking all pairs of states in the denotation of the initial ordering, and trying to update each element of the pair with the CCP. If both states can be successfully updated, then the resulting states are paired in the output ordering. Thus if one state is initially at least as plausible as another, and both states admit the CCP, then in the final ordering the output from the first state will be at least as plausible as the output from the second.
Meaning Postulate MP30

\[ * = \lambda O \lambda \phi \lambda I \lambda J \left[ \exists I' \exists J' \left( I\{O\}J' \land I'\{\phi\}I \land J'\{\phi\}J \right) \right] \]

A reader who had forgotten or never knew who redefined pragmatics, and was taken in by my story, might now be surprised by the fact that it was Bar Hillel who first defined pragmatics to be indexical semantics. Yet even if you were surprised by this fact, or surprised that I knew it, the surprise does not mean that you had any difficulty updating with the previous sentence. This is what would be expected if the structures which you updated were like information orderings, and the mechanism of update was something like the * operation: even apparently unlikely alternatives should be represented somewhere in an information ordering, albeit rather low down.

However, orderings need not contain every state, in the sense that for some states I it will neither be the case that \( \exists J I\{O\}J \) nor that \( \exists J J\{O\}I \). In particular, an ordering produced by updating with a CCP will only contain states which are possible outputs of the CCP, and it need not even contain all of those. Thus, an information ordering registers two kinds of information. Firstly it says which states are possible models of the speaker’s assumptions, and secondly it says what the preferences are amongst those states. Since an initial ordering is reflexive, which means that every state is at least as plausible as itself, it is clear that every state will be contained in an initial ordering. The set of states which a non-initial ordering registers as being possible models of the speaker’s assumptions will be precisely those which the ordering says are at least as plausible as themselves:

Meaning Postulate MP31

\[ \text{contains} = \lambda O \lambda I \left[ I\{O\}I \right] \]

The constant contains relates information orderings to information sets: if O is an information ordering, then \( \text{contains}.O \) is an information set. Thus contains can be used to apply the above definition of support to information orderings: an ordering O supports a CCP \( \phi \) iff \( (\text{contains}.O) \text{supports} \phi \). Clearly this possibility combined with F16 and F17 shows that some natural notions of entailment in terms of information orderings would correspond to the original definition in terms of \( \iota \)-typed states. It is also clear that the potential informativeness of presuppositions is accounted for in just the same way as with information sets. But, as will be demonstrated shortly, there is more to the processing of presuppositions than can be naturally explained in terms of strict entailment, and this is where the additional structure of information orderings comes into play.

In an information ordering, there are some optimal states, in the sense that they
are at least as plausible as all the other states which the ordering contains. Sometimes it will be the case that the optimal states are also minimal, by which I mean that all the non-preferred states are extensions of a preferred state. In this case, the set of propositions supported by the set of optimal states will be just the same as the set of propositions supported by the entire set of states which the ordering contains. However, it may also be the case that the optimal states are non-minimal. In that case, there will be propositions supported by the optimal states that are not supported by the ordering as a whole. If updating some ordering $O$ with a CCP $\phi$ leads to an ordering in which the set of optimal states supports another CCP $\psi$, then I will say that relative to $O$, $\phi$ implicates $\psi$. The formal definitions of optimal and implicates are straightforward:

**Meaning Postulate MP32**

\[
\text{optimal} = \lambda O I \left[ \text{contains} \cdot O I \land \forall J \left[ J \{O\} I \rightarrow I \{O\} J \right] \right]
\]

\[
\text{implicates} = \lambda O \lambda \phi \lambda \psi \left[ \text{optimal} \left( O \ast \phi \right) \text{ supports } \psi \right]
\]

Let us consider a case where strict entailment seems too weak:

**E52** If I go to London, my sister will pick me up at the airport.

As shown in §9.1, in ABLE (and the same clearly holds for KMG) presuppositions in the consequent of a conditional yield conditional presuppositions. Thus the CCP corresponding to E52 presupposes that if I go to London then I will have a sister. The occurrence of these weak, conditionalised presuppositions is one of the most strongly criticised aspects of the CCP model (see e.g. [Gaz79a]). It seems intuitively obvious that somebody hearing E52 would conclude not only that if the speaker were to go to London he would have a sister, but that the speaker actually has a sister. In terms of information orderings, this inference pattern might lead us to come to a certain conclusion about a hearer’s typical information ordering. For the inference to go through, some states in which it was definitely established that the speaker had a sister would have to be more plausible than all states in which the existence of a sister was conditionalised on the speaker’s journeying to London. Here, by saying that one state is more plausible than another, I mean that the first is at least as plausible as the second, but not vice versa. Relative to such an ordering, E52 would implicate that the speaker had a sister, since updating the ordering with E52 would lead to an ordering in which the optimal states all satisfied the existence of a sister.

But why should it be the case that states in which the speaker has a sister are more plausible than certain states in which this proposition is conditionalised? This is a difficult question to answer completely, but I can imagine some potential lines of explanation. In the first place, note that although the CCP corresponding to E52 will
only conditionally presuppose the existence of a sister, it will unconditionally presuppose
the presence of a discourse marker in the input state. The input state will admit the
CCP just in case in the world-sequence pairs where the speaker goes to London, the
discourse marker is constrained to be the speaker's sister. Suppose that the speaker's
immediate family were generally assumed to correspond to rigid designators, and suppose
further that there were a general principle to the effect that states where assumed
discourse markers corresponded to rigid designators were more plausible than those
where assumed markers are non-rigid. In that case, of all the states which contained
a marker that corresponded to the speaker's sister in any world-sequence pair, those
in which the marker corresponded to the speaker's sister in every world-sequence pair
would be most plausible, and this would lead to the observed inference pattern.

Whether or not the reader finds this convincing, it should be clear what I am aiming
for: I would like to justify the particular inference pattern relevant to E52 in terms of
more general principles. In fact, I think that the above markers-are-rigid line of reasoning
is still not general enough, for there are many cases not involving the assumption of a
new marker but where KMG's prediction of a conditional presupposition seems too
weak. Consider the following example (repeated):

E150 If Charles turns up, then everybody will be amazed that both Charles and Diana are
here.

Any state in which it is established that if Charles turns up then both Charles and
Diana will be here would admit the above sentence. However, in a situation where it is
known that Charles and Diana generally try to avoid each other, there is a clear tendency
to come to a stronger conclusion on hearing E150, namely that Diana is already here. For
this result to be predicted, it would have to be the case that a state where Diana's
presence is established is more plausible than every state where Diana's presence is
conditional on Charles' arrival. The source of such orderings must be connected with
the nature of explanation and justification, in particular the fact that if Charles and
Diana habitually avoid each other, then Charles' arrival does not help explain Diana's
presence. On the other hand, it is clear that Charles' arrival does help explain (or
even entails) Charles' presence, so it is not surprising that E150 does not implicate that
Charles is definitely here.

When reading each of the following two examples, ask yourself whether there actually
is any more hot water:

E224 If Jane takes a bath, Bill will be annoyed that there is no more hot water.

\[\text{\footnote{C.f. footnote 14 concerning the role of intonation in this example, concerning which I will not present an analysis.}}\]
E225 If Jane wants a bath, Bill will be annoyed that there is no more hot water.

There is a clear contrast between these two examples. An utterance of E224 does not suggest that there actually is no more hot water, but only that if Jane takes a bath, there will be no more hot water. On the other hand, E225 suggests strongly to me that there is no more hot water. Put another way, E224 is compatible with the standard CCP prediction of a conditional reading, E225 has an interpretation (for me it is the favoured interpretation) which departs from that prediction. It is clear that there exist information orderings that would lead to precisely these predictions. Any ordering satisfying the following two conditions would suffice:

1. At least one state in which it is established that there is no hot water is more plausible than all states in which it is not known whether there is hot water, but in which it is known that if Jane wants a bath then there will be no hot water.

2. A state in which it is not known whether or not there is hot water but in which it is established that if Jane has a bath then there will be no more hot water must be at least as plausible as all states where it is definitely established that there is no hot water.

The general question is, why would it be reasonable to expect information orderings to have such properties? My answer to this question is on the one hand both simple and obvious, and on the other hand both awkward to implement and incompatible with many contemporary theories of presupposition. Many linguists will surely find it unpalatable. The answer is: common sense.

Let me expand on this. The contrast between E224 and E225 results from our ability to find a common-sensical explanation of the lack of hot water in terms of somebody having taken a bath, but in our inability to fully explain a lack of hot water in terms of somebody simply wanting a bath. The simple assumption that there is a finite amount of relevant hot water — about a bathful — is sufficient to allow justification of their being no more hot water in situations where Jane has just taken a bath. However, the same simple assumption would not suffice in the case of E225, and a number of other assumptions would be needed, such as the assumption that if Jane wants a bath then she will definitely take one. Thus it is the relative plausibility of assumptions not explicitly mentioned in the text of the example sentences that determines what is implicated. Here are four more examples (the first two of which were discussed in Part One of this thesis) which illustrate the same point:

E99 If Nixon invites Angela Davis to the Whitehouse, then Nixon will regret having invited a black militant to his residence.
E151 If Spaceman Spiff lands on Planet X, he will be bothered by the fact that his weight is higher than it would be on Earth.

E226 If John has an exam today, Mary will notice that he is smoking more than usual.

E227 If Jane goes into the cave without a light, she will be annoyed that she cannot see much.

In all of these cases, the standard CCP prediction of a conditional presupposition seems unobjectionable. These predicted presuppositions are, respectively, that if Nixon invites Davis he will have invited a black militant, that if Spaceman Spiff lands on Planet X then his weight will be higher than it would be on Earth, that if John has an exam today then he is smoking more than usual, and that if Jane goes into the cave without a light then she will not be able to see much. I would argue that in all three cases, what makes the conditional presuppositions reasonable is the possibility of finding a deeper explanation. Thus, in the first case, the reason why the conditional presupposition makes sense is that it can be justified in terms of common sense physics. We suppose that Planet X is a planet, but we do not know whether Planet X is a big planet or a little planet. The simple assumption that Planet X is big (some readers may have more sophisticated views on the relationship between planets and their gravitational field strength) is sufficient to tell us that if Spiff lands on X then his weight will be higher than it would be on Earth. In the second case the deeper explanation is that Davis is a black militant. I take it that I am not terribly unusual in having constructed this explanation for myself when I first read the example E99 some years ago, although I had no previous knowledge of Angela Davis. I leave it to the reader to construct explanations for the implications of the other two examples. Of course, it might be that the reader deems E226 to implicate that John actually is smoking more than usual, or perhaps thinks E227 is most naturally uttered in a situation where Jane is known to be blind. Yet this would by no means be problematic. On the contrary, it would be grist to my mill, as it indicates something that is quite natural within the current account, namely that the reader's plausibility criteria differ somewhat from my own.

The predictions of the model introduced here stand in sharp contrast to those of various theories that were discussed in Part 1 of this thesis. The accounts which have occupied centre-field in formal presupposition theory, be they multivalent, partial, filtrationalist, cancellationist, dynamic or accommodational, are generally conceived of as solving the presupposition projection problem without any reference to common sense reasoning, whereas I would argue that in principle a theory lacking common sense cannot solve the projection problem. In particular, in Part 1 a number of the examples considered in this section were considered in detail, and it was shown that they are problematic for existing accounts (c.f. especially §3.3.3, §4.6, §6.4 and §6.6). Even if
some authors have been aware that interaction with common sense reasoning is needed in order to account for all the data of presupposition projection, this has generally not resulted in formal models.

Here I would like to mention one prominent research group, namely that of Hobbs, which has provided formal models where common-sense reasoning helps determine presuppositional inferences. The work of this group was not discussed in the first part of this thesis, partly because it is not usually presented as a theory of presupposition per se, but as a complete theory of interpretation. In a series of papers (see e.g. the bibliography in [HSAM90]) Hobbs and colleagues have offered a view of interpretation as abduction, whereby grammar provides only limited absolute constraints on meaning, and a system of weighted abduction allows domain knowledge to supplement grammatical information so as to determine a most plausible interpretation. Systems of abduction typically allow reasoning backwards from some formula to sets of formulae which deductively entail that formula, and also provide some way of selecting the most appropriate such set. Given some observation, an explanation (or set of alternative explanations) for that observation may be generated. In the case of Hobbs' model of interpretation, the observation is an utterance, and the explanation amounts to a theory of the relevant beliefs and intentions of the speaker. And this is just the idea behind the proposal presented in the latter half of the current chapter, albeit that my technical realisation of that idea is quite different from that of Hobbs.

The goals of this thesis have been far more modest than those of Hobbs and his co-workers, in that I have maintained a relatively conservative Frege-derived Montagovian line on the syntax-semantics interface, with compositional rules determining a semantic interpretation (or set of them) for each syntactic constituent in terms of the interpretations of sub-constituents, and in that I have not attempted to provide a computer implementation. The novelty of the current proposal consists principally in two departures from the Frege-Montague tradition. Firstly I have followed a recent trend in semantic theory by suggesting (in Chapters 7–9) that the traditional static view of what meanings are is inadequate, and I have proposed a dynamic alternative. Secondly, in this chapter, a model of the semantics-pragmatics interface has been proposed which allows both common-sense reasoning and semantic content to determine the information which a hearer derives from a particular utterance. Thus I have not gone so far as Hobbs, in that common sense reasoning does not determine content in the model I have developed. But I have shown a way in which a mechanism operating very much in the spirit of Hobbs' proposals can be built on top of a compositional theory of meaning.
Chapter 11

Connections and Directions

In this chapter I will attempt to forge some theoretical links between the model that has been developed in the second part of this thesis and some of the theories discussed in the first part. These links suggest directions that might be taken to tackle phenomena not so far incorporated in the model presented here, but also directions that might be taken to resolve problems which arise in other theories of presupposition.

11.1 Multivalence

The thesis, descending from the work of Frege and Strawson, that presupposition projection should be explained as inheritance of semantic undefinedness, seems to find an antithesis in the suggestion that presupposition projection arises from (pragmatically justified) principles of context change. However, Peters, in [Pe77], provided a synthesis, observing that the presupposition inheritance properties derived in [Kar74] could be duplicated in a system with a trivalent semantics, and thus do not depend on the dynamicity of Karttunen's account. The connectives in Peter's trivalent system, which I will refer to as the Peters' connectives (but which Krahmer [Krah93] terms the Middle Kleene connectives), can be used to show the relationship between the dynamic logics developed in the current work and trivalent logics1. Having made such a connection, it becomes easier to see how insights might be transferred from a trivalent to a dynamic setting or vice versa.

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1The suggestion of drawing a formal connection between a PUL-like system and one based on Peters' semantics was first made to me by Marcus Kracht after a talk I gave in Amsterdam in 1992, and this resulted in the proof presented here. Willem Groeneveld, who was also at the talk, provided a proof of essentially the same property, and his proof runs along similar lines.
11.1.1 A Trivalent Semantics for the Non-epistemic Sub-language of PUL

The Peters' connectives may be likened to the strong Kleene connectives, except that if the left-hand formula under a binary Peters' connective is undefined, then the whole formula is undefined:

**Definition D70 (The Peters' Connectives)** The 3 valued interpretation of a complex formula $\phi$ relative to a world $w$, written $[\phi]_w^3$, is given by recursion over the following truth tables:

<table>
<thead>
<tr>
<th>$\phi \land \psi$</th>
<th>$t$</th>
<th>$f$</th>
<th>*</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>$t$</td>
<td>$f$</td>
<td>*</td>
</tr>
<tr>
<td>$f$</td>
<td>$f$</td>
<td>$f$</td>
<td>$f$</td>
</tr>
<tr>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>$\phi \lor \psi$</td>
<td>$t$</td>
<td>$f$</td>
<td>*</td>
</tr>
<tr>
<td>$t$</td>
<td>$t$</td>
<td>$t$</td>
<td>$t$</td>
</tr>
<tr>
<td>$f$</td>
<td>$f$</td>
<td>$t$</td>
<td>*</td>
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<td>*</td>
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<td>*</td>
<td>*</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\phi \rightarrow \psi$</th>
<th>$t$</th>
<th>$f$</th>
<th>*</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>$t$</td>
<td>$f$</td>
<td>*</td>
</tr>
<tr>
<td>$f$</td>
<td>$f$</td>
<td>$t$</td>
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<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
</tbody>
</table>

The intuition behind the presupposition operator $\partial$ that has been utilised in this thesis is that a formula $\partial \phi$ only defines an update if $\phi$ is true. A natural three valued analogue of this semantics would make $\partial \phi$ undefined whenever $\phi$ is not true, but true otherwise:

**Definition D71 (A Trivalent Presupposition Operator)**

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>$\partial \phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>$t$</td>
</tr>
<tr>
<td>$f$</td>
<td>*</td>
</tr>
<tr>
<td>*</td>
<td>*</td>
</tr>
</tbody>
</table>

The result proved in this section will concern two presentations of the semantics for the language of propositional logic with the addition of the unary $\partial$-operator. This language will be referred to as PL+$\partial$. We assume models $(W, F)$, where $W$ is a set of worlds and $F$ is an interpretation function mapping atomic formulae onto sets of worlds$^2$. The first presentation is the trivalent semantics given by the above truth tables, the meaning of a formula $\phi$ in a world $w$ being denoted by $[\phi]_w^3$, and atomic formulae $\phi$ having interpretation given by $[\phi]_w^3 = t$ if $w \in F(\phi)$, and $f$ otherwise. We

---

$^2$Atomic formulae are thus effectively bivalent: every world is either in the extension of a given atomic formula, or it is not, and there is no third possibility. I will not consider the case where atomic formulae themselves have a *fuzzy* extension, whereby worlds could fall neither into the extension or the anti-extension, or could fall into both.
may then define trivalent entailment as the following relation between formulae in the language:

**Definition D72 (Entailment in the 3-valued system)**

\[ \phi \models_3 \psi \iff \forall w \in W, [\phi]_3^w = t \Rightarrow [\psi]_3^w = t \]

The second presentation of the semantics is just the update semantics introduced in Chapter 7 (as it applies to the non-epistemic sublanguage of PUL), the update semantics of a formula \( \phi \) being denoted \([\phi]_e\). For this language, the update notion of entailment from PUL adapted to the single premise case is easily show to be equivalent to the following (which says that all fixed points of the premise are fixed points of the conclusion):

**Definition D73 (Entailment in the Update System)**

\[ \phi \models_u \psi \iff \forall \sigma \ [\phi]_e \sigma \rightarrow [\psi]_e \sigma \]

It will now be shown that the trivalent and update notions of entailment define exactly the same logic (i.e. the same binary relation between formulae). Firstly, truth and falsity of a formula in a world will be defined for the trivalent and update systems, and an equivalence between these notions established. Truth in a world is analogous to the property of satisfaction in a set of worlds defined for PUL earlier. For truth of \( \phi \) in a world \( w \) in the trivalent system \( w \models_3 \phi \) will be written, with \( w \not\models_3 \phi \) being used to indicate that this property is lacking, and \( w \not\models_3 \phi \) for falsity of \( \phi \) in \( w \) in the trivalent system. Similar notation is used for these notions in the update system.

**Definition D74 (Trivalent Truth in a World)**

\[ w \models_3 \phi \iff [\phi]_3^w = t \]

**Definition D75 (Trivalent Falsity in a World)**

\[ w \not\models_3 \phi \iff [\phi]_3^w = f \]

**Definition D76 (Update Truth in a World)**

\[ w \models_u \phi \iff \{ w \} [\phi]_e \{ w \} \]

**Definition D77 (Update Falsity in a World)**

\[ w \not\models_u \phi \iff \{ w \} [\phi]_e \emptyset \]
Lemma L1 For all contexts w and propositions \( \phi \):

\[ w \models \phi \iff w \models_3 \phi \text{ and } w \nvdash \phi \iff w \not\models_3 \phi \]

Proof: By induction over the complexity of \( \phi \). Note that since disjunction and implication may be defined in terms of conjunction and negation identically in both systems, it is only necessary to consider the clauses for atomic formulae, the presupposition operator, conjunction and negation. As examples, I give (A) the truth clause for the presupposition operator, and (B) the falsity clause for negation.

(A) We want to show that, on the assumption that the above properties hold for some \( \phi \), then for any \( w \), \( w \models_\psi \phi \iff w \models_3 \phi \). First from left to right. We assume \( w \models_\psi \phi \), from which we obtain \( \{w\}\{\phi\}_\psi \{w\} \) and, by the semantics of \( \psi \), \( \{w\}\{\phi\}_\psi \{w\} \), i.e. \( w \models_\psi \phi \). By the inductive hypothesis, \( w \models_3 \phi \), and, by definition of \( \models_3 \) and the truth table for \( \psi \), the result \( w \models_3 \phi \).

From right to left, \( w \models_3 \phi \) gives us \( \{\phi\}_3^w = t \), and by inspection of the truth table \( \{\phi\}_3^w = t \). The inductive hypothesis gives \( \{w\}\{\phi\}_3 \{w\} \), from which \( \{w\}\{\phi\}_3 \{w\} \) follows by update semantics of \( \psi \), and thence, by definition of \( \models_\psi \), the result \( w \models_\psi \phi \).

(B) We want to show that for any \( w \), \( w \models_\psi \neg \phi \iff w \not\models_3 \neg \phi \). From right to left, \( \{w\}\{\neg \phi\}_\psi \{w\} \) gives us \( \{w\}\{\phi\}_\neg \{w\} \), by update semantics of \( \neg \). From this \( w \nvdash_\psi \phi \), and by the inductive hypothesis, \( w \nvdash_3 \phi \), \( \{\phi\}_3^w = f \) and thence \( \{\neg \phi\}_3^w = t \).

All these steps are reversible, so from left to right, we simply get from \( \{\neg \phi\}_3^w = t \) that \( w \nvdash_3 \phi \), using the inductive hypothesis derive \( w \nvdash_\psi \phi \), thence \( \{w\}\{\phi\}_\neg \{w\} \), and \( \{w\}\{\neg \phi\}_\psi \{w\} \).

It must now be shown that the update notion of entailment is equivalent to a definition in terms of update truth and update falsity for this system:

Lemma L2 \( \phi \models_\psi \psi \iff \forall w[ w \models_\phi \psi \iff w \models_\psi \psi] \)

Proof: From right to left definition of \( \models_\psi \) gives us that \( \forall \sigma \ \sigma\{\psi\}_\psi \sigma \rightarrow \sigma\{\psi\}_\psi \sigma \). It follows that this must hold for singleton sets of worlds \( \sigma \), i.e. \( \forall w \ \{w\}\{\psi\}_\psi \{w\} \rightarrow \{w\}\{\psi\}_\psi \{w\} \). But now definition of update truth gives us the left hand side.

The proof from left to right is more involved. Definition of update truth again gives us that (1) \( \forall w \ \{w\}\{\phi\}_\psi \{w\} \rightarrow \{w\}\{\psi\}_\psi \{w\} \). Now, it may be shown that this system has a distributivity property: (2) \( \forall \sigma \ \sigma\{\phi\}_\psi \psi \iff \forall w \in \)
The trivalent and dynamic entailment notions are extensionally equivalent for the language of PL+$\&$: $q \models_3 \psi$ iff $\phi \models_3 \psi$

Proof:

By Lemma 2, $\phi \models_u \psi$ is equivalent to $\forall w [w \models \phi \rightarrow w \models \psi]$. This is equivalent to $\forall w [w \models_3 \phi \rightarrow w \models_3 \psi]$ by Lemma 1. Now according to the definition of truth in the trivalent system this is in turn equivalent to $\forall w [[\phi]_w = t \rightarrow [[\psi]_w = t]$, which is just the right-hand side of the definition of three valued entailment.

This establishes the main result. Now it is easily shown that according to the definition of presupposition given for PUL, and given the restriction to the non-epistemic sub-language, the presuppositions of a formula are all and only those formulae which are update entailed both by the formula and by its negation. Furthermore, a standard definition of presupposition in a trivalent logic would be entailment by both the formula and its negation. But given that the notion of update entailment is extensionally equivalent to the notion of trivalent entailment for this system, it follows as a corollary that the notions of presupposition in the two systems are equivalent, which is essentially Peters' result. Another corollary is that for the sublanguage without the $\theta$-operator, the logic is classical. This follows since, by assumption, atomic formulae have bivalent interpretations, and for all of the remaining connectives in the trivalent system, if their arguments are bivalent then they return the classical value.

Clearly there is a tight correspondence between the dynamic systems which have been introduced in Part II of this thesis and trivalent logics. Although I doubt whether for the epistemic and first-order systems such a strong extensional equivalence could easily be demonstrated, the equivalence demonstrated here is highly suggestive of a general similarity between the traditional semantic approach to presupposition and the dynamic semantic approach.

11.1.2 Assertion and Denial in Dynamic Logic

The operators in the following table are Bochvar's assertion operator $(A)$, Bochvar's (presupposition-canceling) external negation $(\#)$, and Seuren's presupposition-denying
negation ($S$):

**Definition D78 (Assertion and Denial Operators)**

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>$A\phi$</th>
<th>$\neg\phi$</th>
<th>$S\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>$t$</td>
<td>$f$</td>
<td>$f$</td>
</tr>
<tr>
<td>$f$</td>
<td>$f$</td>
<td>$t$</td>
<td>$f$</td>
</tr>
<tr>
<td>$*$</td>
<td>$f$</td>
<td>$t$</td>
<td>$t$</td>
</tr>
</tbody>
</table>

Both the external negation and Seuren's negation can be defined in terms of the assertion operator and connectives from the Peters system above. Thus $\neg\phi \equiv \neg A\phi$ and $S\phi \equiv \neg\phi \land \neg\neg\phi$. The following definition provides an appropriate dynamic interpretation for the assertion operator, and thus in the process enables the negation operators to be defined:

**Definition D79 (Assertion in PUL)**

$$\sigma[A\phi]\tau \iff \tau = \{w \in \sigma \mid \{w\}[\phi]\{w\}\}$$

**Fact F19** The trivalent and dynamic entailment notions are extensionally equivalent for the language of PL+$\theta + A$.

**Sketch of Proof:** The proof requires us to check that the induction in lemma L1 still goes through, and that the $A$-operator does not affect the distributivity property used in the proof of lemma L2.

If an $A$-operator could be defined as an extension to ABLE, then it would be possible to augment the account given here with a Linkian floating-$A$ type theory as described in §3, which would allow treatment of presupposition-denying negation without direct postulation of an ambiguous negation. Alternatively, it would be possible to define negation to be ambiguous between internal and external interpretations, or between internal and Seuren negation.

### 11.1.3 Formalising Backward Suppletion

As mentioned in Part One of this thesis (§3.3), Seuren [Seu85] suggested that his trivalent logic (in fact Kleene's strong logic with the addition of Seuren's presupposition-denying negation) should be augmented with an accommodation-like mechanism which

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2One could, of course, define the external negation and Seuren negation directly in the dynamic setting:

$$\sigma[\neg\phi]\tau \iff \tau = \{w \in \sigma \mid \neg\{w\}[\phi]\{w\}\}$$

$$\sigma[S\phi]\tau \iff \tau = \{w \in \sigma \mid \exists v\{w\}[\phi]v\}.$$
he termed *backward suppletion*, although some details of this mechanism have remained unformalised. It will now be shown that the approach to accommodation developed in this thesis could also be applied to a trivalent system like Seuren’s.

Let us define an *epistemic alternative*, a view of what the speaker takes to be the common ground, to be a set of worlds. Then we may approximate the hearer’s knowledge of the speaker’s assumptions by a set of epistemic alternatives, that is, a set of sets of worlds. Suppose the speaker utters a sentence with meaning \( \phi \). Let us take it that the presuppositions of \( \phi \) reflect the speaker’s assumptions about the common ground, so that if in some epistemic alternative there is a world for which \( \phi \) is undefined, then that alternative is not a good picture of the speaker’s assumptions. On the other hand, if an alternative is compatible with the speaker’s presuppositions, it can be updated simply by taking the intersection of the worlds in the alternative with the set of worlds for which \( \phi \) is true.

**Definition D80 (Updating an Information Set with a Trivalent Meaning)**

\[
I + \phi = \{ \sigma \mid \exists \sigma' \in I \\
\forall w \in \sigma' [ \phi]_w \neq \star \land \\
\sigma = \{ w \in \sigma' \mid [\phi]_w = t \}\}
\]

One can think of the above definition as comprising two principles, a principle of update, and a principle of accommodation. The principle of update says that (a) the information which the speaker takes to be established is modeled as a set of worlds, and (b) an appropriate assertive utterance should divide this set cleanly into two disjoint covering subsets, such that the utterance is true in all worlds in the first, and false in all worlds in the second. Just this view on how trivalent meanings should determine updates is espoused by Stalnaker in [St79, pp. 325–326]. On his view, such a principle “connects semantic presupposition with pragmatic speaker presupposition”, and he gives the following rationale: “The point of an assertion is to reduce the context set in a certain determinate way. But if some proposition is not true or false at some possible world, then it would be unclear whether that possible world is to be included in the reduced set or not.” The second principle, the principle of accommodation, carries Stalnaker’s view one stage further. It states that (a) the hearer’s knowledge of what the speaker takes to be established is modeled as a set of sets of worlds, and (b) an assertive utterance

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4However, I have my doubts as to whether the system to be described is true to Seuren’s intentions, since the architecture of the semantics-pragmatics interface which I propose differs from that of Seuren. Seuren envisages quite general inferential mechanisms operating at a level of mental representation, and has used such devices to explain, for instance, instances of implicature denial. The divergence from this picture in the current work could be thought of as resulting from simplifications made so as to facilitate formalisation.
divides this set cleanly into two disjoint covering subsets, such that the utterance is appropriate in the sets of worlds in the first set, and inappropriate in the sets of worlds in the second. The two principles combine into a single principle of hearer’s context incrementation: given an utterance, update those sets of worlds where it is appropriate, and discard those where it is not.

A more sophisticated approach to modeling incrementation of a hearer’s information states is to use information orderings like those discussed earlier. An ordering can be defined as a set of pairs of epistemic alternatives. Intuitively, a pair \((\sigma, \tau)\) will be in the ordering just in case \(\sigma\) is considered at least as plausible a model of the speaker’s assumptions as \(\tau\). Update proceeds by filtering out those pairs for which at least one of the alternatives is incompatible with the speaker’s presuppositions, and otherwise updating both members of each pair with the new proposition, again simply by intersection with the set of worlds where the proposition is true:

**Definition D81 (Updating Information Orderings with Trivalent Meanings)**

\[
O + \phi = \{ (\sigma, \tau) \mid \exists (\sigma', \tau') \in O \\
\forall w \in (\sigma' \cup \tau') [\phi]^w_{3} \neq \star \wedge \\
\sigma = \{ w \in \sigma' \mid [\phi]^w_{3} = t \} \wedge \\
\tau = \{ w \in \tau' \mid [\phi]^w_{3} = t \}\}
\]

One characteristic of the strong Kleene connectives is their tendency to yield conditional presuppositions. For instance, the following example (reproduced from the previous chapter), is predicted (on a standard Strawsonian definition of presupposition) to presuppose that if I go to London then I have a sister:

E52' If I go to London, my sister will pick me up at the airport.

Now we can apply the same argumentation as used earlier to allow for the tendency to come to a stronger conclusion, namely that I have a sister. If it were the case that alternatives containing only worlds where I have a sister were higher in the plausibility ordering than states where it is not established whether I have one but it is established that in all the going-to-London worlds I have a sister, then the stronger inference would be predicted. Of course, whether the model actually does predict strengthening will depend on the particular common sense theory of the world underlying the information ordering.

Note that the same argumentation need not produce strengthening of a non-presuppositional conditional as in ‘If I go to London then I have a sister.’ Provided that there are some alternatives which are more plausible than all those where I assume that
To further work to determine what further properties such a notion of entailment would have.

In Keutmann, different models fit the generalised formal semantics. We refer to the appendix for the details.

There is an additional consideration, however, that is worth noting. The model-theoretic semantics of the propositional logic are not unique. There are many models that satisfy the same set of axioms. This means that the truth-value of a formula in a given model can depend on the particular model chosen. In the case of the propositional logic, there is only one truth-value for each propositional formula. However, in the case of the predicate logic, there are many possible models for a given set of axioms. This means that the truth-value of a formula in a given model can depend on the particular model chosen.

This raises the question of whether there is a preferred model for a given set of axioms. There are several criteria that could be used to determine a preferred model. One criterion is the size of the model. A smaller model is generally preferred over a larger one.

Another criterion is the complexity of the model. A simpler model is generally preferred over a more complex one.

Yet another criterion is the accuracy of the model. A model that accurately reflects the intended meaning of the axioms is generally preferred over a model that does not.

In conclusion, the choice of a preferred model is a matter of personal preference. However, there are some criteria that can be used to guide the selection of a preferred model.
cancellation models. As a result, it could be argued that Soames’ theory needs to be shored up with a pragmatic account of how weak presuppositions may be strengthened. The theory of accommodation presented in the last chapter provides a natural way of achieving this strengthening, and I will briefly show how the Local Satisfaction Preferred model may be combined with an account of global accommodation.

11.2.1 Global Satisfaction Preferred

A system will now be defined in which presuppositions are interpreted as providing global preferences, which is essentially the idea of cancellationist accounts. I will use the language of PL+Ω.

Models are just as for PUL, pairs \((W, F)\), where \(W\) is a set of worlds and \(F\) is an interpretation function mapping atomic formulae onto sets of worlds. It may be assumed that \([\cdot]\) is an essentially classical valuation, mapping formulae onto the set of worlds where they would be classically true, and giving formulae \(\emptyset \phi\) the same valuation as \(\phi\). An information state will just be a set of worlds, the update of an information state \(\sigma\) with a formula \(\phi\), written \(\sigma + \phi\), will just be given by \(\sigma \cap [\phi]\), and \(\sigma\) will be said to satisfy \(\phi\), written \(\sigma \models \phi\), if \(\sigma\) is a fixed point of \(\phi\), i.e. \(\sigma + \phi = \sigma^T\). If \(\mu\) is a pair of states \(\langle \sigma, \tau \rangle\), I will write \(\mu + \phi\) to mean \(\langle \sigma + \phi, \tau + \phi \rangle\).

The preferences of a formula \(\phi\), written \(\uparrow \phi\), will be defined in terms of an information ordering, which, as before, is a set of pairs of states (epistemic alternatives). Definition D82 below specifies the set of good orderings (GO) relative to some model. Firstly, the minimally constrained ordering \(\ominus\), which is the set of pairs of any two non-absurd states, is a good ordering (the set of non-absurd states defined over a set of worlds \(W\) being written \(W^\ast\)). Secondly, an ordering \(\omega(\phi)\), which is one where there is a preference for states where \(\phi\) holds, is a good ordering. For a pair \(\langle \sigma, \tau \rangle\) to be in \(\omega(\phi)\), it must be the case that if \(\tau\) satisfies \(\phi\), then \(\sigma\) does too. Thirdly, the intersection of any two good orderings is also a good ordering. I will not explore the algebraic structure of the Good Orderings here.

7In fact I will not here take advantage of the generality of this definition of satisfaction, and for current purposes could equally well have defined \(\sigma \models \phi\) as equivalent to \(\sigma \subseteq [\phi]\).
Definition D82 (Good Orderings) The set of good orderings is the smallest set $GO$ such that (1) $\odot$ is in $GO$, (2) for any $\phi$, $\omega(\phi)$ is in $GO$, and (3) for any $O$ and $O'$ in $GO$, $O \cap O'$ is in $GO$, where $\odot$ and $\omega(\phi)$ are defined by:

$$W^* = \varphi(W) \setminus \emptyset$$

$$\odot = W^* \times W^*$$

$$\omega(\phi) = \{(\sigma, \tau) \in \odot \mid \tau \models \phi \rightarrow \sigma \models \phi\}$$

It is natural to write $\sigma \geq_{O} \tau$ for $(\sigma, \tau) \in O$, and $\sigma > \tau$ to mean that $\sigma \geq_{O} \tau$ but $\tau \not<_{O} \sigma$. Since any formula which is satisfied by the minimally informative state $W$ is a tautology, and is thus also satisfied by every other state, it follows that every non-absurd state is at least as preferable as $W$ in any ordering $\omega(\phi)$. Indeed, this property holds for any good ordering $O$: $\forall \sigma \in W^*, \sigma \geq_{O} W$.

For each formula $\phi$ an associated set of preferences $\dagger \phi$ will now be recursively stipulated. Here the only clause which actually introduces preferences is that for presuppositional formulae, a formula $\partial \phi$ setting up a preference for states where $\phi$ holds. Atomic formulae carry the trivial preference ordering $\odot$, which means that no state is strictly preferred to any other. The negation of a formula carries the same preferences as its argument, and the conjunction of two formulae carries a combination of the input preferences of the two conjuncts. Here two orderings are combined simply by taking their intersection.

Definition D83 (GSP Orderings)

$$\dagger \phi = \odot \text{ for atomic } \phi$$

$$\dagger \neg \phi = \dagger \phi$$

$$\dagger (\phi \land \psi) = \dagger \phi \land \dagger \psi$$

$$\dagger (\phi \rightarrow \psi) = \dagger \neg (\phi \land \neg \psi) \quad \underline{=} \dagger \phi \land \dagger \psi$$

$$\dagger (\phi \lor \psi) = \dagger \neg (\neg \phi \land \neg \psi) \quad \underline{=} \dagger \phi \land \dagger \psi$$

$$\dagger \partial \phi = \omega(\phi)$$

When defining a notion of entailment it will be important to consider the case of formulae containing explicitly denied presuppositions. So the set of preferred inputs for a formula $\phi$, written $\dagger \phi$ will be defined as the set of the preferred inputs out of those that are actually consistent with $\phi$. I write $c(\phi)$ for the set of states which are consistent with $\phi$, and $\sigma \not<_{O} \tau$ to mean that $\tau$ is not higher in the ordering $O$ than $\sigma$: 
Definition D84 (Consistency, Non-Preference, and Preferred Inputs)

\[
c(\phi) = \{\sigma \mid (\sigma + \phi) \supset \emptyset\}
\]

\[
\sigma \not\supset_{\emptyset} \tau \text{ iff } \tau \supset_{\emptyset} \sigma \supset_{\emptyset} \tau
\]

\[
\models \phi = \{\sigma \in c(\phi) \mid \forall \tau \in c(\phi) \sigma \not\supset_{\emptyset} \tau\}
\]

Now one formula \( \phi \) can be defined to presuppositionally entail another \( \psi \), written \( \phi \leadsto \psi \), if and only if with respect to all of \( \phi \)'s preferred inputs \( \phi \) contextually entails \( \psi \):

Definition D85 (Presuppositional Entailment)

\[
\phi \leadsto \psi \text{ iff } \forall \sigma \in \models \phi (\sigma + \phi) \models \psi
\]

A logician peering at this definition for the first time might perhaps be worried. What strange new fangled relation is \( \leadsto \), and why is it being described as a notion of logical consequence? I believe it could be termed a **conservative** notion of consequence in at least two senses.

Firstly, for the sublanguage without the \( \partial \)-operator, \( \leadsto \) is equivalent to the classical relation of consequence. This is so because none of the operators apart from \( \partial \) introduce non-trivial preferences if their arguments have only trivial preferences, so that the set of preferred inputs for a consistent non-presuppositional formula is just the set of all states. In that case \( \phi \leadsto \psi \) holds if and only if the set of worlds in the extension of \( \phi \) is a subset of those in the extension of \( \psi \), which is just the condition for classical entailment. Inconsistent formulae have an empty set of preferred states, and will presuppositionally entail all other formulae, just as in classical propositional logic.

Secondly, under this consequence relation all classical entailment patterns should remain valid for the full language of PL+\( \partial \) with entailment defined by \( \leadsto \). To see this, initially let us consider the conclusion. Observe that since the preference ordering associated with the conclusion is not involved in the definition of \( \leadsto \), and since the valuation of \( \partial \phi \) is just that for \( \phi \), we may safely erase all occurrences of \( \partial \) on the right of an entailment. Now the premise. Suppose that some formula \( \phi \) has a sub-formula \( \psi \). Clearly replacing \( \psi \) by \( \partial \psi \) will not affect the semantic interpretation of \( \phi \), and *a fortiori* will not affect the set of states with which \( \phi \) is consistent. Now suppose that \( \phi \) is classical, i.e. does not contain any instances of the \( \partial \)-operator. Then, as shown above, all its classical consequences will also be presuppositional entailments. Now modifying any number of sub-formulae by enclosing them in (any number of) \( \partial \)-operators can only restrict the set of preferred inputs to \( \phi \). It is easily seen that the set of entailments of the modified premise must be a strict superset of the entailments of the original premise. So, \( \leadsto \) is a strictly weaker notion of consequence than classical entailment, in that more
formulae are presuppositionally entailed than classically entailed.

Next we consider how the presuppositions of a formula may be defined using the ideas underlying presuppositional entailment. The following definition says that one formula presupposes another if and only if all preferred states of the first satisfy the second:

**Definition D86 (GSP Presupposition)**

\[ \phi \gg_{GSP} \psi \iff \forall \sigma \in \downarrow \phi \quad \sigma \models \psi \]

If \( \phi \) is consistent (i.e. \( c(\phi) \supseteq \emptyset \)) then for any state \( \sigma \) in \( \uparrow \phi \), \( \sigma + \phi \) must be nonempty. Further, since \( + \) is defined in terms of intersection, and since the set of propositions which are satisfied in \( \sigma \cap \tau \) is just the union of those which are satisfied in \( \sigma \) and those which are satisfied in \( \tau \), it follows that the earlier definition of presuppositional entailment could be rewritten: \( \phi \sim \psi \iff \forall \sigma \in \downarrow \phi \quad (\sigma \models \psi \text{ or } [\phi] \models \psi) \). Given that additionally \( [\phi] \models \psi \) holds if and only if \( \phi \models \psi \), where \( \models \) is the classical entailment notion (extension of consequent containing extension of premise), it follows that \( \sim \) can here be expressed in terms of preferential presupposition and classical entailment: \( \phi \sim \psi \iff \phi \gg_{GSP} \psi \text{ or } \phi \models \psi^8 \). The following relatively weak properties of the system show that it is natural to think of \( \partial \)-formulae as potential presuppositions in Gazdar’s sense, for they emerge as presuppositions when nothing conflicts, and disappear otherwise:

1. The presupposition operator yields presuppositions even when embedded: if \( \phi \) contains a subformula \( \partial \psi \) and no other \( \partial \)-formulae, and the conjunction of \( \phi \) and \( \psi \) is consistent, then \( \phi \gg_{GSP} \psi \).

2. Presuppositions persist if there is no conflict: if \( \phi \gg \psi \) and \( \chi \) is a non-presuppositional formula such that the conjunction of \( \phi, \psi \) and \( \chi \) is consistent, then \( \phi \land \chi \gg_{GSP} \psi \).

3. Presuppositions disappear when conflicting material is added: if \( \phi \gg \psi \), and although the conjunction of \( \phi \) and \( \chi \) is consistent, that of \( \phi, \psi \) and \( \chi \) is inconsistent, then \( \phi \land \chi \gg_{GSP} \psi \).

What this shows is that we have here the beginnings of a cancellationist account of presupposition. Of course, much would need to be shown before we could really be confident about this system, and much needs to be added (e.g. cancellation by

\[ ^8\text{This provides another demonstration that } \sim \text{ is conservative with respect to classical entailment in the senses discussed above.} \]
implicatures) before the system would be very potent as a theory of presupposition. But the system still manifests at least the most basic property of a cancellationist theory. That is, presuppositions project if there is no conflict and are canceled otherwise. Given that the developments in the current chapter are only intended to be proofs of concept, this provides sufficient foundation to move on to the next stage, of showing how a preference for global satisfaction can be replaced by a preference for local satisfaction.

11.2.2 Local Satisfaction Preferred

In the Global Satisfaction Preferred (GSP) system above, conjunction is symmetrical, its preferences not reflecting any ordering of the conjuncts. The only change now made to that system is an alteration of these preferences associated with conjunction\(^9\). In the following definition, whilst the presuppositions of the left conjunct become preferences of the whole conjunction, those of the right conjunct only impose a preference for satisfaction in a context which has been first augmented with the left conjunct:

**Definition D87 (Local Satisfaction Preference for Conjunction)**

\[
\uparrow (\phi \text{ AND } \psi) = \{(\sigma, \tau) \in \uparrow \phi \mid \langle \sigma + \llbracket \phi \rrbracket, \tau + \llbracket \phi \rrbracket \rangle \in \uparrow \psi \}
\]

It is easily checked that this operation does not introduce any non-good orderings\(^{10}\). Let the relation of presupposition in the earlier PUL system be denoted by \(\ggPUL\), and presupposition in the LSP system \((\ggLSP)\) be defined as for the GSP system excepting that the preferences associated with conjunction are altered. The following should now hold:

1. When there is no inconsistency, PUL presuppositions are LSP presuppositions: if \(\phi\) has just one presuppositional subformula, \(\phi \ggPUL \psi\), and the conjunction of \(\phi\) and \(\psi\) is consistent, then \(\phi \ggLSP \psi\).

\(^9\)The preferences associated with implication and disjunction are taken to be still defined in terms of the equivalences with conjunction and negation used above. Thus disjunction becomes asymmetric once conjunction is asymmetric. But in this system it might make sense to define the preferences for disjunction symmetrically, as the intersection of the preferences of the disjuncts. For now problematic cases of cancellation of presuppositions occurring in a disjunct need not be handled by defining disjunction so as to enforce local satisfaction, but can instead be handled by the cancellation mechanism.

\(^{10}\)Define the localisation of \(\sigma\) to \(\phi\) as \(L(O, \phi) = \{(\sigma, \tau) \in \sigma \mid \langle \sigma + \phi, \tau + \phi \rangle \in Q\}\). Now the above clause for conjunction may be written: \(\uparrow (\phi \text{ AND } \psi) = \uparrow \phi \cap L(\uparrow \psi, \phi)\).

To determine whether this produces only good orderings, we need to check that if \(O\) is a good ordering then \(L(O, \phi)\) is a good ordering. But if \(O\) is a good ordering then either \(O = \emptyset\), or \(O = \omega(\chi)\) for some \(\chi\), or \(O\) is an intersection of two other good orderings \(O_1\) and \(O_2\). The three cases break down as follows:

1. If \(O = \emptyset\) then \(L(O, \phi) = \emptyset\)
2. If \(O = \omega(\chi)\) then \(L(O, \phi) = \omega(\phi \rightarrow \chi)\)
3. If \(O = O_1 \cap O_2\) then \(L(O, \phi) = L(O_1, \phi) \cap L(O_2, \phi)\)

It follows that the preferences associated with the LSP system are still good orderings.
2. Non-PUL presuppositions are not LSP presuppositions: if $\phi$ has just one presuppositional subformula, and $\phi \not\subseteq PUL \psi$, then $\phi \not\subseteq LSP \psi$

3. Inconsistency with presuppositions causes cancellation: if $\phi$ has just one presuppositional subformula, $\phi \not\subseteq LSP \psi$, and although the conjunction of $\phi$ and $\chi$ is consistent, that of $\phi$, $\psi$ and $\chi$ is inconsistent, then $\phi \land \chi \not\subseteq LSP \psi$.

These results, which I hope with further study could be strengthened, show that the LSP system is PUL-like when there is no inconsistency, and thus typically produces weaker presuppositions than the cancellationist GSP model. However, under threat of inconsistency, LSP presuppositions vanish into thin air, in the manner of a cancellationist rather than a dynamic semantic account of presupposition\textsuperscript{11}.

There are a number of modifications that it might be of interest to make to the LSP system, such as adding a treatment of implicatures, or replacing the notion of a context with a more constructive notion, such as a set of formulae. One possibility that might be quite significant would be replacing the classical interpretation of $\sigma + \phi$ with a dynamic interpretation. I can see no reason why such a system should not be extended to first-order: presupposed open propositions would then place preferences on their local contexts, but preferences which could be canceled. If I am right, then what would result would be a cancellationist model of presupposition which, unlike the models of Gazdar [Gaz79a], van der Sandt [vdS88], Mercer [Me87, Me92] and Soames [So82], would allow a treatment of the interaction of presupposition and quantification. I will not investigate this possibility here. Rather, I will consider another line of development, the addition of a mechanism of accommodation which can allow the weak presuppositions of the Soames-like LSP model to be strengthened.

\textsuperscript{11}One rather puzzling aspect of LSP concerns the fact that whilst the presuppositions of a formula may be canceled by explicitly denying them, denying the truth of the actual arguments of $\partial$-operators can still leave non-trivial conditional presuppositions. For instance, let $\alpha$ be $(\phi \rightarrow \neg(\partial(\psi) \land \chi)) \land \neg \psi$. The left-hand conjunct introduces a preference for states where $\phi \rightarrow \psi$ holds. But this preference is not canceled by the conjunction with $\neg \psi$. So the formula still ends up presuppositionally entailing more than it would classically entail. In particular $\alpha$ presuppositionally entails $\neg \psi$.

An example may clarify. Consider the discourse 'If Mary closed her eyes then she did not realise that Bill was smiling. He wasn't.', which I find rather odd. If we take the two sentences to be conjoined, and we understand the 'realise' clause as $(\partial(\text{bill-is-smiling}) \land \text{mary-realises-bill-is-smiling})$, then the discourse has the logical form of $\alpha$, and LSP predicts that it entails that Mary did not close her eyes. Given the oddity of this discourse, I am not sure whether this result is problematic. But it does show that there may be cases where LSP predicts stronger presuppositions than GSP, for in this case LSP predicts a conditional presupposition, and GSP predicts no presupposition. I would note that, at least in this case, the inference could presumably be removed by stipulating, as is standard, that the conditional introduced an implicature that the truth of its antecedent was unresolved. Whether we could then produce a variant of LSP that did produce strictly weaker presuppositions than (some similar variant of) GSP, and whether this would be desirable, I cannot say.
11.2.3 Adding Common Sense to LSP

This thesis is not about common sense. But it does provide a method for a theory of common sense reasoning to interact with a theory of presupposition, this being based on the assumption that whatever method is used to say that one theory is more in line with common sense than another, the output of that method can be encoded in an information ordering. It must now be determined how such a plausibility ordering may be combined with the LSP system, which involves its own ordering determined not by common sense but by a preference for satisfaction of conventionally stipulated presuppositions. This combination will take the form of a definition of presuppositional entailment which is sensitive to an external plausibility ordering, so that $\phi \sim_0 \psi$ if $\psi$ is a presuppositional consequence of $\phi$ relative to the ordering $O$.

One obvious possibility would be to take the intersection of the two orderings, and use that in the new definition of consequence. But I do not think this would give the right results empirically. Suppose that states are preferred where Mary does not own a rhinoceros, and I say 'I didn’t realise that Mary owned a rhinoceros.' Taking the intersection of an ordering encoding a preference for states where Mary owns a rhinoceros with one where there is a preference for states where Mary does not own a rhinoceros will have a canceling effect. The set of maximal states in the resulting ordering would contain both states where Mary had a rhinoceros and states where she did not, and so there would be no net preference for states where Mary did own a rhinoceros. However, my strong impression is that even though it is prima facie unlikely that Mary owns a rhinoceros, if I say ‘I didn’t realise that Mary owned a rhinoceros’ and do not explicitly mention any caveat, then I am committed to Mary’s having a rhinoceros. Thus, when interpreting what someone has said, satisfaction of the presuppositional defaults could be said to have a higher priority than satisfaction of common sense preferences about how the world ought to be.

We can model this by firstly finding the maximal states associated with the presuppositions of the utterance, and then using the plausibility ordering to select amongst the resulting states.

**Definition D88 (Plausible Presuppositional Entailment)**

\[
\begin{align*}
\max(O, S) & = \{ \sigma \in S \mid \forall \tau \in S \sigma \not\prec \tau \} \\
\phi \sim_0 \psi & \iff \forall \sigma \in \max(O, \uparrow \phi) \; (\sigma + \phi) \models \psi
\end{align*}
\]

This defines a notion of entailment which allows for filtering of entailed presuppositions, cancellation of presuppositions that are explicitly denied, and strengthening of weak presuppositions such as those generated when a presupposition is located in the consequent of a conditional.
11.3 Local Accommodation

11.4 Non-determinism and local accommodation

Although PUL, ABLE and KMG each involve a relationally specified semantics, none of them take full advantage of that denotation space. In particular, in none of these systems is it possible for a formula to be non-deterministic, in the sense that for a given input there is more than one output. Yet I can conceive of a number of potential applications for such non-determinism. One such application was presented in the version of KMG appearing in [Bea93a], in which fully relational CCPs are used to model DRT style non-deterministic pronoun resolution. On this basis a semantics is given for a fragment of unindexed English. In the system presented there, given an input state which has two discourse markers in its domain, both of which are established to correspond to singular females, update with the relational CCP corresponding to the unindexed sentence 'she is walking' could yield either of two output states, one in which the first discourse marker was asserted to be walking, and one in which the second was. This is an attractive approach to pronoun resolution, since the assumption that syntax somehow determines pre-indexed logical forms always struck me as unsatisfying. It is clear that syntax imposes strong constraints on coreference, but it is equally clear that syntax does not completely determine the issue.

Non-determinism could also be used in the modeling of local and intermediate accommodation. The trouble is that it is clearly inappropriate to allow arbitrary local accommodation, so the question is, how can it be constrained. One way of constraining it is by restricting what can be accommodated structurally, as in van der Sandt's model of accommodation. However, this account on the one hand is still too unconstrained (witness the inappropriate readings which intermediate accommodation was shown to generate in §6.5) and on the other hand is too constrained. For instance structural accounts fail to generate conditional presuppositions when appropriate (c.f. §6.6) and cannot possibly account for the phenomenon of bridging:

E155’ If I go to a wedding then the rabbi will get drunk

This example has a reading where the rabbi is understood to be somehow related to the wedding mentioned in the antecedent, rather than being a globally salient rabbi. The nature of the conceptual bridge that can be built between a wedding and a rabbi is essentially non-structural, and relies on world-knowledge rather than any detail of the form of the sentence. Only a common-sense reasoning based approach like that developed in the previous section could account for the phenomenon of bridging.

Most importantly, local accommodation supplies solutions to (amongst many others) the following traditional presuppositional riddles:
The King of France is not bald because there is no King of France.

Either the King of Buganda will open parliament or the President of Buganda will.

In the first case, local accommodation of the existence of a King of France (and an appropriate discourse marker) within the scope of the first negation would allow for the fact that the sentence does not implicate that there is a King of France\textsuperscript{12}. In the second case, local accommodation of the existence of a King of Buganda in the first disjunct and the existence of a President in the second would allow the sentence to be used in cases where it was not established whether Buganda was a monarchy or a republic. However, accepting that there is such a thing as local accommodation is only part of what is needed. I believe that local accommodation is a highly constrained process, and there is still no adequate theory of precisely what the constraints are.

Let me indicate briefly how we might introduce a non-structural notion of local accommodation using a radical (in terms of traditional semantic theory rather than AI natural language systems) alteration to the simple PUL system from Chapter 7. Suppose that we replaced the PUL definition of a context as a set of worlds with a notion of a context as a pair of a set of worlds and a cost: the cost will be used to constrain local accommodation. For a context $\sigma$, let us say that $\sigma_0$ is a set of worlds and $\sigma_1$ is a cost. The context is supposed to represent the implausibility of a given accommodated piece of information, so that the more implausible the propositions that have to be accommodated, the higher the cost\textsuperscript{13}. Let us then say that there is some relation of plausible extension between contexts, $\rightarrow$, which is determined by world knowledge, and which has at least the following properties:

1. If $\sigma \rightarrow \tau$, then $\tau_0 \subseteq \sigma_0$
   Plausible extension cannot cause a downdate.

2. If $\sigma \rightarrow \tau$ and $\tau_0 \subseteq \sigma_0$, then $\tau_1 > \sigma_1$.
   Plausible extension increases the cost.

3. If $\sigma \rightarrow \tau$ then there is no $\upsilon$ such that $\sigma \rightarrow \upsilon$, $\upsilon_0 = \tau_0$ and $\upsilon_1 \neq \tau_1$
   Extending some initial state with any given proposition has only one cost.

4. $\forall \sigma \; \sigma \rightarrow \sigma$
   Extending with a tautology (i.e. not at all) is free.

\textsuperscript{12}But see [vdS:MS] for an alternative view of how this type of example should be treated.

\textsuperscript{13}C.f. Sperber and Wilson's [SW84] use of relevance, which they describe as a processing cost. The cost utilised above measures unexpectedness (of extra assumptions). I have sometimes surmised that this cost might be measured in millimetres, a cost of, e.g., 2 mm. corresponding to a surprisingness which would cause raising of the eyebrows by this amount.
Here are the first few clauses showing what the semantics might look like:

\[ \sigma[p_{atomic}] \tau \quad \text{iff} \quad \tau_1 = \sigma_1 \land \tau_0 = \{w \in \sigma_0 \mid w \in F(p)\} \]
\[ \sigma[\phi \land \psi] \tau \quad \text{iff} \quad \exists u \sigma[\phi][v[\psi]] \tau \]
\[ \sigma[\neg \phi] \tau \quad \text{iff} \quad \exists u \exists u' \sigma \leadsto v[\phi][u'] \land \tau_1 = u' \land \tau_0 = \sigma_0 \setminus u_0' \]
\[ \sigma[\phi \implies \psi] \tau \quad \text{iff} \quad \sigma[\neg (\phi \land (\neg \psi))] \tau \]

For atomic propositions the cost of the input is simply identified with the cost of the output, whilst conjunction and implication are defined in the normal way. The interesting clause is that for \( \neg \phi \), which allows for plausible extension before update with \( \phi \).

Normally, the fact that not accommodating anything is cheaper than accommodating something will mean that the cheapest update with a given formula is equivalent to the standard PUL update. However, in some circumstances, failure to accommodate would yield presupposition failure. Consider the following example:

E230 Perhaps Bertha is hiding. However, if Anna discovers that Bertha is not hiding, then she will be upset.

The input context to the second sentence will not satisfy the proposition that Bertha is not hiding, and thus updating can only proceed if extra information is accommodated into the antecedent of the conditional, a possibility which arises from the definition of implication in terms of the new negation. On quite reasonable assumptions about the nature of plausible extension, the cheapest update with the sentence will be one corresponding to the local accommodation into the antecedent of the proposition that Bertha is not hiding.

This is a far from complete story, but it does suggest at least the possibility that the theory of meaning described in this paper could be extended with an account of local accommodation. Personally, I would be happier with an account that did not involve postulation of explicit costs and relied instead on a preference ordering mechanism like that invoked to describe global accommodation in Chapter 10. However, in the absence of any detailed proposal, I think it wise to keep an open mind, with there being clear potential for solutions using orderings, costs, or probabilities. Given recent formal convergence in the field of non-monotonic reasoning, it seems likely that a solution depending on any one of these might eventually be recast in terms of either of the others.
11.5 A Note on Presupposition Denial

In making various connections in this chapter, my eye has been on one phenomenon in particular. This is the cancelibility of presuppositions when the trigger is embedded under a negation — what could be called the problem of presupposition denial. In the CCP systems introduced in Chapters 7–10 of this thesis, if a sentence (formula) carries some presupposition, then the negation of the sentence will carry that same presupposition. Yet numerous examples have been cited in the first part of this thesis for which this result seems inappropriate. One of the goals of this chapter has been to illustrate that there is no reason why insights from elsewhere in the presupposition literature should not be drawn into the kinematic model so as to resolve this difficulty. It is clear that, there is no shortage of solutions to the problem of presupposition denial, but rather an embarrassment of riches.

Firstly there is the possibility of adding an extra presupposition-canceling negation. Then there are three approaches which are more general in their scope, applying not only to cases of canceled presuppositions embedded under negations, but also canceled presuppositions under other operators as well. These three are: introducing assertion operators (plus some pragmatic mechanism for determining when to use them), making presuppositions defeasible, and adding mechanisms of local and intermediate accommodation. The connection between the last two options has already been discussed in §6.8.1, and it is clear that the assertion operator approach is not that distant. All three have the effect that the constraint associated with a presupposition is not absolute, whether what removes that constraint is co-assertion (in the terminology of [Li86]), cancellation, or local accommodation.

However, it is not clear to me that any of these yet provide the proper way to deal with cases of presupposition denial. The following data is drawn from [Bea92], but Horn [Horn89, Horn85] and Burton-Roberts [Bu89a] have both provided voluminous quantities of similar data:

E231 John doesn't know that Bill is happy, since Bill is not happy.
E232 John doesn't know that Bill is happy, he merely believes it.
E233 John doesn't think that Bill is happy, he is totally convinced.
E234 John doesn't believe that Bill is happy, he knows it.

The various approaches outlined above provide a treatment of the first two of these examples, but not the last two. Indeed, the last two examples present a major problem for such approaches since if the negation is taken to map true onto false, then they are semantically inconsistent. I believe that an account which offers a treatment of the
It's rare when one reads a page that's not part of a conversation or an explanation of a concept. The idea that there are canceling mechanisms of either an implicit or an explicit nature that aren't explained is taken as a premise for a prescriptive propositional analysis to respond to examples, but if it's not clear, then the problem of non-assertive speech acts could also be addressed in the chapter to the CoC model with any available mechanisms such as discussed in the chapter. Significant information often means that the whole thing builds to a point in either of the case for a conversational analysis of the understanding of the basic unit of conversation. The understanding of the whole thing builds to a point in either of the case for a conversational analysis of the basic unit of conversation. However, if the reader wants to understand the point in either of the case for a conversational analysis of the understanding of the basic unit of conversation, they would have a conceptual explanation. Of all these examples, one that might explain why this feeling comes from one of E331, link of them, has not really go to the heart of the problem. My full feeling is that the first two examples, but not the last two (E-2) the prescriptive propositional theories of coercion,
matter, there are clear counterexamples to every existing theory of presupposition which is formally detailed enough to make definite predictions, and I am unable to see why Wilson felt that cancellation type accounts would differ; perhaps hindsight has obscured the point she was trying to make. In his presentation Gazdar chose to excise the final part of the above passage, that concerning the "coming from someone else" suggestion arising in cases of cancellation, but it is this part to which I wish to draw attention.\(^{15}\)

It seems to me that Wilson is absolutely right that an adequate theory of cancellation is not merely one which allows presuppositions to be canceled, but one which indicates in what circumstances cancellation can occur. If a theory does not distinguish between assertive and non-assertive utterances, but certain types of cancellation are typically or entirely restricted to just one of those, then the theory is inadequate\(^{16}\). In this sense all of the extensions to the dynamic semantic model which I have proposed in this chapter would fail, in and of themselves, to produce an adequate theory. In principle any of the three extensions, introducing assertion operators, making local presuppositional constraints defeasible, and adding local accommodation, could produce a model in which denied presuppositions did not project, but this would not make sense of what it is to deny, would not explain or account for the distribution of such denials in discourse, and further would not account for the observation that what can be denied is not limited to presuppositions and assertions.

I think it a promising start that from a technical point of view there are methods of extending the coverage of the dynamic semantic model such that cases of presupposition denial are treated at least as well as in presuppositional theories which take the possibility of such denial as their central datum (e.g. the cancellation theories). But it is clear that the revisions called upon in order to adequately treat the types of discourse in which denial occurs are of such a fundamental nature, including a move from monologue to dialogue and a rejection of the simplifying assumption that discourse contexts are incremented monotonically, that it would be premature to champion any one of the extensions I have proposed, for it remains unclear which extension will fit in with the final picture most felicitously.

\(^{15}\)Prince [Pri78] also discusses the way in which presuppositions can appear to be suspended when they are attributed to someone other than the speaker.

\(^{16}\)Whilst the cancellation theories I described in part I do not account for dialogue and do not account for variation in speech act type, it should be mentioned that Blok [Blo93] has proposed extensions in both these directions, and his model of presupposition denial might well answer Wilson’s objection.
In [Gaz79a], Gazdar concludes of Karttunen's "plugs, holes and filters" account of presupposition, the theory which formed the basis of the CCP model:

The...theory...has had a long and distinguished career. First formulated in 1971, published in 1973, modified and reconceptualized in 1974, and formalised and reterminologized in 1975. But the theory as of 1978 is in poor shape, enmeshed in its own epicycles, beset by counterexamples and constantly in need of "conversational implicatures" to unclog the filters and explain the leakage from its plugs. The time for euthanasia has arrived.

Fifteen years later, the CCP model is alive and kicking, and although it has developed considerably from Karttunen's original proposals, it seems to me that many of the central ideas of the current account are already to be found in Karttunen's work. In this light, one could say that as of 1979 Karttunen's account of presupposition was not a theory five years past its prime, but one fifteen years ahead of its time. Much the same could be said of Karttunen's even earlier account of discourse referents [Kar76]. For it is only in the last few years that the technical methods have been developed with which to give adequate expression to the essentially dynamic model that Karttunen was advocating.

The combination of Heim's work with the further developments in this thesis have provided solutions to many of the problems with Karttunen's account. Yet the resulting theory is by no means just an ad hoc collection of repairs. On the contrary, as I have gone to great lengths to demonstrate, the same analysis of internal sentence dynamics which is appropriate to the requirements of quantification, anaphora and epistemic modality, is also at the heart of the CCP theory. Some of the developments in this thesis will now be summarised, and, where relevant, these developments will be related to problems arising elsewhere in the presupposition literature as discussed in part one of this thesis.
12.1 Quantification, Anaphora and Modality

In Chapter 8, a theory of quantification was been presented which accounts for both internal and external dynamic properties of generalised quantifiers, together with a theory of anaphora that exploits the properties of the quantifiers to yield a treatment of singular and plural donkey and discourse anaphora.

A theory of epistemic modality was been described in Chapter 9 which accounts for some modal identity problems and for the logic of epistemic modality within quantified contexts in dynamic logics.

12.2 Compositionality

The KMG system described in the first half of Chapter 10 provides the first integration of dynamic accounts of quantification, anaphora, modality and presupposition into a compositional grammar fragment. Note that in the past the CCP type presuppositional meanings have only been stipulated for sentential expressions [Kar74], or for sentential expressions with free variables [Hei83a], so that KMG is the first system to demonstrate how such meanings may be derived from lexical meaning assignments in a Montagovian fashion.

12.3 Presupposition and Quantification

In Chapters 8 and 9 a solution was given for the projection problem for presuppositions arising within compound and quantified sentences, and it was argued that the results with respect to quantified sentences improve on those found in the existing literature.

To begin with, as discussed in Chapter 4, none of the cancellation accounts (e.g. [Gaz79a, Me92, vdS88]) have offered any account of the interaction of presupposition and quantification, nor is it obvious how this might be done (but c.f. one possible direction for research mentioned in §11.2.1). Thus no comparison with these accounts is possible.

With regard to theories which do account for this interaction, relevant examples are the following:

E235 Somebody curtsied.

E112 A fat man was pushing his bicycle.

E200a No woman regrets that she is married

E201a Every woman who regrets that she is married is sane
E134? I don’t know whether the Pope has measles. But every protestant who realises that the Pope has measles is converting.

E139 How many team members and cheerleaders will drive to the match?

* Few of the 15 team members and none of the 5 cheerleaders can drive, but every team member will come to the match in her car. So expect about 4 cars.

In §3.2, it was shown that for examples like E235 and E112 Karttunen and Peters’ system [KP79] makes incorrect predictions because it does not allow for binding between presupposition and assertion. On the other hand, as discussed in §5.3, Heim’s system [Hei83a] does allow for such binding, but mistakenly predicts a universal presupposition. Although she suggested avoiding this problem using a mechanism of accommodation, it has been shown [So89] that such a solution would be problematic. The developments in Chapter 9 yielded a system where binding is not only possible, but produces intuitively reasonable results: these examples produce existential rather than universal presuppositions.

As discussed further in Chapter 9, the Heimian system without accommodation also produces incorrect results for sentences like E201a and E200a. Once again the presuppositions she predicts are far too strong. In Chapter 9 it was shown how a modification to the Heimian semantics produces a system with intuitively reasonable existential presuppositions for these examples, and this modified system provided the basis of the KMG fragment.

Van der Sandt’s DRT based account [vdS92] was first discussed in Chapter 5 and more fully in Chapter 6. Unlike his earlier cancellation account, it does allow for a treatment of the interaction between presupposition and quantification, and indeed it behaves quite respectably. The presuppositional inferences predicted for all of E235, E112, E201a and E200a are uniformly weak. However, it was shown in §6.3 that the DRT accommodation model fails to predict the oddity of E134 due to the presence of an underconstrained mechanism of local accommodation. On the other hand this type of example should be no problem for KMG, which lacks such a mechanism. In the case of E139, it was shown in §6.5 that van der Sandt’s model incorrectly predicts domain restriction to the set of car-owning team members, and does not predict any oddity for the example. KMG does not allow this type of accommodation, and will successfully predict incoherence of E139, this incoherence arising from a truth conditional inconsistency.

1See [Bea94a] for a more detailed discussion of data concerning presupposition and quantification.
12.4 Accommodation and Common Sense

It was shown in Chapter 10 how global accommodation can be formalised within an account which is both dynamic and pragmatic, by using preference orderings over information states, and has been demonstrated that this resolves both the general problem of how presuppositions can be informative, and one particular problem with the CCP model concerning the prediction of overly weak conditional presuppositions from compound sentences. It was argued in §10.2 that all purely semantic theories of presupposition, and all theories of presupposition involving structural accounts of accommodation or cancellation make systematically incorrect predictions by failing to account for the importance of common sense in determining exactly what is accommodated (c.f. also §3.3.3, §4.8, §6.4 and §6.6). Here is some relevant data:

E236 We regret that children cannot accompany their parents to commencement exercises.

E52 If I go to London, my sister will pick me up at the airport.

E224 If Jane takes a bath, Bill will be annoyed that there is no more hot water.

E225 If Jane wants a bath, Bill will be annoyed that there is no more hot water.

E237 If Jane takes a bath and Bill is annoyed that there is no more hot water, he won’t let her have a towel.

E99 If Nixon invites Angela Davis to the Whitehouse, then Nixon will regret having invited a black militant to his residence.

E151 If Spaceman Spiff lands on Planet X, he will be bothered by the fact that his weight is higher than it would be on Earth.

Karttunen’s example, [Kar74], E236 demonstrates informativity of presuppositions. The model he proposed provided no direct treatment of these cases, which he explained as being deviant, arising from imperfections in the functioning of natural discourse. Heim [Hei83a] did provide a line of explanation for these cases, by adding a mechanism of global accommodation to the CCP model. In this thesis it has been seen how this explanation may be further formalised.

Example E52 is the type of case where a conditional presupposition is generated in the dynamic models of [Kar74] and [Hei83a], in multivalent accounts based on (e.g.) Kleene’s strong logic and a conventional Strawsonian definition of presupposition, and also in the two dimensional model of Karttunen and Peters [KP79] and the hybrid account of Soames [So82]. This type of claimed presupposition, here that if I go to London then I have a sister, has been widely attacked. It is clear that an utterance E52 would
normally only occur in situations where the speaker actually has a sister, and that a
hearer would be able to infer this even if unfamiliar with the speaker's family back-
ground. Karttunen and Peters' provided a pragmatic explanation for the strengthening
of conditional presuppositions, but this justification has come under attack — see es-
pecially [Geu95] — and is anyway informal. I have offered a formal model of global
accommodation whereby the form of an utterance places minimal semantic constraints
on what must be accommodated, but does not determine what is accommodated: world
knowledge and plausibility criteria determine this. It has been shown that this can yield
the appropriate strengthening. Furthermore, the method is of sufficient generality that
it can be applied not only to the dynamic semantic model presented in this thesis, but
also, as shown in Chapter 11, to other models, such as those based on a multivalent
semantics.

The next two examples, E224 and E225, demonstrate why the interaction with world
knowledge is so important. Hearsers may come to quite different conclusions when en-
countering the two sentences. If knowing that there are many households where having
a bath tends to exhaust available hot water supplies, a hearer may conclude from E224
that the speaker is referring to such a household, and that if Jane takes a bath the hot
water reservoir will indeed be emptied. But a hearer faced with the structurally similar
example E225 will generally not be able to conclude that there is a general rule linking
the antecedent of the conditional ('Jane wants a bath') to the presupposition that there
is no hot water. The most plausible explanation of why the speaker is able to take for
granted that there is no hot water in the local context of the consequent of E225 is that
he also takes it to be globally true that there is no hot water, and takes this to be an un-
controversial assumption. So in the case of E224 a hearer will tend not to conclude that
there is a lack of hot water, but in the case of E225 the hearer will tend to conclude this.
The global accommodation mechanism which I have proposed provides a way in which
common sense knowledge of what is plausible, encoded as an information ordering, can
interact with the CCP model of presupposition so as to yield just this difference.

No model of presupposition which fails to detail the mechanism by which knowledge
of plausibility can affect what is accommodated will be able to predict the contrast.
Multivalent semantic theories (e.g. [vF75, Ha76, Seu85, Bu89a]) cannot predict the
difference without an extra pragmatic component, and up until now there has been no
serious attempt to formalise such a pragmatic component. Cancellation theories (e.g.
[Gaz79a, vdS88, Me92]) and existing theories of structural accommodation (e.g. [vdS92,
Ze92]) fail on two counts. Firstly, there is what might be called the inconsistency problem:
it is determined whether or not a presupposition is projected/globally accommodated
using a consistency test. But I have argued that the difference between E224 and
E225 concerns plausibility and not consistency. Both examples could consistently be
uttered when the speaker was assuming that there was an absence of hot water, so cancellation and structural accommodation theories will uniformly predict projection of this presupposition.

Secondly, the cancellation theories and the structural accommodation theories do not allow for the fact that what is accommodated is in general not a simple yes-no proposition, but an explanation, a theory which incorporates the conventionally signalled presupposition as just one element. Information orderings provide many such alternative explanations, encoded abstractly in terms of CCP states. In cases such as E224, and also E237–E151, I have argued that the CCP model is right in predicting that a local constraint for presupposition satisfaction yields a global constraint that a conditional holds, e.g. that if Spiff lands on our planet then his weight will be higher than on Earth in the case of E151. Further, I have shown how given that the form of the utterance places such a constraint, the information ordering mechanism allows an appropriate explanation containing that conditional to be accommodated. Conditional presuppositions are thus at the core of the explanations accommodated not only for E224, but also for the remaining examples E237–E151. But current cancellation and structural accommodation theories cannot in general generate conditional presuppositions. Thus, as was shown at length in §4.8 and §6.6, cancellation and structural accommodation theories not only predict overly strong presuppositions in cases like E224 and E237–E151, because of the inconsistency problem (i.e. that in these models inconsistency is what causes cancellation or non-accommodation, and not implausibility), they also lack any method for producing appropriate presuppositions if the inconsistency problem was overcome.

12.5 What Comes Last

Having shown that the greater part of the CCP theory can be independently motivated in terms of extraneous semantic phenomena, what remains that is specific to a theory of presupposition is just that it really is a theory of presupposition. If you convert the intuitive idea that presuppositions come before other aspects of meaning into the technical claim that presuppositions are constraints on input contexts in dynamic semantics, what results, broadly, is the CCP model. Thus presupposition is what comes first in dynamic semantics.
Appendix A

Properties of ABLE

The main facts demonstrated in this appendix are:

1. All ABLE formulae are eliminative. Without this property the definitions of satisfaction and entailment would not be appropriate.
2. The non-modal presuppositions of an ABLE formula are also entailments.

Definition D89 (Extension) A state $I$ is an extension of a state $J$ iff:

$$\forall w \forall f \ I.w.f \rightarrow (\exists g \ g \supset g \land J.w.g)$$

Definition D90 (Eliminativity) A CCP $F$ is eliminative iff for any states $I$ and $J$, if $I\{F\}J$ then $J$ is an extension of $I$.

Fact F20 All ABLE formulae are eliminative.

Proof: An induction over ABLE formulae. The most difficult cases are the quantificalional determiners, which I will leave until last. Firstly, the postulate on the denotation of ABLE predicates, MP8, forces the output to be a subset of the input, from which eliminativity follows. Conjunction is defined as relational composition, and it is clear that if two CCPs are eliminative then their composition will be. Negation is defined such that the output is the input minus some other set, which means that once again the output must be a subset of the input. The output of a definite determiner, if defined, is the input updated with the scope condition, so it suffices that, by the induction hypothesis, the scope condition will be eliminative. Since tests are eliminative, the output either being the input or the absurd state, facts D61 and F11 guarantee that the epistemic modalities are eliminative.
The definition of the semantics of existential determiners, D65, is of the following form:

\[
[D.i.\phi.\psi] = \lambda I \lambda J \\
\ldots \land \\
J = \lambda w \lambda g \\
[\exists f \exists c o \exists G_{sco} \ldots I.w.f \land \ldots \land G_{sco} = \lambda h [h \supset f \land sco.w.h] \land \\
\ldots \land G_{sco}.g]
\]

Thus we have that for existential determiners \( D \), if \( I[D.i.\phi.\psi]J \), then:

\[
\forall w \forall g [J.w.g \rightarrow \exists f \exists c o \exists G_{sco} I.w.f \land \\
G_{sco} = \lambda h [h \supset f \land sco.w.h] \land \\
G_{sco}.g]
\]

From this it follows that:

\[
\forall w \forall g [J.w.g \rightarrow \exists f I.w.f \land g \supset f]
\]

This is the required result.

Similarly, it follows from definition D64 that if \( D \) is a non-existential determiner and \( I[D.i.\phi.\psi]J \), then:

\[
\forall w \forall g [J.w.g \rightarrow \exists f \exists c o \exists G_{sco} I.w.f \land \\
G_{sco} = \lambda h [h \supset f \land sco.w.h] \land \\
g = \lambda D \lambda x [f.D.x \lor \\
\exists h G_{sco}.h \land h.D.x]
\]

Once again, it is straightforward to verify that \( g \) must be an extension of \( f \), and the result follows. This completes the proof.

Definition D91 (Singleton)

\[
\langle (w, f) \rangle = \lambda w' \lambda g [w' = w \land f = g]
\]
Definition D92 (Distributivity) A CCP $F$ is distributive iff

$$\forall I \forall J \{ F \} J \leftrightarrow J = \lambda w \lambda f [$$

$$\exists g I.w.g \wedge \langle \langle w, g \rangle \} \{ F \} \{ \langle w, f \rangle \} ]$$

Fact F21 The non-modal formulae of ABLE are distributive.
Sketch of Proof: Another induction on (non-modal) formula complexity, starting with the fact that MP8 guarantees that ABLE predications are interpreted distributively.

Definition D93 (Persistence) A CCP $F$ is persistent iff

$$(I \text{satisfies } F \land I; J \text{is an extension of } I) \rightarrow J \text{satisfies } F$$

Fact F22 Non-modal ABLE formulae are persistent.
Proof: ABLE predicates have the property of relevance, so that their denotation is only sensitive to the values the input state gives to the predicated markers. Thus if an ABLE predication is satisfied in a state, then adding new discourse markers to the state will produce another state in which the predication is satisfied. Since predications are distributive, it holds that if a predication $F$ is satisfied in some state $I$, then it must be satisfied in every subset of $I$. It follows that ABLE predications are persistent.

Inspection of the remaining semantic clauses show that they all preserve relevance, so that if any ABLE formula is satisfied in a state then adding new discourse markers will yield another state which satisfies the formula. Since all non-modal formulae are distributive, it follows that all non-modal formulae are persistent.

Fact F23 If $\phi$ is a non-modal formula and $\psi$ presupposes $\phi$, then $\psi \models \phi$.
Proof: Since $\phi$ is non-modal, it must be persistent, so that if it is satisfied in a state then it must be satisfied in any extension of the state. Since $\psi$ presupposes $\phi$, it follows that every state that admits $\psi$ satisfies $\phi$. Since all ABLE formulae are eliminative, it follows that an output of $\psi$ must be an extension of the input. Thus if an input of $\psi$ satisfies $\phi$, then the output must also satisfy $\phi$. This yields the required result, since every output of $\psi$ satisfying $\phi$ is the condition for $\psi \models \phi$. 
Appendix B

ABLE in Ty₃

The following postulate defines Ty₃ constants for all the ABLE operators, as well as the pronominal abbreviations:

**Meaning Postulate MP33**

1. \( \text{IS} = \lambda \lambda' D \lambda' I \lambda J \left[ \text{t-domain.}I.D \land \text{t-domain.}I.D' \land J = \lambda w \lambda f \left[ I.w.f \land f.D = f'.D' \right] \right] \)

2. \( \text{AND} = \lambda \lambda' F \lambda' I \lambda J \left[ \exists K I\{F\} K \{F'\} J \right] \)

3. \( \text{NOT} = \lambda \lambda' I \lambda J \left[ \exists K I\{F\} K \land J = \lambda w \lambda f \left[ I.w.f \land \neg (\exists g \geq f \land K.w.g) \right] \right] \)

4. \( \text{IMPLIES} = \lambda \lambda' F \lambda' I \lambda J \left[ \exists I' \exists I'' I\{F\} I'\{F'\} I'' \land J = \lambda w \lambda f \left[ I.w.f \land \forall g (g \geq f \land I'.w.g) \rightarrow (\exists h \geq g \land I''.w.h) \right] \right] \)

5. \( \text{OR} = \lambda \lambda' F \lambda' I \lambda J \left[ \exists K \exists K' I\{F\} K \land I\{F'\} K' \land J = \lambda w \lambda f \left[ I.w.f \land \exists g g \geq f \land (K.w.g \lor K'.w.g) \right] \right] \)

6. \( \text{THE} = \lambda \lambda' F \lambda' I \lambda J \left[ \text{t-domain.}I.D \land I\{\text{satisfies}\} F \land \exists K I\{F\} K\{F\} J \right] \)

7. \( \text{SHE} = \lambda \lambda' F \left[ \text{THE.}(\text{SINGULAR}^{u}.D \text{ AND FEMALE}^{u}.D).F \right] \)

8. \( \text{HE} = \lambda \lambda' F \left[ \text{THE.}(\text{SINGULAR}^{u}.D \text{ AND MALE}^{u}.D).F \right] \)
\[
(9) \quad \text{IT} = \lambda D \lambda F [\text{THE}.D.(\text{SINGULAR}^u.D \ \text{AND} \ \text{NEUTER}^u.D).F]
\]

\[
(10) \quad \text{THEY} = \lambda D \lambda F [\text{THE}.D.(\text{PLURAL}^u.D).F]
\]

\[
(11) \quad \text{MIGHT} = \lambda V(\delta,\tau) \lambda F \lambda I \lambda J [\exists K \ I(F)K \ \land \\
J = \lambda w \lambda f [I.w.f \ \land \ \exists w' \exists g \ K.w'.g \ \land \ g \equiv V \ f]]
\]
(12) If $D$ is one of EVERY, MOST, FEW or NO then:

$$D = \lambda D \lambda F \lambda F' \lambda I \lambda J$$

If $V$ is one of EVERY, MOST, FEW or NO then:

$$J = \lambda w \lambda g$$

$$\exists f \exists G_{res} \exists G_{sco} \exists X_{res} \exists X_{sco}$$

$$\begin{array}{l}
I \{+D\} \text{init} \{F\} \text{res} \{F'\} \text{sco} \wedge \\
G_{res} = \lambda h [h \geq f \wedge \text{res.w.h}] \wedge \\
G_{sco} = \lambda h [h \geq f \wedge \text{sco.w.h}] \wedge \\
X_{res} = \lambda x [\exists h G_{res}.h \wedge h.d_{i}.x] \wedge \\
X_{sco} = \lambda x [\exists h G_{sco}.h \wedge h.d_{i}.x] \wedge \\
D'.X_{res} \cdot X_{sco} \wedge \\
g = \lambda D \lambda x \left[ f.D.x \vee \exists h G_{sco}.h \wedge h.D.x \right]
\end{array}$$

(13) If $D$ is SOME or EXACTLY-ONE, then:

$$D = \lambda D \lambda F \lambda F' \lambda I \lambda J$$

If $V$ is SOME or EXACTLY-ONE, then:

$$J = \lambda w \lambda g$$

$$\exists f \exists G_{res} \exists G_{sco} \exists X_{res} \exists X_{sco}$$

$$\begin{array}{l}
I \{+D\} \text{init} \{F\} \text{res} \{F'\} \text{sco} \wedge \\
G_{res} = \lambda h [h \geq f \wedge \text{res.w.h}] \wedge \\
G_{sco} = \lambda h [h \geq f \wedge \text{sco.w.h}] \wedge \\
X_{res} = \lambda x [\exists h G_{res}.h \wedge h.d_{i}.x] \wedge \\
X_{sco} = \lambda x [\exists h G_{sco}.h \wedge h.d_{i}.x] \wedge \\
D'.X_{res} \cdot X_{sco} \wedge \\
G_{sco}.g
\end{array}$$
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